6 Essais sur les enchères: Approches théorique et empirique. Application aux marches de l'électricité.

Laurent Lamy

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THESE

Pour obtenir le grade de
Docteur de l'Ecole Nationale des Ponts et Chaussées : Sciences Economiques

Présentée et soutenue publiquement par

Laurent Lamy

le 28 juin 2007

Titre :

6 ESSAIS SUR LES ENCHÈRES: APPROCHES THÉORIQUE ET EMPIRIQUE. APPLICATION AUX MARCHÉS DE L'ÉLECTRICITÉ.

Directeur de thèse :M. Philippe Jehiel

JURY :

M. Bernard Salanié,	(rapporteur)
	Administrateur de l'INSEE, Professeur Columbia University
M. Jean Tirole,	(rapporteur)
	Ingénieur Général des Ponts et Chaussées, Directeur d'Etudes EHESS
M. Philippe Jehiel,	(directeur de thèse)
	Ingénieur en Chef des Ponts et Chaussées, Professeur UCL London

Résumé

Cette thèse est consacrée à l'étude théorique et empirique de mécanismes d'enchères. Motivée par les problèmes d'allocation dans les industries de réseau, en particulier par la libéralisation du secteur de l'électricité, elle s'intéresse aux enchères avec externalités (allocative et informationelle) ainsi qu'aux enchères multi-objets.

Après une introduction qui parcoure l'utilisation et les analyses de mécanismes d'enchères dans les marchés de l'électricité, cette thèse est constituée de six chapitres. Le premier d'entre eux analyse l'impact pour le vendeur de ne pas pouvoir s'engager à ne pas participer à l'enchère de manière anonyme comme n'importe quel enchérisseur, ceci en présence d'externalités informationelles. Le deuxième et le troisième chapitre s'intéressent au mécanisme d'enchères combinatoires proposé par Ausubel et Milgrom, un mécanisme qui choisit une allocation dans le coeur. Le premier de ces deux chapitres propose une légère modification du mécanisme originel d'Ausubel-Milgrom et précise le statut théorique de ces formats d'enchères, en particulier les conditions telles que reporter ses préférences soit une stratégie dominante. Motivé par des critères de robustesse pour les généralisations de l'enchère d'Ausubel-Milgrom et de l'enchère de Vickrey à des environnement avec des externalités allocatives entre co-acquéreurs, le second de ces chapitres analyse la condition de substituabilité entre les enchérisseurs dans un environnement avec des externalités allocatives entre les coacquéreurs. Dans un environnement en information complète, le quatrième chapitre analyse le problème général de mécanisme optimal lorsque les pouvoirs d'engagement du principal sont réduits, à savoir s'il ne peut pas s'engager sur des mécanismes simultanés de participation. Le cinquième chapitre s'intéresse à l'analyse structurelle d'un modèle asymétrique à partir de données d'enchère où les enchérisseurs ont été anonymisés. Il est montré que le modèle IPV (Independent Private Value) asymétrique est identifié. Une méthode d'estimation optimale asymptotiquement et reposant sur plusieurs étapes utilisant des estimateurs à noyaux est analysée. Le dernier chapitre utilise les données sur les enchères aux interconnexions électriques frontalières entre la France et le Royaume-Uni pour une analyse sous forme réduite d'une enchère ascendante multi-unité.

Discipline : Sciences Economiques (05).
Mots-clés : Enchères, Externalités, Enchères multi-unités, Enchères combinatoires, Econométrie structurelle des enchères, Marchés de l'électricité, Enchères explicites pour les transmissions
Laboratoires : CREST-LEI, 28 rue des Saints-Pères, 75007 Paris,
ENPC-PSE, 48 Bd Jourdan, 75014 Paris

6 Essays about auctions: a theoretical and empirical analysis. Application to power markets.

This thesis is devoted to a theoretical and empirical analysis of auction mechanisms. Motivated by allocation issues in network industries, in particular by the liberalization of the electricity sector, it focus on auctions with externalities (either allocative or informational) and on multi-objects auctions.

After an introduction which provides a survey of the use and the analysis of auctions in power markets, six chapters constitues this thesis. The first one considers standard auctions in Milgrom-Weber's model with interdependent valuations when the seller can not commit not to participate in the auction. The second and third chapters study the combinatorial auction mechanism proposed by Ausubel and Milgrom. The first of these two studies proposes a modification of this format with a final discount stage and clarifies the theoretical status of those formats, in particular the conditions such that truthful reporting is a dominant strategy. Motivated by the robustness issues of the generalizations of the Ausubel-Milgrom and the Vickrey combinatorial auctions to environments with allocative externalities between joint-purchasers, the second one characterizes the buyer-submodularity condition in a general model with allocative identity-dependent externalities between purchasers. In a complete information setup, the fourth chapter analyses the optimal design problem when the commitment abilities of the principal are reduced, namely she can not commit to a simultaneous participation game. The fifth chapter is devoted to the structural analysis of the private value auction model for a single-unit when the econometrician can not observe bidders' identities. The asymmetric independent private value (IPV) model is identified. A multi-step kernel-based estimator is proposed and shown to be asymptotically optimal. Using auctions data for the anglo-french electric interconnector, the last chapter analyses a multi-unit ascending auctions through reduced forms.

Keywords : Auctions, Externalities, Multi-unit auctions, Combinatorial Auctions, Structural Econometrics of Auction Data, Power Markets, Explicit auctions for electricity transmission
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Chapitre Introductif

1 Introduction générale

Un des points de départ de cette thèse sont les questions relatives à l'ouverture à la concurrence des industries de réseaux. Sous l'impulsion des directives européennes, la France a dû tour à tour ouvrir à la concurrence les marchés des télécommunications, de l'énergie et enfin celui du transport ferroviaire. L'application de ces directives varie beaucoup d'un pays à l'autre. La mise en vente des licences U.M.T.S. de téléphones portables de 3ème génération a eu lieu par le biais d'une enchère multi-unité dans de nombreux pays européens comme l'Angleterre et l'Allemagne, suivant ainsi l'exemple des Etats-Unis qui met aux enchères depuis une dizaine d'années l'ensemble de son spectre radio-électrique, en ayant fait appel à des consultants appartenant au monde universitaire pour mettre en place les détails des mécanismes d'enchères. En revanche, en France, c'est un processus administratif de qualification -qualifié de 'concours de beauté' dans le jargon économique- qui a été retenu.

La libéralisation des industries de réseaux s'articule autour de trois types de marchés. En amont, les marchés de gros et l'allocation des infrastructures essentielles sont le lieu des interactions entre les producteurs, les distributeurs et les détenteurs du réseaux d'infrastructure. En aval, les marchés de détails sont le lieu des interactions entre les distributeurs et les consommateurs. Enfin, les problèmes de sécurité, les missions de service public et les préoccupations environnementales nécessitent la mise en oeuvre de marchés auxiliaires. L'objectif de ces marchés est l'allocation efficace des resources entre ces acteurs. Historiquement, les industries de réseaux dans la seconde moitié du vingtième siècle étaient organisées autour d'une entreprise publique monopolistique régulée par la puissance publique. Ainsi les problèmes d'allocation étaient résolues par des procédures d'optimisation effectuées en interne par le monopole et par un contrôle de certaines variables clefs par le régulateur. Même pour le traffic aérien, qui était structurellement ouvert à la concurrence depuis ces origines, ce ne sont pas des procédures de marché, guidées par des prix, qui étaient à l'origine de l'allocation des créneaux aéroportuaires mais plutôt des processus de négociation entre les compagnies aériennes dans un environnement où le 'droit du grand-père' prévalait, c'est à dire où les créneaux sont reconduits d'une année sur l'autre aux compagnies. Cette règle était légèrement amendée dans la mesure où la règle 'use it or lose it' imposait une utilisation minimale d'un créneau (80% en Europe) pour disposer du droit de le conserver l'année suivante. Cette logique a pu conduire certaines compagnies à faire du 'baby-sitting', c'est à dire à utiliser un slot avec des appareils chargé à vide pour atteindre cette limite des 80%. Notons que l'allocation des capacités électriques de transmission trans-frontalière suivait jusqu'à très récemment ce type de logique non monétaire. La logique de l'ouverture à la concurrence consiste à donner des incitations monétaires afin d'allouer les resources de manière efficace, logique dont les systèmes d'enchères en sont la version la plus épurée.

L'objet de cette introduction est, dans un premier temps, de motiver nos investigations théoriques et empiriques ultérieures par une industrie particulière qui est très riche entre terme d'applications: le marché de l'électricité. De part les spécificités de ce bien que sont l'inélasticité de court terme, la quasi-impossibilité de stockage et une technologie de production susceptible de butter sur des contraintes de capacité dans un environnement où la demande subit de fortes variations saisonnières, la théorie de l'organisation industrielle nous enseigne que les problèmes de concurrence sont susceptibles d'y être exacerbé.[1] Ainsi, les outils standards d'analyse et de régulation des marchés ne sont pas toujours appropriés dans les marchés de l'électricité. Par exemple, Borenstein et al [12] déconseillent l'utilisation de l'indice HHI pour mesurer la concentration du marché, comme cela est traditionnellement fait par les autorités de la concurrence pour mesurer la compétitivité d'un secteur.[2] Nous présentons d'abord brièvement les coûts et les bénéfices économiques à attendre de la libéralisation du secteur. Ensuite, nous donnons quelques éléments historiques sur la libéralisation de cette industrie. Enfin, nous passons en revue quelques mécanismes de marché mis en place pour dynamiser le marché de gros.

Dans un second temps, nous nous consacrons à un résumé de chacun des chapitres de cette thèse. Une partie importante de ces travaux sont à caractère théorique. Nous insistons sur les liens ténus entre le monde économique réel et les analyses académiques de la théorie des enchères. Dans la mesure où les règles des mécanismes d'enchères sont transparentes, contrairement à des mécanismes de fixation des prix par négociation, les modèles reposent sur des hypothèses raisonnables proche du contexte réel. Les activités de conseils de théoriciens des enchères témoignent de ce succès, en particulier dans la mise en oeuvre de mécanismes d'enchères dans le marché de l'électricité.[3] C'est pour ces même raisons que la théorie des enchères est aux avant-postes de l'économétrie structurelle des modèles issus de la théorie des jeux et que de plus en plus, les échanges entre la théorie et les analyses empiriques sont fructueux. L'analyse structurelle des enchères a débuté il y a une dizaine d'années et reste encore très active comme en témoigne le livre de Paarsch et Hong [76].

[1] Se référer à l'ouvrage de Tirole [87]

[2] Notons que la Commission de Régulation de l'Energie (CRE) reporte cet indice dans son rapport trimestriel, 'l'observatoire des marchés'.

[3] La société Market Design est un bon exemple. Voir http://www.market-design.com/.

2 L'ouverture à la concurrence des Industries de Réseaux: un exemple, le marché de l'électricité

Cette section part des perspectives les plus générales sur l'architecture de l'ensemble du secteur pour aller jusqu'à des questions plus précises que peuvent se poser un régulateur pour la mise en oeuvre des marchés de gros.

2.1 Les bénéfices à attendre de la libéralisation

Le processus de libéralisation du secteur électrique, dont un des points de référence est l'expérience britannique avec le British Electricity Act (1989), ne répond pas vraiment à une logique interne au secteur électrique mais suit plutôt le mouvement général que connaît l'ensemble des industries de réseaux à travers le monde. L'argument classique qui justifiait la situation de monopole était les économies d'échelle: les coûts fixes pour la construction du réseau et des centrales nécessitant une activité importante de recherche et développement qu'il serait inefficace de dupliquer. En réalité, ces arguments ne s'appliquaient qu'à des activités spécifiques de ces monopoles historiques et sont devenus caducs dans certains cas pour des technologies qui sont devenues mûres.

L'activité de transmission bénéficie ainsi sans ambiguïté de tels rendements d'échelle croissants. D'une part, les coûts correspondent presque exclusivement aux coûts fixes d'installation et de maintenance des lignes qu'il serait aberrant de dupliquer. D'autre part, les aléas auxquels font face le système électrique signifient qu'il y a un gain à mutualiser l'ensemble des offres et des demandes sur le réseau pour réaliser l'équilibre en temps réel. Pour ces raisons, les activités de transmission sont restées sous un régime monopolistique. Sous l'égide du Forum de Florence, des voix s'élèvent pour une gestion de la transmission à l'échelle continentale en vue de créer une grande plaque de cuivre européenne.

Pour la production d'électricité, il semble en revanche qu'une fois dépassée l'échelle de la centrale, il n'y a pas de gain d'échelle en terme d'efficacité. Conjointement, la taille des centrales a aussi été réduite avec l'apparition de nouvelles technologies comme les centrales à gaz à cycles combinés laissant la possibilité à des acteurs de petites tailles d'entrer sur ce marché. Cet absence de rendement d'échelle peut sembler en contradiction avec les vagues de fusions et acquisitions qu'a connu le secteur en Europe. Cependant il ne faut pas confondre les rendements d'échelle en terme technologique avec ceux résultant de pouvoirs de marché accrus: un meilleur pouvoir de négociation pour l'achat des combustibles dans le meilleur des cas et une hausse des prix de gros.

L'activité de distribution consiste à proposer des menus de contrats aux différents types de consommateurs, à assurer la maintenance des équipements de distribution (les boucles locales) et à contrôler la consommation. Elle semble donc relativement peu sensible aux gains d'échelle, excepté éventuellement pour l'activité de relevé des compteurs des consommateurs résidentiels.

Ainsi ce sont effectivement la production et la distribution, avec dans un premier temps les consommateurs industriels, qui ont été ouvert à la concurrence.

Derrière ce mouvement de libéralisation se trouve la conviction qu'un monopole historique verticalement intégré de la production à la distribution et régulé de près par la puissance publique est moins efficace qu'un système partiellement concurrentiel avec une régulation des marchés. Un système de marché se voit attribuer des vertus en terme de productivité de court terme et de politique d'investissement. De manière très générale, ces deux vertus correspondent à deux types d'objectif en terme d'efficacité productive:

- L'efficacité de court terme: la minimisation des coûts de production avec le parc existant et l'égalisation des coûts marginaux de production avec l'utilité du consommateur marginal.
- L'efficacité de long terme: l'optimisation des investissements concernant le réseau, les centrales et les choix technologiques des consommateurs.

Tant du point de vue théorique qu'empirique, la grande majorité des travaux adopte la perspective de court terme en se restreignant en plus à l'efficacité dans une dimension particulière. Par rapport à l'ensemble de la problématique que sont les bénéfices de la libéralisation, il s'agit d'un champ très étroit. Avant de rentrer dans les détails de mécanismes mis en place, mentionnons brièvement les rares travaux qui s'intéressent directement à mesurer les améliorations en terme d'efficacité du système productif dans son ensemble d'abord du côté de l'offre (les producteurs) et ensuite du côté de la demande (les consommateurs).

2.1.1 Efficacité vis-à-vis de l'offre

En ce qui concerne l'efficacité de court terme, l'optimisation s'effectue à plusieurs niveaux. Au niveau de chaque unité de production, il s'agit de minimiser les coûts pour un niveau de production donné. Un régime libéral est supposé conduire à une meilleure maîtrise des coûts. Markiewicz, Rose et Wolfram [66], Bushnell et Wolfram [21] et Wolfram [94] ont étudié, à partir d'un panel de producteurs sur l'ensemble des Etats-Unis, l'évolution du rendement de chaque centrale après des sessions d'actif et des restructurations du mode de régulation qui ont touché les différents Etats de manière très hétérogène. Ces travaux analysent pour chaque unité de production l'évolution des inputs et des outputs: ils contrôlent les effets spécifiques à chaque unité, mais aussi les effets résultant de variations de la demande.

On peut retenir que les restructurations ont eu un impact significatif sur la productivité: elles conduisent à une baisse de 8% des coûts salariaux et de 14% des dépenses opérationnelles auxquelles on a soustrait le coût des combustibles. Bushnell et Wolfram [21] ont montré que le rendement des centrales -les ratio entre l'output et les combustibles- ont significativement augmenté de 2% dans les centrales après une session d'actif par rapport à la même centrale au sein d'une entreprise de service public. Ce dernier résultat est encore plus intéressant du point de vue du bien-être car il ne présente aucune ambiguïté: un meilleur rendement augmente le bien-être tandis que les évolutions des coûts salariaux peuvent représenter à court terme un transfert de rente entre le producteur et ses employés.

Au niveau supérieur du choix des centrales, la concurrence est susceptible d'exacerber certaines formes d'inefficacité: un gros producteur efficace peut préférer restreindre son offre pour que les prix s'envolent tandis que de nouveaux entrants moins efficaces produisent à un coût plus élevé. Borenstein, Bushnell et Wolak [15] ont estimé l'ampleur de l'inefficacité productive en Californie pendant les périodes estivales de 1998 à 2000. Cette inefficacité productive est passée de 44 millions de dollars en 1998 à 347 millions pendant l'été 2000 qui a marqué le début de la crise californienne. Ces chiffres sont à relativiser par rapport à l'explosion conjointe de la facture énergétique de 1.7 milliard à 9.0 milliards de dollars en raison du pouvoir de marché exercé par les producteurs sur les distributeurs comme nous le détaillerons par la suite.

En ce qui concerne l'efficacité des investissements, l'effet de Averch-Johnson [9] prédit que les monopoles régulés par la puissance publique surinvestissent dans les technologies avec des coûts fixes élevés et des coûts marginaux faibles comme le nucléaire. Le parc français est une très bonne illustration: un excédent considérable de capacité nucléaire qui amène EDF à exporter environ 15% de sa production, tandis que le parc est mal adapté pour faire face des fluctuations de demande de court terme qui exigent des centrales flexibles ce qui conduit à ce qu'environ 6% des injections soient des importations. A l'opposé, l'ouverture à la concurrence, avec en particulier de nouveaux entrants, conduit au développement de centrales de petites tailles avec des faibles coûts fixes. Newbery et Pollitt [75] estiment que l'ouverture à la concurrence a eu un impact positif sur la structure des portefeuilles de centrales. En se basant sur les plans d'investissement à la fin des années 1980 du monopole historique CEBG, les estimations contre-factuelles de Horton et Littlechild [64] concernant les gains en terme de politique d'investissement réévaluent à 13,3 milliard de livres les estimations de Newbery et Pollitt [75].

2.1.2 Efficacité vis-à-vis de la demande

L'ouverture à la concurrence de la distribution devrait conduire à l'émergence de tarifications innovantes. A ce titre, la France est une exception: sous l'égide de Marcel Boîteux, EDF a développé des options tarifaires originales. Actuellement, EDF propose aux particuliers des grilles tarifaires où le prix de l'électricité dépend de l'heure et de la journée où s'effectue la consommation. Avec l'Option Tempo, l'année est ainsi divisée en six périodes pendant lesquelles des prix s'échelonnant de 0.045 à 0.47 euro par kWh sont appliqués. En revanche, la norme dans le reste du monde est plutôt une tarification de la somme de la consommation annuelle avec un prix unique comme c'est le cas avec l'Option de Base d'EDF. Dans le cadre d'un modèle où les marchés de gros sont en concurrence parfaite, Borenstein et Holland [16] analysent de manière théorique l'impact de la proportion des consommateurs soumis à une tarification en temps réel. En théorie, certains effets sont indéterminés: par exemple il n'apparaît pas que le bien-être social croisse de manière monotone avec la proportion des consommateurs soumis à une tarification en temps réel, alors qu'en revanche, si tous les consommateurs sont soumis à cette tarification, le problème d'incomplétude des marchés est résolu et le premier théorème du bien-être s'applique. Toutefois, le modèle prédit très généralement qu'une hausse de la proportion des consommateurs avec une tarification en temps réel induit une baisse du prix des consommateurs qui restent avec un tarif uniforme et ainsi une hausse du bien-être de cette frange de consommateurs. En pratique, pour des calibrations raisonnables du modèle, la tarification en temps réel prédit une baisse de la capacité de pointe installée et une hausse du bien-être. Calibrant leur modèle sur la Californie, Borenstein et Holland [16] évaluent différents scénarii: les effets les plus notables sont sur l'introduction d'un premier tiers des consommateurs avec la tarification en temps réel avec une baisse de 44% des capacités de pointe et une hausse du bien-être s'élevant à 3-11% de la facture énergétique. La leçon à retenir de l'exercice est qu'avant de réfléchir au problème de la rémunération des capacités de pointe, qui concoure à la fiabilité du réseau, il faudrait se demander si les fluctuations vertigineuses de la demande conduisant à la construction de centrales qui ne sont utilisées que quelques jours par an ne sont pas un artefact de la tarification uniforme dans le temps et d'une inélasticité de la demande qui serait ainsi purement fictive. Notons que l'estimation de la fonction de demande des consommateurs face à des fluctuations en temps réel des prix et les gains d'efficacité potentiels d'un telle tarification est un champ de recherche actif. Wolak [92] utilise une expérience auprès d'une centaine de consommateurs résidentiels, tandis que Borenstein [11] utilise des données sur des consommateurs industriels.

Dans une perspective de long terme, la baisse des prix de détails de l'électricité escomptée par la libéralisation comme cela s'est produit en Angleterre modifie les technologies de production en faveur de technologies électro-intensives. Ainsi la courbe de demande d'électricité est le résultat de choix plus en amont sur la nature de l'input utilisé par les consommateurs. Par exemple, les particuliers font le choix irréversible du gaz, de l'électricité ou du fuel pour le chauffage.

2.2 Éléments historiques

2.2.1 Le Pool britannique (1990-2001): l'échec d'un marché centralisé

A la suite du British Electricity Act (1989), l'Angleterre a démantelé le monopole historique qu'était CEBG, où les activités de production et de transmission étaient intégrées, en trois producteurs indépendants et un monopole s'occupant de la transmission et de l'adéquation à chaque instant entre l'offre des producteurs et la demande des consommateurs par le biais d'un marché journalier centralisé auprès duquel tous les acteurs devaient impérativement s'adresser. Ce système est qualifié de Pool. Parallèlement à la mise en place d'un système concurrentiel pour la production, l'activité de distribution, qui était historiquement aux mains de 12 entreprises régionales en position de monopole dans les zones géographiques qui leur étaient attribuées, s'est libéralisée. De 1990 à 1998, les consommateurs finaux ont été progressivement déclaré éligible pour choisir librement leur fournisseur. Les prix de détails ont baissés de 30% entre 1990 et 1998 tandis que 46% des consommateurs n'était plus lié à leur fournisseur historique.

La libéralisation du secteur n'a pas connu de heurt important comme en Californie. Cependant, on ne peut pas dire que le Pool ait été un succès dans la mesure où pendant dix ans, le régulateur (l'OFFER) a suivi continûment et de très près son fonctionnement et fixait de facto quasiment les prix de gros. Le secteur a connu ainsi une seconde grande réforme en 2001: les accords du NETA (New Electricity Trading Arrangements) ont mis fin au Pool au profit d'un système décentralisé où les producteurs s'assurent eux-même de l'équilibre avec les distributeurs, par exemple par le biais de bourses d'électricité comme APX qui fonctionnent en temps continue. Le reste de cette section donne des précisions sur le fonctionnement du Pool et les outils utilisés par l'OFFER pour le réguler. En tant qu'expérience pionnière, le Pool anglais a été largement étudié dans la littérature: on peut se référer aux travaux de Green et Newbery [39], Wolfram [95, 96] et Green [38] pour plus de détails.

Le Pool est une institution centralisée auquel doit s'adresser impérativement tous les acteurs du marché: soit pour vendre de l'électricité soit pour en acheter. Dans la matinée du jour J-1 est organisée une 'enchère double'. De manière schématique, pour chaque tranche demi-horaire, chaque producteur (distributeur) soumet une courbe d'offre (de demande) reflétant ses coûts pour une quantité donnée à produire (reflétant sa valorisation).[4]

[4] Plus précisément, l'enchère d'un producteur consiste en de nombreux éléments: un prix pour l'allumage d'une centrale pour la journée, une fonction linéaire par morceaux convexe avec trois paliers reflétant les coûts de production avec un coût éventuellement strictement positif pour le fait de maintenir en activité la centrale, ainsi que des caractéristiques techniques de la centrale.

L'allocation qui égalise l'offre et la prévision de la demande basée à partir des enchères des distributeurs et qui minimise les coûts pour l'ensemble de la journée est alors calculée. Pour l'instant, notons que les contraintes de transmission ne sont pas prises en compte et que, jusqu'à maintenant, le programme d'optimisation peut appeler en théorie des centrales qu'il ne pourra utiliser par la suite. Ainsi pour chaque demi-heure, le prix (le System Marginal Price, SMP) qui correspond au coût marginal de la 'dernière' unité de production appelée (en théorie) est déterminé. Le prix payé par le Pool aux producteurs (Pool Purchase Price, PPP) est la somme du SMP et d'une compensation qui est faite aux producteurs pour le fait de déclarer leur capacité disponible, compensation qui dépend de la probabilité estimée de pénurie. Le Pool prend en compte les contraintes spatiales dues à la transmission dans un second temps. Cela l'amène à modifier la planification de la production: les centrales initialement programmées sont compensées au SMP tandis que les centrales, qui sont finalement rappelées, sont rémunérées au prix correspondant à leurs propres enchères. Ces ajustements représentent un coût supplémentaire pour le Pool. Ils se traduisent par le fait que le prix de vente de l'électricité par le Pool (Pool Selling Price) soit supérieur au PPP. Ce surcoût (qualifié d'Uplift) traduit l'ensemble des contraintes dues à la complexité du problème d'allocation, contraintes qui ne peuvent pas être intégrées dans l'enchère initiale.

En théorie, si les acteurs de ce marché sont de taille négligeable, alors ils ont intérêt à reporter de manière sincère leurs fonctions de coût ou de demande et un tel mécanisme de marché conduit à l'allocation efficace sans intervention du régulateur. La réalité est tout autre et les acteurs sont stratégiques. En fait ce sont surtout les producteurs qui le sont, les distributeurs reportant peu ou prou les estimations de leur demande. Un producteur a intérêt à réduire sa production ou à surestimer ses coûts afin de faire monter les prix, ceci d'autant plus que la demande résiduelle auquel il fait face, égale à la différence entre la courbe de demande agrégé et la courbe d'offre agrégé de ses concurrents, est inélastique. Ces comportements stratégiques ont pris des propensions vertigineuses dans la crise Californienne que l'on discutera dans la prochaine section. En fait, en Angleterre, de nombreux garde-fous avaient été mis en place par le régulateur pour limiter la marge de manoeuvre des producteurs.

Premièrement, l'entreprise Nuclear Energy issue du démentèlement de CEBG et qui avait hérité de son parc de centrales nucléaires était contrainte de soumettre au Pool des enchères nulles. Pour cette raison, on a parlé de duopole anglais au lieu de triopole. Deuxièmement, pour les premières années de la privatisation, le duopole avait hérité de contrats à terme couvrant 85-90% de sa capacité de production. Sans ambiguïté, un producteur qui détient de tels contrats est moins incité à faire monter les prix journaliers dans une perspective de court terme. Néanmoins, la perspective des négociations futures lors du renouvellement des contrats à terme peut l'inciter à faire monter les prix spots. L'OFFER qui surveillait constamment l'évolution des prix se plaignait régulièrement dans ses rapports de prix spots trop élevés. Finalement, l'OFFER a demandé en 1994 au duopole National Power et PowerGen de céder 6,000 MW de capacité de production, soit 10-15% de leur capacité totale de production, sous la menace de faire appel aux autorités de la concurrence. Plus précisément, les centrales qui ont été vendues avaient des coûts marginaux 'moyens': il s'agissait ainsi d'unités qui fixaient souvent le prix marginal (SMP). En plus d'un prix plafond, le régulateur a imposé au duopole anglais de s'engager sur des prix moyens à partir du 1er avril 1994. Plus précisément, le duopole s'est engagé à enchérir de manière à ce que, sauf circonstances exceptionnelles, le prix moyen soit inférieur à 2.4 pence par kWh en faisant une moyenne horaire sur les prix journaliers et 2.55 pence par kWh avec une pondération relative aux quantités vendues.

Le Pool britannique a ainsi été marqué par une forte présence des autorités de régulation: instauration de prix plafonds, surveillance continue des prix, obligation de contrats à terme et enfin exigence de sessions d'actif. En dehors de ces interventions directes, l'influence du régulateur était aussi très importante par l'intermédiaire de son pouvoir de menace comme l'a montré Wolfram [96]. Elle a estimé les marges des producteurs autour du mois de mars 1994, date à laquelle le régulateur a émis un rapport concernant le niveau des prix sur le pool et qui a conduit à l'instauration d'une régulation des prix à partir du 1er avril 1994. Quatre semaines avant la publication du rapport les marges sont estimées à 32,9 % alors que les marges se situent autour de 25% dans les périodes antérieures. Quatre semaines après cette publication et avant l'instauration des plafonds sur les prix moyens, ces marges sont tombées à 15,6% alors que les marges remontent ensuite autour de 20% sous le régime du système de régulation étroite des prix. Ces rapides fluctuations des marges sont expliquées par Wolfram de la façon suivante. Juste avant la publication du rapport du régulateur, les producteurs n'ont rien a perdre à rehausser leurs marges puisque ces prix ne seront pas pris en compte dans le rapport du régulateur alors qu'ils avaient déjà une bonne idée du contenu du rapport qui allait sortir et de son caractère irréversible. En revanche, juste après la décision du régulateur, les prix du pool sont au coeur de l'actualité et les producteurs ont intérêt à adopter un profil bas et de plutôt faire démentir le rapport pour éviter des mesures plus drastiques par la suite. D'une certaine manière, on peut se demander si l'on peut vraiment parler de libéralisation et le contrôle continue des prix s'interprète comme un échec du Pool en tant que système décentralisé de formation d'un prix de marché même si la restructuration du secteur n'a pas été un échec en Angleterre.

2.2.2 La crise Californienne: le pouvoir de marché illimité des producteurs

La facture des dépenses d'électricité dans l'Etat de Californie est passée de 2.04 Milliard à 8.98 Millard pour la période estivale (juin à octobre) de 1999 à 2000. Des facteurs conjoncturels, à savoir une sécheresse exceptionnelle qui a conduit conjointement à une hausse de la demande et à une baisse de l'offre, ont déclenché la crise alors qu'en raison d'une absence d'investissement pendant les années 90 tant en capacité de production qu'en capacité de transmission, l'excédent de capacité entre la capacité totale du parc et la demande pendant les heures pleines était passé de 20% à 7%. Parallèlement, les coûts de production des dernières centrales appelées ont effectivement explosés en raison du triplement du prix du gaz ainsi que de la multiplication par 50 du prix de certains permis à polluer en raison d'un mauvais design des marchés correspondants. En exploitant les données publiques sur les coûts de production des centrales avant la libéralisation du secteur, Borenstein, Bushnell et Wolak [15] ont pu décomposer les facteurs explicatifs quant à l'explosion de la facture énergétique, à savoir: les coûts de production sous l'hypothèse d'un marché de gros concurrentiel, les rentes qui sont dues aux producteurs infra-marginaux et que l'on peut qualifier de rentes concurrentielles et les rentes résultant du comportement non-concurrentiel des producteurs. Ils obtiennent que la hausse de 6.94 milliard de dollars de la facture énergétique entre 1999 et 2000 est due pour 21% à la hausse des coûts de production, pour 20% à la hausse des rentes concurrentielles et enfin pour 59% à la hausse du pouvoir de marché des producteurs. La Californie a subi le pouvoir de marché des producteurs -qualifié de réduction de la demande dans la littérature sur les enchères- de manière particulièrement aiguë dans la mesure où la demande des distributeurs était quasiment inélastique sachant que les grands distributeurs s'étaient engagé à adresser toute leur demande sur le marché journalier et que les contrats à terme étaient prohibés. A la limite, si un producteur est nécessaire pour satisfaire une partie de la demande -on dit alors qu'il est pivotal- il est parfaitement libre de fixer le prix final. Pour éviter cet écueil, un outil de régulation est utilisé: les prix plafonds. Il a été réajusté progressivement pour faire face à la crise en passant de 750$ MWh à 150 $ par MWh.

La 'réduction de la demande' peut prendre deux formes qu'il convient de comprendre si l'on veut réguler les comportements stratégiques des producteurs sur ces marchés.

La première forme de réduction de la demande, qualifiée de 'strategic bidding', correspond à une hausse des enchères par rapport aux coûts marginaux. En théorie, les incitations d'un producteur à augmenter ou réduire ses marges dépendent de nombreux facteurs qui ont été étudié empiriquement par Wolfram [95] pour le pool anglais sur la période de 1992-1994 pendant laquelle la compétition était duopolistique entre National Power et PowerGen. L'incitation à enchérir de manière plus ou moins agressive dépend du nombre d'unités infra-marginales: pour une unité de production donnée, si un producteur a plus d'unités infra-marginales, alors une hausse du prix d'équilibre représente un bénéfice plus important. Cet effet se traduit par les prédictions empiriques suivantes: les unités les moins efficaces, qui ont plus d'unités infra-marginales, ont des marges plus élevées ; les producteurs les plus gros ont des marges plus élevées ; enfin si un producteur a déclaré beaucoup de son parc hors service, ses incitations à hausser ces marges sont plus faibles. De même, si cette unité de production qui est à la marge est de taille plus élevée, alors le producteur a intérêt à enchérir de manière moins agressive car réussir à être retenu représente dans ce cas un gain plus élevé. A partir de la base de donnée des enchères journalières du Pool Anglais mais aussi grâce à des données sur les coûts marginaux des unités de production, Wolfram [95] donne une validation empirique à l'ensemble de ces prédictions. En particulier, elle met en évidence que, pour chaque type de centrale, les marges de National Power, qui avait hérité de la plus grosse part du parc de CEBG, étaient plus élevées que celles de PowerGen : par exemple, pour les centrales thermiques à charbon, PowerGen enchérissait 18.8 livres par MWh là où National Power enchérissait 20.1 livres. Cette forme de distorsion signifie une forme d'inefficacité productive : des unités de PowerGen sont susceptibles d'être appelées alors qu'il existe sur le marché des unités de production plus efficaces mais qui sont dans les mains de National Power qui préfère faire monter les prix. La réduction de la demande conduit aussi à une autre forme d'inefficacité qui est classique dans l'analyse des marchés: elle conduit à un niveau de consommation sous-optimal. Ce dernier impact est très difficile à quantifier dans la mesurer où l'on connaît assez mal la fonction de demande. En revanche, Borenstein, Bushnell et Wolak [15] ont estimé l'ampleur de l'inefficacité productive en Californie pendant les étés de 1998 à 2000. Cette inefficacité productive passe de 44 millions de dollars en 1998 à 447 millions pendant l'été 2000 qui a marqué le début de la crise californienne. L'inconvénient de cette stratégie de distortion des enchères vis-à-vis des coûts marginaux est sa visibilité. En allant au bout de sa logique, une telle stratégie implique aussi des enchères très différentes en fonction du niveau général de la demande. Pour cette raison, les producteurs ont souvent choisi la stratégie plus discrète qui consiste à déclarer une centrale hors service.

La seconde forme de réduction de la demande, qualifiée de 'capacity withholding', correspond à déclarer une partie de son parc de production hors service, pour des raisons de maintenance par exemple. Les travaux de Wolak et Patrick [93] et Green [37] soulignent que les temps des opérations de maintenance sur les centrales étaient plus longs sur le Pool Anglais après la libéralisation du secteur, comme ce fut également le cas en Californie. En écartant la question de la visibilité vis-à-vis des autorités de régulation, cette stratégie peut sembler équivalente à celle qui consiste à proposer de produire à un prix très élevé, bien au dessus du prix d'équilibre, afin d'être sûr de ne jamais être appelé. En fait, retirer une centrale permet aussi de manipuler les prix sur les marchés de capacités où les producteurs sont rémunérés pour le simple fait d'avoir de la capacité disponible afin d'assurer la sécurité du réseau en cas de pénurie.

L'explosion de la facture d'électricité en Californie correspond pour l'essentiel à un transfert de rente entre les producteurs et les distributeurs. Ces derniers se sont trouvés au bord de la faillite en étant pris en tenaille entre des prix qui explosent sur les marchés spot et un système de régulation où les prix de détails étaient rigides et des consommateurs qu'ils devaient satisfaire. Cette rigidité condamnait les distributeurs ne disposant pas de capacité de production à subir le pouvoir de marché des producteurs. Les enjeux de ces transferts de rente ont attiré sur les différents marchés de gros (le marché à terme au jour J-1 et le marché d'ajustement en temps réel) des acteurs s'adonnant à de petit jeux spéculatifs analysés par Borenstein et al [14]. Alors que le marché en temps réel était destiné à des ajustements de dernière minute relatifs à des écarts de faible amplitude entre les estimations de la demande et sa réalisation, tandis que c'était le marché au jour J-1 qui devait être le lieu de la confrontation de l'offre et de la demande, fixant ainsi le prix de marché, certains acteurs ont adopté des comportements spéculatifs: certains acheteurs se sont mis à délibérément sous-estimer leur demande sur le marché au jour J-1 afin de faire baisser les prix et à reporter leur demande sur le marché en temps réel. Les investigations de la Federal Energy Regulatory Commission (FERC) ont mis en évidence que la compagnie Pacific Gas & Electric a adopté de telles stratégies. De même l'entreprise Enron a spéculé sur les différences de prix entre ces deux marchés (en sur- ou sous-estimant sa demande effective) alors que ces pratiques étaient interdites par les règles édictées par ces marchés. Cependant, la publicité qu'a fait les médias sur ces comportements qui étaient prohibés par les règles établies par ces marchés ne doit pas cacher le véritable problème et pourquoi une telle crise semblait inéluctable face à un aléa climatique relativement important: la répartition du risque entre les acteurs, thèse centrale de Wilson [90]. Contrairement aux marchés spots dans le reste du monde qui ne couvrent qu'un faible pourcentage des achats des distributeurs, la législation Californienne avait interdit aux distributeurs de détenir des contrats à terme, les rendant ainsi otage du marché journalier. La crise Californienne a connu son épilogue le 31 janvier 2001 lorsque le California Power Exchange a suspendu son marché au jour J-1, que l'Etat est venu au secours des distributeurs.

La crise Californienne va être le point de départ pour la FERC d'une nouvelle politique de régulation. Initialement, il s'agissait de réguler la structure du marché et ensuite de laisser faire le marché, méthode qui s'oppose à la véritable tutelle qu'exerçait l'OFFER sur le Pool britannique. On peut parler de régulation ex-ante. Dans un premier temps, ce sont les indices de concentration qui ont guidé les décisions, par exemple celles relative à l'approbation des fusions. La FERC avait repris les lignes directrices de la Federal Trade Commission (FTC) qui s'appliquent à tous les secteurs. Dans un second temps, des améliorations ont été apportées pour répondre aux spécificités du marché de l'électricité. Une nouvelle mesure était utilisée pour caractériser le pouvoir de marché d'un acteur: elle consiste à calculer la proportion des périodes où celui-ci est pivotal, c'est-à-dire est nécessaire pour satisfaire la demande. Dans un esprit un peu similaire, certains auteurs ont proposés de calculer l'indice HHI en se restreignant aux unités qui fixent le coût marginal. Suivant cette méthodologie et pour le Pool anglais sur la période 1995-2000, Sweeting [86] trouve des indices HHI plus élevés de 50% lorsqu'il se restreint aux unités fixant le coût marginal. Enfin, désormais la FERC a mis en place un nouveau protocole de régulation qui concerne le comportement des acteurs sur le marché. On peut qualifier ce nouveau type de régulation sur les courbes d'offre des producteurs comme une régulation ex-post. Certains comportements sont déclarés comme non-concurrentiels: par exemple, une unité de production qui modifie sa courbe d'offre dans le temps alors même que ces coûts de production n'ont pas varié. Ceci fait référence à la stratégie d'enchérir de manière agressive pendant les heures creuses tandis, qu'en période de pointe dans lequel le producteur se trouve pivotal, il fait monter les prix. Cependant, comme l'indique Bushnell [20], aucun producteur n'a été sanctionné sur ces termes jusqu'à présent.

2.2.3 La France: l'application progressive des directives européennes

Sans enthousiasme, la France a transposé dans sa législation les directives européennes visant à réformer le secteur électrique. En particulier, la France a exercé son droit de veto en 2001 pour le premier projet de directive concernant la libéralisation de la distribution pour les consommateurs résidentiels. La libéralisation s'articule autour de deux directives.[5] La directive du 19 décembre 1996 sur la libéralisation du marché intérieur de l'électricité prône la séparation des activités de production et de transmission, exige la fin des positions de monopole pour les activités de production et de distribution auprès des consommateurs industriels et que l'activité de transmission soit basée sur des tarifs non-discriminatoires. Enfin une agence de régulation indépendante doit être créée pour veiller au bon déroulement de la libéralisation du secteur. Les conséquences de cette directive qui est transposée en février 2000 sont peu visibles pour le grand public: cela conduit à la création de RTE (Réseau de Transport d'Electricité) une filiale d'EDF qui est responsable de la transmission ainsi que de la CRE (Commission de Régulation de l'Electricité). Seul les consommateurs ayant une consommation

[5] Se référer à Morin [73] pour plus d'information sur ces directives et leurs transpositions dans la législation française.

annuelle de plus de 16GWh sont éligibles pour s'approvisionner auprès des concurrents de leur opérateur historique. La seconde directive du 26 juin 2003 impose un calendrier impératif d'ouverture des marchés de détails avec pour date butoir le 1er juillet 2007 et exige une séparation comptable des activités de production et de distribution, séparation qui s'est traduite par la création d'EDF-Distribution. En pratique, sur les 3500 gros consommateurs (>7GWh), 3300 ont exercé leur éligibilité et 23% ont opté pour un fournisseur alternatif. A partir de juillet 2004, plus de la moitié de la demande d'électricité était libre de choisir son fournisseur.

Sur le marché de gros, les grandes dates visant à dynamiser la concurrence sont:

- début 2001: premiers achats des pertes par RTE (représente 5% de la consommation finale)
- Mai 2001: premières cotations pour le marché français de l'électricité concernant les échanges de gré à gré
- Septembre 2001: mise en place des enchères de production virtuelles (6% du parc d'EDF)
- Novembre 2001: création du marché Powernext *Spot*
- Juin 2004: lancement des marchés à terme sur Powernext (*Powernext Futures*)
- Janvier 2006: mise en place d'enchères de capacités explicites aux frontières

2.2.4 Bilan: les différences entre ces expériences

Mettons brièvement en perspective ces expériences. De manière surprenante, c'est le marché le moins concentré au regard des indices standards, la Californie, qui a failli. La prise en tenaille des distributeurs n'a pas eu lieu en France ou en Angleterre pour diverses raisons. En France, production et distribution sont intégrés verticalement au sein d'EDF. Poweo, le principal concurrent d'EDF pour la distribution, se lance aussi dans la production d'énergie et des contrats de très long terme comme celui signé avec EDF au début de l'année 2007 pour une durée de 15 ans, évite aussi toute situation de 'prise d'otage' vis-à-vis du prix de marché. En Angleterre, les distributeurs ont pu progressivement détenir de la capacité de production à partir du milieu des années 90. Les interventions du régulateur sur les contrats à terme permettaient aussi de réduire le pouvoir de marché des producteurs.

Enfin, un des motifs pour accélérer le processus de libéralisation a été souvent l'apport pour la puissance publique de fonds privés pour des investissements en terme de capacité de production ou de transmission, alors que comme le souligne Wilson [90] c'est plutôt dans un contexte d'abondance de capacité, où les producteurs sont rarement pivotaux, qu'il faudrait mener une libéralisation du secteur.

2.3 Mise en oeuvre et régulation des marchés de gros

Dans cette section, nous examinons les apports de la littérature économique vis-à-vis des grandes étapes citées précédemment pour la construction des marchés de gros en France, étapes qui sont mis en exergue par la CRE: les marchés spots, les marchés à termes, les enchères de capacité virtuelle et l'allocation des capacités de transmission. D'autres enjeux sont importants dans l'architecture des marchés de gros et en particulier la gestion de la fiabilité du réseau analysé par Joskow et Tirole [52]. Les coupures d'électricité de très grande ampleur mettent le doigt sur une spécificité de la production d'électricité: si un producteur fait brutalement défaut, il ne remet pas en cause sa seule production, ce qui dans un marché standard aurait pour conséquence de tirer les prix à la hausse et donc d'exercer une externalité positive sur ses concurrents, mais il est susceptible de remettre en cause l'ensemble du système productif par le biais d'un effondrement du réseau et donc d'exercer une externalité négative sur ces concurrents.

2.3.1 Les marchés spots

Si les marchés de gros ne peuvent reposer exclusivement sur un marché spot comme l'a montré l'expérience Californienne, un tel marché est essentiel pour ajuster les fluctuations de court terme. Une des étapes essentielles de la libéralisation du secteur est la mise en place de bourses d'électricité qui reposent sur des enchères. Plusieurs questions animent alors le débat quant aux mécanismes adéquats.

La question du format du mécanisme d'enchères a souvent occulté les autres questions. Ainsi, une question centrale qui occupe depuis longtemps les économistes avec la mise aux enchères des bons du trésor est de savoir s'il faut préférer une enchère discriminatoire, où une offre acceptée est payée à son prix, à l'enchère à prix uniforme, où toutes les offres sont rémunérées au même prix, celui correspondant au prix de la dernière offre acceptée. Nombreux travaux mettent en avant que la réduction de la demande est beaucoup plus aiguë pour l'enchère uniforme. Du point de vue théorique, Back et Zender [10] caractérisent des équilibres et montrent que l'enchère uniforme peut être dominée par l'enchère discriminatoire pour des biens divisibles tandis que Engelbrecht-Wiggans et Kahn [31], pour des biens non-divisibles, montrent que les enchérisseurs ont intérêt à sous-enchérir à partir du deuxième bien. Par la suite, la question de la modélisation précise de l'enchère uniforme par Kremer et Nyborg [57] a montré que les équilibres très défavorables dans la littérature pouvaient être un artefact de la modélisation, en particulier avec des variables continues. Dans cette veine, Kremer et Nyborg [56] et McAdams [68] ont proposé de légères modifications de l'enchère uniforme. Dans un environment incertain, l'enchère uniforme est toutefois vantée pour sa meilleure capacité à intégrer des entrants non informés et fournir un signal aux investisseurs. Notons que pour les bons du Trésor, le débat fait rage depuis trente ans sans qu'une réponse empirique claire n'ait tranché la question: Umlauf [88] trouve par exemple que l'enchère discriminatoire domine l'enchère uniforme.

La perception du marché spot comme celui pour un bien homogène, qui est le modèle de base dans toute la littérature, omet une dimension importante du système électrique: les contraintes de transmission et les pertes énergétiques dues à l'effet Joule. En France, ces pertes représentent environ 5% de la consommation totale et RTE finance ces pertes par le biais d'une tarification timbre poste. Ainsi, en France, le coût du transport d'électricité est indépendant du point d'injection et de soutirage. En pratique, le coût marginal des pertes électriques serait difficile à calculer en raison des effets externes sur les autres acteurs du système impliqués par un tel transport. En particulier, les lois de Kirchhoff induisent des externalités complexes: même si la capacité de transmission entre deux noeuds A et B n'est pas saturée et en négligeant les pertes, injecter 1MW en A et soutirer 1MW en B peut avoir des effets externes sur d'autres noeuds du réseau et les congestionner. Joskow et Tirole [53] consacrent ainsi une partie de leur analyse à ces effets de bouclage dans les réseaux de plus de trois noeuds. Green [37] explique que produire de l'électricité en certains noeuds peut imposer une externalité négative sur l'ensemble du système électrique.

Pour donner de bonnes incitations, le système tarifaire pour l'électricité doit donc dépendre du lieu géographique des points d'injection et de soutirage: on parle de tarification nodale décrite précisément dans l'ouvrage de Schweppe et al. [84]. C'est le système de tarification qui prévaut sur le marché PJM (zone couvrant plusieurs état du Nord Est des Etats-Unis Pennsylvannie-New Jersey-Maryland) avec environ 1700 noeuds. Cette approche a aussi été mise en place au Chili et en Nouvelle-Zélande. Lorsque les noeuds sont regroupés en de vastes zones dans lesquelles la tarification est uniforme, on parle de tarification zonale comme c'est le cas dans le Pool Nordique regroupant les trois pays scandinaves. Un des enjeux centraux quant à l'organisation des marchés spots est ainsi l'intégration des autres contraintes du système comme celle de la transmission afin de restaurer des signaux monétaires qui étaient internalisés au sein des monopoles historiques. Schweppe et al. [84] décrivent un marché spot en passant en revue l'ensemble des contraintes propres au système électrique. On peut aussi citer les contraintes à l'allumage qui créent des marchés pour plusieurs gammes de flexibilité comme le décrit Wilson [90].

2.3.2 Les marchés à terme

Afin de limiter l'incitation d'un gros producteur à faire monter les prix, le régulateur peut lui imposer de signer des contrats déconnectant du prix du marché spot le revenu d'une grande part de son parc de production. Il s'agit de contrats à terme (*forward*) entre un producteur et un distributeur pour se couvrir sur l'incertitude du prix de marché. Le contrat le plus simple correspond à s'entendre sur un prix fixe p: ainsi, par exemple, si le prix de marché est $p_m > p$, le producteur paye au distributeur la différence $p_m - p$. Sans arriver à de telles extrémités en terme de régulation, comme dans le Pool britannique, on peut se demander dans quelle mesure le développement d'un marché à terme peut limiter le pouvoir de marché de producteurs en situation oligopolistique. Nous passons en revue quelques travaux théoriques récents qui sont souvent motivés par le marché de l'électricité. En revanche, le caractère confidentiel de ces contrats n'est pas propice à des analyses empiriques. La littérature a débuté avec la formalisation par Allaz et Vila [2] de l'intuition qu'un marché à terme réduit le pouvoir de marché des producteurs. Cependant, d'autres travaux théoriques ont remis en question leur approche suggérant plutôt que l'ouverture de marchés à terme est une politique insuffisante ou même potentiellement nuisible et qu'il faudrait alors imposer aux producteurs des quotas de production qu'ils devraient vendre à l'avance.

Allaz et Vila [2] considèrent le modèle standard d'un duopole symétrique en compétition à la Cournot. A ce modèle standard qui conduit à un équilibre inefficace, ils ajoutent des périodes antérieures pendant lesquelles les producteurs peuvent signer des contrats à terme. Une période sur le marché à terme est modélisée encore sous la forme d'un jeu à la Cournot: les producteurs annoncent des quantités qu'ils vendent à terme et le prix qui s'établit est celui qui offre un profit nul aux détenteurs de tels contrats. Ils font aussi l'hypothèse que les contrats à terme sont observés par tous les acteurs, hypothèse très critiquable en pratique dans la mesure où les prises de position sur les marchés à terme sont anonymes et seules les positions agrégées peuvent être observées mais aussi dans la mesure où des contrats bilatéraux secrets peuvent être signés entre un producteur et un distributeur.[6] Le résultat central est que si l'on fait tendre vers l'infini le nombre de périodes où l'on peut échanger à terme alors on converge vers l'issue de la concurrence à la Bertrand: la quantité efficace est produite au coût marginal. L'intuition vient du fait que si l'on ajoute une période où l'on peut vendre à terme, les producteurs ont intérêt à vendre à terme pour se positionner en leader de Stackelberg: chaque période de marché à terme correspond à un dilemme du prisonnier où chacun a intérêt à vendre et à exercer une externalité négative sur l'autre producteur.

Ferreira [33] reprend le modèle d'Allaz-Vila en considérant une infinité de périodes où l'on peut échanger sur le marché à terme. La dimension

[6] Se référer à Hughes et Kao [46] pour une critique d'Allaz et Vila [2] au regard de cette hypothèse d'observabilité avec des motifs d'assurance contre le risque. Dans tous les cas, sans motifs d'assurance, si ni les contrats ni les positions agrégées ne sont observables, alors les marchés à terme n'ont aucun effet.

répétée du jeu est telle que toutes les issues entre Bertrand et Cournot sont soutenables par des équilibres parfait en sous-jeux. Cependant, le critère de 'renegociation-proofness' sélectionne les mauvais équilibres. Dans un cadre où les producteurs peuvent tant vendre qu'acheter sur les marchés à terme -hypothèse réaliste au regard de l'historique de certains marchés à terme comme celui du café à la fin des années 70 comme le rapporte Mahenc et Salanié [65], il y a un unique équilibre renegociation-proof: l'équilibre monopoliste où le profit joint est maximisé. Ainsi l'ouverture d'un marché à terme pourrait avoir un impact anti-concurrentiel. Liski et Montero [67] obtiennent des résultats similaires avec un modèle où sont répétés le jeu de marché à terme ainsi que le jeu de marché spot: le marché à terme apparaît comme un outil permettant de soutenir un panel plus riche de stratégies de collusion. Mahenc et Salanié [65] remettent en cause le bénéfice concurrentiel des marchés à terme en reprenant le modèle d'Allaz-Vila avec un duopole qui est en concurrence à la Bertrand différencié sur le marché spot final. Une autre direction consiste à considérer le modèle d'Allaz-Vila auquel on ajoute dans une première étape un jeu d'investissement en capacité qui fixe les quantités maximales qui peuvent être produites. Adilov [1], Grimm et Zoettl [41] et Murphy et Smeers [74] montrent que les marchés à terme réduisent les investissements.

2.3.3 Les enchères de capacité virtuelle

Pour ouvrir le marché à la concurrence, la méthode la plus drastique consiste à démanteler les opérateurs historiques comme cela a été fait avec CEBG en Angleterre qui a été coupée en trois entités ou à obliger certains producteurs à des sessions d'actifs comme l'a fait le régulateur anglais. A moindre échelle, en France, les deux producteurs que sont la CNR (Compagnie Nationale du Rhône) et la SNET (Société Nationale d'Electricité et de Thermique), auparavant fortement liés à EDF, ont retrouvé leur indépendance et représentent une production annuelle de 20TWh ce qui représente de l'ordre de 3-4% de la production annuelle française.

Une mesure de régulation plus 'légère' consiste à obliger certains opérateurs à vendre leur capacité de production de manière transparente au sein d'un mécanisme d'enchères. Tandis qu'une session d'actif consiste à vendre physiquement l'unité de production -ce qui peut poser problème pour la technologie nucléaire que l'on ne voudrait pas voir disséminée au sein de nombreuses entreprises, il s'agit de vendre le droit d'utiliser une centrale, on parle alors de capacité virtuelle. Plus précisément, 1MW de capacité virtuelle est une option d'achat jusqu'à 1MW de puisssance à un prix prédéterminé, qui est typiquement fixé au coût marginal. Les détenteurs de telles capacités virtuelles sont ensuite en concurrence sur les marchés de gros, en particulier en concurrence avec le producteur que l'on a contraint à effectuer de telles retro-sessions. Cet outil de régulation a été utilisé à de nombreuses

Pays	capacité totale GW (2004)	capacité virtuelle (GW)
France	116.7	6
Irlande	5.7	0.6
Belgique	15.8	1.2
Pays-Bas	21.3	0.9
Danemark	12.8	0.6

Table 1: Enchères de capacité virtuelle en Europe

reprises par la commission européenne. La mise en vente a lieu par le biais d'enchères séquentielles dont le revenu revient au producteur initial. Ainsi, suite à la prise de position d'EDF dans le capital de EnBW (un producteur allemand dans le land frontalier) qui a été perçu par la Commission Européenne comme une démarche pour augmenter son pouvoir de marché et bloquer l'ouverture du marché français, celle-ci a exigé en contrepartie qu'EDF mette aux enchères 6GW de capacité. Ces enchères de capacité virtuelle (Virtual Power Plant) ont lieu depuis 2001 à un rythme trimestriel et ceci pour une durée de cinq ans avec une prolongation de ce système dans le cas où le niveau de concurrence sur le marché français serait jugé non satisfaisant par la commission. Le Tableau 1 fait un récapitulatif de la mise en place de telles enchères en Europe afin de promouvoir la concurrence.

L'intuition grossière suggère que les deux outils de régulation que sont les sessions d'actifs et les VPP sont très voisins sinon équivalents. Du point de vue des marchés de gros et dans un cadre statique, une fois que la capacité est dans les mains de nouveaux entrants, que ce soit de façon physique ou de façon virtuelle, il semble ne pas y avoir de différences. En revanche, si les marchés de capacité virtuelle et de gros sont enchevêtrés, comme c'est le cas actuellement avec des contrats de courte durée vendus de manière séquentielle pour les VPP, l'analyse est modifiée. Schultz [83] montre ainsi que les VPP sont susceptibles d'induire moins de concurrence que des sessions d'actifs dans la mesure où, sur les marchés de gros, le producteur dominant a intérêt à adopter un comportement de maximisation de son profit joint à celui des producteurs ayant acquis de la capacité virtuelle. De cette façon, il s'engage à ce que les détenteurs de capacité virtuelle ait un profit élevé ce dont il bénéficie par le biais du revenu des enchères de capacité futures. Ce canal stratégique est le résultat de la dynamique des différents marchés et n'a pas lieu d'être si la capacité virtuelle était vendue en une seule fois.

2.3.4 La gestion des capacités de transmission transfrontalière

La promotion des importations est un autre instrument d'ouverture du marché. Le développement des interconnexions est la méthode mise en oeuvre: cela passe par une gestion plus transparente et une ouverture à tous les producteurs suivant des critères non discriminatoires. Ainsi, les 2GW de capacité que représente l'interconnexion France-Angleterre (IFA) pour lequel EDF avait signé des contrats d'exclusivité jusqu'en 2001, est désormais alloué par le biais d'enchères simultanées ascendantes. Il y a plus généralement un grand mouvement européen sous l'égide de la Commission Européenne dans le cadre du Forum de Florence qui suit les propositions formulées par les travaux d'*ETSO* en vue de la mise en place de mécanismes transparents et non-discriminatoires pour l'allocation des capacités de transmission: il s'agit souvent de mécanismes d'enchères, qualifiées d''enchères explicites' en opposition au terme 'enchères implicites' qui est employé pour qualifier les systèmes de tarification nodale ou zonale. Ces mécanismes sont censés donner de bonnes incitations pour l'allocation des resources et fournir un signal pour les politiques d'investissement.[7] Les travaux théoriques émettent de nombreuses réserves.

En terme d'allocation des capacités à court terme, Joskow et Tirole [53] montrent que, pour préserver leur pouvoir de marché, les producteurs en situation oligopolistique ont intérêt à racheter la capacité de transmission pour bloquer l'entrée sur le marché. Pour atténuer le pouvoir de marché des producteurs, il faut alors obliger les détenteurs de capacité à l'utiliser. En fait, en pratique, une telle règle est difficile à mettre en place car il est normal que la capacité achetée ne soit pas systématiquement utilisée en raison de chocs conjoncturels. Par exemple, sur l'interconnexion France-Angleterre, cette règle semble n'avoir qu'un caractère théorique et il n'existe pas de critères précis pour établir qu'un acteur sous-utilise de manière structurelle la capacité achetée. Cette capacité de préemption est réminiscente de la capacité d'un monopole à bloquer l'entrée, comme Gilbert et Newbery [36] l'ont établi. Néanmoins, les possibilités de préemption dépendent du mécanisme à l'oeuvre comme le suggère Krihna [58] qui rétablit la possibilité d'entrée avec une enchère séquentielle au lieu d'une enchère unique sur les opportunités pour entrer. Alors que Joskow et Tirole [53] centrent leur analyse sur la nature des droits sur la capacité de transmission (droits physiques, comme c'est le cas avec des enchères explicites, versus droits financiers, qui correspondent aux enchères implicites; caractère optionnel ou impératif de l'utilisation de la capacité achetée), Gilbert, Neuhoff et Newbery [35] insistent sur le fait que le processus d'allocation est crucial et mettent en exergue les différences entre l'enchère uniforme et l'enchère discriminatoire tant un tel contexte avec des externalités.

En terme d'allocation des capacités à long terme, l'intuition grossière suggère que les revenus des enchères aux interconnexions devraient guider les investissements. Ainsi, si λ est le coût marginal de construction d'une

[7] Se référer à Brunekreeft et al [18] pour un survey des différents enjeux de la politique de transmission.

unité de transmission, alors la procédure de décision pertinente serait de comparer λ au différentiel de prix entre les deux zones et de mettre en oeuvre l'investissement si ce coût marginal est inférieur au différentiel de prix. Outre le fait qu'il s'agit d'une analyse à la marge et que les investissements en transmission représentent surtout des coûts fixes, ce raisonnement est pertinent uniquement si la concurrence est parfaite dans les deux marchés. En revanche, si la concurrence est imparfaite, un investissement en transmission conduit à un autre dividende: celui de réduire le pouvoir de marché des producteurs et ainsi de faire baisser les prix dans les deux marchés. L'analyse théorique de Borenstein, Bushnell et Stoft [13] révèle des surprises qui montrent que la problématique est complexe et qu'une politique basée sur les revenues des enchères est inadéquate. Ainsi augmenter la capacité de transmission peut faire basculer la situation d'un équilibre où la capacité est utilisée à un équilibre où la capacité n'est plus utilisée. Le paradoxe est qu'augmenter la capacité conduit à ce qu'elle ne soit plus utilisée alors que justement cet investissement en capacité est désirable socialement. En pratique, il peut être alors difficile au comité exécutif d'un gestionnaire de réseau composé en général des différents acteurs du secteur de résister aux arguments des producteurs expliquant que les investissements en capacité sont inutiles dans la mesure où la capacité construite ne va pas être utilisée. Pour éviter ces problèmes de gouvernance, les producteurs en place ne peuvent pas bloquer la construction de nouvelles lignes dans l'Etat de l'Ontario au Canada.

3 Les chapitres de cette thèse

Motivé par les problèmes d'allocation dans les industries de réseaux, les travaux présentés dans cette thèse s'intéressent à des problèmes soulevés par l'étude des enchères avec des externalités (allocatives ou informationnelles) et des enchères multi-objets. Ces questions sont abordées à la fois d'un point de vue théorique et empirique. Cette partie est consacrée à l'exposé de ces travaux, aux liens par rapport à nos motivations initiales et à la manière dont ils s'inscrivent dans la littérature.

3.1 Chapitre 1: The Shill Bidding Effect versus the Linkage Principle

Ce chapitre s'intéresse à l'impact sur les mécanismes d'enchères de la possibilité de participer sous un faux nom ou un prête nom. C'est un thème très peu exploré dans la littérature. Les exceptions sont Yokoo et al [98], qui étudie la robustesse des mécanismes multi-objets lorsque ce sont les acheteurs potentiels qui peuvent utiliser plusieurs identités, et Chakraborty et Kosmopoulou [23] qui considèrent la possibilité pour le vendeur de faire monter les prix en se faisant passer pour un enchérisseur dans l'enchère anglaise. Lorsque le bien en vente est à valeur commune, faire croire que certains enchérisseurs sont prêt à payer cher induit les 'vrais' enchérisseurs à surestimer de manière fallacieuse la valeur qu'ils se font du bien. C'est ce dernier phénomène qui nous intéresse dans ce chapitre dans le cadre d'un modèle standard ancré dans la littérature et pouvant donner lieu à des estimations structurelles comme cela sera suggéré dans le chapitre 5, contrairement à Chakraborty et Kosmopoulou [23] qui se restreignent à des signaux binaires.

Les témoignages de cette pratique dans la vie réelle sont rares par essence mais aussi parce que cette pratique est fortement prohibée. En particulier, aux Etats-Unis, il s'agit d'une violation de la règle du fonctionnement des marchés à savoir que le vendeur adopte un comportement déloyal qui donne une fausse information sur le prix de marché aux acheteurs potentiels. Néanmoins certains scandales sur eBay ou sur le marché de l'immobilier en Australie où les ventes se font généralement par le biais d'enchères et les témoignages correspondants suggèrent qu'il s'agit d'un phénomène récurrent auquel il semble très difficile de remédier.[8] Cette pratique semble encore plus facile à mettre en oeuvre avec de grandes entreprises qui peuvent utiliser, parfois de manière légale, une filiale pour participer à l'enchère. Par exemple, dans les enchères aux interconnexions étudiées dans le chapitre 6, EDF peut participer à l'enchère organisée par RTE qui est une de ces filiales. De plus, il s'agit d'un mécanisme qui conserve l'anonymat des enchérisseurs. Dans le même esprit, lorsque le mécanisme de marché peut être modélisé par une

[8]On peut se référer aux articles de presse: par exemple Dobrzynski [29] ou http://www.morrellandkoren.com.au/news.html

enchère sans que celle-ci ait lieu de manière formelle comme c'est le cas pour les questions de rachat d'entreprises ou de capacité de production par des banques d'investissement, le vendeur a la possibilité d'exercer une activité qui s'apparente à ce que l'on appelle le shill bidding: le fait de soumettre une enchère comme n'importe quel enchérisseur. Cette activité est aussi apparentée à l'utilisation d'un prix de réserve secret. Dans certains environnement, les deux activités peuvent être considérées comme stratégiquement équivalentes.

A la suite du travail empirique d'Ashenfelter [4] qui souligne l'importance en pratique de ces prix de réserve secret, quelques travaux théoriques s'y sont attelé. Horstmann et LaCasse [44] analysent la possibilité pour le vendeur d'annuler une enchère dans un modèle où le vendeur peut remettre en vente le bien et montre le bénéfice d'une telle option dans la mesure où elle lui permet de révéler de manière crédible une information positive qu'il disposerait sur la qualité du bien. Tan et Li [61] montrent l'intérêt de prix de réserve secret lorsque les enchérisseurs sont averses au risque. Enfin, Vincent [89] montre par le biais d'un exemple que les prix de réserve secret peuvent accroître le revenue espéré du vendeur en moyenne dans la mesure où il accroissent le niveau de participation. Ces travaux théoriques mettent en lumière pourquoi les prix de réserve secret peuvent être optimaux pour le vendeur dans certains cas et expliquent ainsi pourquoi ceux-ci sont couramment utilisés, par exemple dans les enchères de bois en France [30, 62]. Néanmoins ces travaux ne formalisent pas l'aspect de la manipulation informationnelle qui est souvent invoqué dans les articles de presse. Il s'agit du coeur de l'analyse de ce chapitre.

La quasi-absence de travaux antérieurs où le vendeur peut enchérir de manière stratégique dans l'enchère est principalement due à des difficultés techniques qui ont conduit Vincent [89] a présenté le problème comme intractable dans la mesure où il implique un système de deux équations différentielles asymétriques. En faisant l'hypothèse que le vendeur ne dispose pas d'information privée et grâce à l'ajout d'hypothèses techniques adéquates, nous parvenons à caractériser l'ensemble des équilibres avec un vendeur stratégique dans le modèle de Milgrom, Weber [71] où les enchérisseurs reçoivent des signaux affiliés et où leurs valuations dépendent non seulement de leur propre signaux mais aussi des signaux des autres enchérisseurs. La résolution de ce type de jeux bayésiens avec deux classes d'acteurs est la contribution technique du papier qui pourrait être utile pour d'autres travaux économiques. Ensuite, la contribution économique consiste à remettre en question le célèbre linkage principle lorsque le vendeur ne peut pas s'engager à ne pas participer à l'enchère. Le 'Linkage Principle' est un des résultats les plus influents de la théorie des enchères: il établit en particulier que l'enchère ascendante et l'enchère au second prix dominent l'enchère au premier prix en terme de revenu espéré. Nous montrons que ce résultat n'est plus valide lorsque le vendeur ne peut pas s'engager à ne pas participer à l'enchère. En particulier, lorsque les signaux des enchérisseurs sont indépendants, l'effet provenant du shill bidding conduit à l'optimalité de l'enchère au premier prix par rapport à ces dernières. Alors que c'est l'affiliation qui est le terreau du linkage principle, un résultat de statique comparative montre que c'est le caractère à 'valeur commune' du bien mis aux enchères qui est le terreau du 'shill bidding effect'. Le chapitre comporte aussi une discussion de l'impact du shill bidding sur le bien être social. Alors que les objectifs que sont le revenu du vendeur et le bien-être social sont généralement antagonistes dans la théorie des enchères, le premier incitant à faire monter le prix de réserve par rapport au second objectif qui plaide pour un prix de réserve nul, nous montrons ici que le 'shill bidding effect' est néfaste pour les deux points de vue. Enfin, d'autres environnements où le shill bidding modifie considérablement l'équilibre de l'enchère sont présentés brièvement.

3.2 Chapitre 2: The Ausubel-Milgrom Proxy Auction with Final Discounts

Ce chapitre s'intéresse au design d'enchères multi-objets et propose une légère modification du mécanisme d'enchères combinatoires proposé par Ausubel et Milgrom [8].[9] Ce chapitre se place dans le cadre de l'allocation de biens à valeurs privées. La portée du mécanisme s'inscrit dans la littérature sur les mécanismes d'allocation sans externalités informationelles qui a pour départ le mécanisme de Vickrey-Clarkes-Groves formalisé dans les années 70 et qui est au coeur de la théorie microéconomique. Ce mécanisme bénéficie de la propriété d'incitation à révéler ces préférences de la part des enchérisseurs indépendamment des reports des autres enchérisseurs tout en implémentant l'allocation qui est efficace socialement relativement aux préférences qui sont reportées. Malgré ces 'bonnes' propriétés qui en font le candidat théorique naturel, le mécanisme de Vickrey n'est jamais mis en oeuvre dans la vie réelle comme le remarque Rothkopf et al [82].

Une des critiques du mécanisme de Vickrey est sa forme sous pli-scellés qui demande aux enchérisseurs de reporter leurs préférences, ce pour quoi ils peuvent être réticent. Ajoutons qu'un tel mécanisme de prix n'est pas intuitif et difficilement compréhensible par un non-spécialiste. Dans cette perspective des mécanismes ascendants ont été proposé dans différents cadres plus ou moins restrictifs: Ausubel [6] dans le cadre multi-unités avec des demandes marginales décroissantes, Ausubel [7], de Vries et al [28] et Mihsra et Parkes [72] pour des enchères multi-objets. Ces mécanismes proposent à la fin d'une première phase ascendante une phase de 'price discounts' [28, 72] ou une 'clinching rules' [6, 7], qui déconnecte la dynamique des prix affichés avec les prix payés par les enchérisseurs. Cette phase n'est pas dénuée de similitude avec celle proposée dans ce chapitre. D'un point de vue théorique, le statut de ces nouveaux mécanismes n'est pas clair. Il s'agirait essentiellement d'implémenter l'allocation de Vickrey avec des prix ascendants et plus intuitifs: le caractère 'ascendant' étant assez ambiguë lorsqu'il y a une phase de price discounts et le caractère intuitif étant très subjectif. En revanche, il ne s'agirait que d'implémenter à nouveau le mécanisme de Vickrey sans remédier à certaines réticences à savoir la question de la stabilité de l'allocation d'équilibre, problèmes développés par Ausubel et Milgrom [8] et Milgrom [70]. La notion générale de stabilité qui est retenue ici est celle de coeur par rapport au jeu de coalition naturellement associé au problème d'allocation. L'appartenance au coeur de l'allocation finale garantie, en particulier, la robustesse du mécanisme vis-à-vis de certaines déviations jointes. Une famille de mécanisme intéressante sont alors les mécanismes

[9] Notons qu'Ausubel est le consultant à l'origine du mécanisme d'enchère pour les Virtual Power Plant mis en place par EDF. Les enchères VPP sont en revanche des enchères multi-unités ascendantes standards et n'offrent pas la possibilité d'enchères à caractère combinatoire.

dont l'allocation finale est dans le coeur vis-à-vis des préférences qui lui sont reportées. L'enchère ascendante d'Ausubel-Milgrom [8] est un exemple. Malheureusement, l'allocation de Vickrey n'est pas en général dans le coeur et, dans un cadre général, les enchérisseurs ne sont alors pas inciter à reporter leurs véritables valorisations monétaires vis-à-vis des différentes allocations proposées mais plutôt à les tronquer. Ausubel et Milgrom [8] établissent une condition suffisante pour que leur mécanisme de coeur implémente l'allocation de Vickrey. Néanmoins, ils ne proposent pas de réciproque: la condition nécessaire la plus générale étant seulement que l'ensemble des préférences soit tel que l'allocation de Vickrey soit dans le coeur.

Premièrement, nous montrons par le biais d'un exemple que cette dernière condition n'est pas suffisante pour inciter les enchérisseurs à reporter les vraies préférences dans l'enchère d'Ausubel-Milgrom. Deuxièmement, nous proposons une modification de l'enchère sous la forme d'une phase finale de 'price discounts' qu'il faut adjoindre au mécanisme originel d'Ausubel-Milgrom et telle que cette condition devienne effectivement suffisante. Au demeurant, cette phase de price discount ne dépend que de l'allocation finale lorsque le mécanisme d'Ausubel-Milgrom s'arrête et s'implémente dans le même esprit que les autres mécanismes ascendants en n'exigeant qu'une révélation partielle des préférences. Le mécanisme que nous proposons a le statut de mécanisme de coeur qui offre les meilleurs incitations à reporter ces préférences: l'allocation finale est le point dans la frontière supérieure du coeur qui est le plus proche de l'allocation de Vickrey, tandis que le mécanisme d'Ausubel-Milgrom garantie seulement le fait que l'allocation finale soit dans la frontière supérieure du coeur. Au passage, notre analyse donne ainsi des précisions sur le statut du mécanisme originel d'Ausubel-Milgrom.

3.3 Chapitre 3: Contingent Auctions with Allocative Externalities: Vickrey versus the Ausubel-Milgrom Proxy Auction

Dans la lignée du chapitre précédent, ce chapitre poursuit l'analyse du mécanisme d'enchères combinatoires d'Ausubel-Milgrom, mécanisme qui a longtemps été discuté pour la mise aux enchères du spectre radioélectrique aux Etats-Unis. Une question centrale est la caractérisation de l'ensemble des préférences pour lesquelles il soit une stratégie dominante de reporter ces préférences. Grossièrement, Ausubel et Milgrom [8] établissent que ceci correspond à une propriété dite de bidder submodularity qui équivaut à ce que les enchérisseurs perçoivent les biens comme des substituts. Cependant, Ausubel et Milgrom considèrent un cadre sans externalités allocatives. La motivation générale de ce chapitre est d'étendre cette analyse dans un cadre avec externalités allocatives. Il existe une littérature importante sur les enchères pour un seul bien avec externalités allocatives, en particulier les travaux de Jehiel et Moldovanu [47, 48] pour les mécanismes standards. La présence d'externalités négatives imposées sur un enchérisseur qui n'obtient pas le bien en vente et qui soient fonction de l'identité de l'acquéreur final ne garantie plus l'existence d'une allocation stable. Jehiel et Moldovanu [47] établissent que pour tous les concepts naturels de coeur formalisant un tel problème d'allocation, le coeur est susceptible d'être vide. Cette forme d'instabilité peut se traduire dans certains environnements par des incitations à ne pas participer à l'enchère. Ainsi contrairement au cadre général sans externalités, l'existence de mécanismes de coeur n'est pas garantie sans restrictions supplémentaires dans le cadre général avec externalités allocatives. Dans ce chapitre, on s'intéresse à une famille de préférences avec externalités allocatives qui est 'orthogonale' à celle analysée par les travaux de Jehiel et Moldovanu: il s'agit d'externalités entre les coacquéreurs. L'idée est qu'un acheteur n'est pas indifférent au nombre et à l'identité des autres acheteurs. En revanche, on considère qu'un enchérisseur qui n'obtient aucun bien est indifférent quant à l'allocation finale.

La contribution de ce chapitre est double. D'une part, nous proposons un modèle simple pour des enchères multi-unités avec des externalités allocatives et des demandes multi-unitaires, modèle qui généralise des travaux existants (Hoppe, Jehiel et Moldovanu [43], Ranger [80], Segal [85]). Ces travaux précédents se limitent à des externalités où un enchérisseur ne se soucie que du nombre de ses co-acquéreurs ou bien de la quantité totale mis en vente. Un tel modèle correspond bien à la mise en vente d'une capacité pour un usage homogène comme cela était le cas pour les enchères UMTS. Ainsi, Hoppe, Jehiel et Moldovanu [43] relèvent qu'une grande banque d'investissement estimait à respectivement 14.75, 15.88, and 17.6 milliards d'Euros la valeur d'une licence UMTS sur le marché allemand s'il y avait 6, 5, and 4 acquéreurs. En revanche, pour un usage hétérogène de la capacité mis en vente, comme c'est le cas pour l'allocation des sillons ferroviaires avec des usages comme le fret, le transport longue distance ou le transport de proximité qui sont des marchés cloisonnés, l'extension naturelle des modèles précédents consiste à considérer qu'un acquéreur se soucie exclusivement de la quantité totale de capacité allouée aux concurrents qui en font le même usage. Ceci conduit à un modèle plus général où les externalités dépendent de l'identité des coacquéreurs. Ce modèle a l'ambition d'être utile pour des travaux ultérieurs théoriques ou expérimentaux dans ce domaine. Dans un autre papier [59], j'analyse la performance des enchères standards et des dynamiques de revente dans le cadre de ce modèle mais avec la restriction de demande unitaire (par exemple pour des licences).

D'autre part, nous introduisons le concept d'enchères contingentes une généralisation des enchères combinatoires en présence d'externalités allocatives où les offres des enchérisseurs peuvent dépendre pas seulement de sa propre allocation mais de l'allocation de l'ensemble des biens. Cela conduit à une généralisation de l'enchère d'Ausubel-Milgrom qui demeure un mécanisme de coeur. Au sein de notre modèle avec des externalités allocatives négatives entre des agents d'un même groupe, nous caractérisons l'ensemble des préférences telles que l'allocation de Vickrey soit dans le coeur, c'est à dire telle qu'il soit une stratégie dominante de reporter ces préférences dans l'enchère d'Ausubel-Milgrom généralisée. Cette condition qui correspond ainsi à l'analogue de la condition de substituabilité, mais dans un cadre avec externalités et avec des biens homogènes, est satisfaite par un ensemble de mesure nulle dans la classe de préférences que l'on considère. Néanmoins dans certains exemples comme celui développé par Ranger [80], ce sous-ensemble peut être une bonne approximation de la réalité. En guise d'application, notre analyse permet d'identifier quelles sont les conditions cruciales et celles qui peuvent être relâchées dans l'exemple de Ranger [80]. Nous caractérisons aussi l'ensemble des préférences telles que l'enchère de Vickrey soit robuste par rapport aux déviations de perdants, classe de préférences qui, en revanche, n'est pas de mesure nulle.

Ausubel et Milgrom [8] ont établi une sorte d'équivalence entre les environnements dans lesquels l'enchère de Vickrey et l'enchère d'Ausubel-Milgrom sont 'robustes'. Dans un cadre sans externalités allocatives, il s'agit de la classe des préférences où les biens sont perçus comment substituts. Avec externalités allocatives, cette équivalence n'est plus vraie et c'est l'enchère de Vickrey qui est plus robuste au regard des critères d'Ausubel-Milgrom [8].

3.4 Chapitre 4: Individual Rationality under Sequential Decentralized Participation Processes

Ce chapitre s'intéresse aux incitations d'acteurs à participer à un mécanisme économique. Jusqu'à présent, l'outil de modélisation, que constitue la 'boite noire' mechanism design et qui se développe dans la littérature depuis une trentaine d'années, fait l'hypothèse que les acteurs décident simultanément et indépendamment les uns des autres de participer ou non au mécanisme. Cela conduit à des équilibres contre-intuitifs où certains groupes de participants préféreraient ne pas participer de manière jointe au mécanisme comme dans Jehiel, Moldovanu et Stachetti [50, 51] pour des mécanismes optimaux et Jehiel et Moldovanu [47, 48] ainsi que Das Varma [26] pour des enchères standards.

Ce chapitre s'intéresse à l'implementation de mécanismes lorsque l'organisateur (ou principal) du mécanisme ne peut pas s'engager sur un jeux simultané de participation mais uniquement sur une classe de jeux de participation séquentiel où les acteurs se voient toujours offrir une nouvelle option de participer après que certains acteurs aient décider de participer. Notons que certains travaux récent comme McAdams et Schwarz [69] ou Compte et al [24] considèrent dans des environnements particuliers d'enchères la possibilité qu'un enchérisseur refasse une offre après la clôture du mécanisme. La motivation générale de notre concept d'implementation est un plus grand réalisme vis-à-vis des capacités d'engagements du principal.

Outre la définition du concept d'implémentation, la contribution consiste à caractériser les mécanismes optimaux dans un environnement sans asymétrie d'information et de montrer que le principal n'arrive pas à extraire toute la rente qu'il serait en mesure d'extraire s'il pouvait imposer la pire des menaces aux participants qui choisirait de ne pas participer à l'instar de Jehiel et al [50]. L'espace des mécanismes implémentables n'est plus convexe et les mécanismes optimaux reposent des strategies 'divide and conquer': le principal s'assure de la participation de certains agents sans utiliser la pire des menaces, qui reposeraient sur la participation des autres agents, afin de pouvoir utiliser ces agents de manière crédibles pour menacer les autres.

Notre analyse considère le cas où les décisions de participation sont publiquement observées comme dans les travaux à caractère positif [69, 24]. Nous traitons aussi le cas général où celles-ci ne sont pas forcément observées mais où les participants peuvent fournir une preuve (non contractualisable) aux autres participants s'ils ont déjà pris la décision de participer au mécanisme.

Par rapport à des problèmes concrets, ce chapitre insiste sur l'importance des capacités d'engagement du principal. Ceci peut expliquer l'utilisation de grandes banques d'investissement pour la mise en vente de capacité de production comme les centrales électriques comme l'indique Ye [97]. Il invite aussi à un examen des questions relatives à la révélation des offres faites par les enchérisseurs à ceux qui n'ont pas encore pris la décision de participer. Par exemple, Das Varma [27] cite le gouvernement indien qui s'engage à une discrétion totale vis-à-vis des décisions de participation des enchérisseurs, tant leur nombre que leur identités.

3.5 Chapitre 5: Nonparametric Identification and Estimation of the Private Value Auction Model under Anonymity

Ce chapitre s'intéresse à étendre l'analyse structurelle des enchères lorsque l'identité des acteurs n'est pas observable par l'économètre, une contrainte sur les données qui est courante en pratique. Préserver la confidentialité des enchérisseurs qui ne remportent pas l'enchère est souvent utilisée pour se prémunir vis-à-vis de stratégies collusives. La littérature empirique sur la collusion dans les enchères comme Porter et Zona [78, 79] et Pesendorfer [77], qui utilisent des données où les identités des perdants sont révélées, insiste sur le facteur stabilisant de cette divulgation des identités vis-à-vis de stratégies collusives.

La méthode structurelle consiste à estimer un modèle dans son ensemble et pas uniquement des prédictions de ce modèle par le biais de formes réduites, qui ne reposent pas sur un modèle sous-jacent qui soit consistant. Longtemps, on a pensé que les modèles issus de la théorie des jeux qui mettent en jeux des acteurs disposant d'informations privées n'étaient pas propre à l'estimation structurelle. Néanmoins sous des hypothèses raisonnables, en particulier dans le cadre à valeurs privées, les modèles d'enchères sont identifiables comme l'ont montré Athey and Haile [5]. De plus, on peut aussi mettre en oeuvre assez facilement des estimateurs nonparamétriques comme l'on fait Guerre, Perrigne et Vuong [42] pour l'enchère au premier prix. Le point de départ de l'économétrie structurelle des enchères sont les formats standards d'enchères avec un seul bien mis en vente. Récemment, des travaux ont cherché à étendre l'analyse structurelle à des enchères multi-unités. L'enchère uniforme et l'enchère discriminatoire ont été étudiée par exemple par Hortaçsu [45] dans un cadre à valeur privé et nonparamétriquement et par Février et al. [34] dans un cadre à valeur commune et paramétriquement tous deux s'attaquant à des modèles avec information incomplète. Les travaux de Wolak [91], qui a analysé les enchères uniformes sur le Pool électrique en Australie, considèrent le modèle d'enchère de Supply Function Equilibrium (Klemperer, Meyer [55]) sans asymétrie d'information entre les enchérisseurs et où l'incertitude vient de chocs exogènes de demande. Il s'agit du modèle théorique qui sert de référence pour les marchés de l'électricité.

L'analyse structurelle est souvent effectuée sous l'hypothèse de symétrie des agents, hypothèse qui semble nécessaire a priori si l'on n'observe pas l'identité des enchérisseurs. Cette hypothèse est bien souvent maladroite. L'objet de ce chapitre est de montrer que celle-ci n'est en fait pas nécessaire si l'on est disposé à faire l'hypothèse d'indépendance qui est très souvent faite dans cette littérature même si cette hypothèse n'est pas toujours nécessaire (voir Li, Perrigne et Vuong [63] et Campo et al [22] pour un traitement du cadre à valeurs affiliées).

Dans un premier temps, nous étendons la notion d'identifiabilité d'un modèle structurel sous anonymat : elle exige l'unicité des 'primitives' du modèle à partir des ‘observables’ à une permutation près des identités des joueurs. Ensuite, nous montrons que le modèle asymétrique affilié à valeurs privées n’est pas identifiable sous anonymat. En revanche, le modèle asymétrique indépendant à valeur privée est identifiable sous anonymat. Il s’agit de la résolution d’un problème inverse non linéaire et non standard qui ne peut se rattacher à des méthodes préexistantes utilisées en économie. En particulier, la notion d’identifiabilité sous anonymat ne permet pas d’appliquer les résultats généraux d’identification à partir de conditions locales à la Roehring [81]. La clé de la résolution consiste à utiliser les restrictions issues de l’hypothèse d’indépendance et qui permet de présenter une des étapes de la résolution comme l’identification des N racines d’un polynôme de degré N. Dans un second temps, nous adaptons la méthode d’estimation nonparamétrique développée par Guerre, Perrigne et Vuong [42]. Il s’agit d’une méthode en plusieurs étapes utilisant des estimateurs dits de ’noyaux’. Nous montrons qu’un choix adéquat des noyaux et des paramètres de lissage, en fonction de la régularité des distributions sous-jacentes des valeurs privées des agents, permet d’obtenir le même taux de convergence uniforme que celui obtenue par Guerre, Perrigne et Vuong [42] lorsque l’identité des agents est observée, taux de converge dont ils ont prouvé l’optimalité dans le cadre symétrique lorsque les identités des enchérisseurs sont observées, tandis que nous prouvons son caractère optimal dans le cadre asymétrique lorsque les identités des enchérisseurs sont observées. Dans un troisième temps, nous développons des extensions pour expliquer comment cette méthodologie peut s’adapter si on n’observe pas certaines soumissions ou si l’économètre n’observe pas certaines identités ou avec des prix de réserves liants.

La méthodologie mise en oeuvre pourrait aussi être utilisée pour identifier de manière structurelle des modèles avec shill bidding comme celui développé dans le premier chapitre. Nous réservons l’utilisation et l’adaptation de cette méthodologie avec des données empiriques à des recherches ultérieures.

3.6 Chapitre 6: Bidder Behavior in Multiunit Ascending Auctions: Evidence from cross-border capacity Auctions

Ce chapitre s'intéresse au mécanisme d'enchères ascendantes utilisé pour allouer la capacité de transmission électrique entre la France et le Royaume-Uni: les enchères IFA allouent depuis l'année 2001 la capacité suivant différentes fréquences, en particulier journalière. Contrairement au chapitre précédent qui adopte l'approche dite structurelle, ce chapitre adopte une approche dite sous forme réduite faute de mieux dans la mesure où le caractère dynamique des enchères semble écarter toute possibilité d'une caractérisation d'équilibres ou de conditions d'équilibre sur lesquelles on pourrait se baser.

L'utilisation de tels mécanismes dynamiques et anonymes pour allouer plusieurs biens de manière simultanée est récente et reste largement à explorer. D'un point de vue théorique, les travaux d'Ausubel et Schwartz [60] et de Brusco et Lopomo [19] suggèrent la possibilité d'équilibres à caractère fortement non concurrentiels où la réduction de la demande y serait exacerbée et le revenu pour le vendeur très faible. Les travaux expérimentaux ne se sont pas directement intéressé au format des enchères IFA mais à des versions dynamiques de l'enchère uniforme. Alsemgeest et al [3], Engelmann et Grimm [32] et Kagel et Levin [54] confirment les intuitions théoriques: la réduction de la demande est plus importante dans la version dynamique de l'enchère uniforme, les enchérisseurs se coordonnent plus facilement sur des équilibres peu concurrentiels. D'un point de vue empirique, les analyses jusqu'à présent ont souvent un caractère un peu anecdotique puisqu'il s'agit bien souvent d'analyser le déroulement d'une seule enchère. La dynamique de certaines enchères de spectre ont confirmé les intuitions théoriques (Grimm et al [40] et Cramton et Schwartz [25]). En revanche, certaines enchères comme en Allemagne et au Royaume-Uni pour les licences UMTS ont été très compétitives avec des dynamiques singulières que la théorie ne parvient pas à rationaliser (Börger et Dustmann [17], Jehiel et Moldovanu [49]).

Les deux faits marquants sur les enchères IFA journalières sont, d'une part, le faible revenu comparé aux estimations théoriques que l'on peut avoir de la valeur ex post du bien et aussi comparé aux résultats des enchères mensuelles ou annuelles, et d'autre part, les grandes variations dans le temps du rapport entre le revenu et les estimations de la valeur ex post de la capacité. L'objectif du travail empirique est de comprendre ces changements de régime tel que sur des fenêtres d'un trimestre notre estimation de la valeur ex post puisse expliquer parfois plus de 80% de la variance des prix tandis que pendant d'autres périodes c'est seulement 20% et pourquoi les prix sont très peu concurrentiels par rapport à la valeur ex post -de l'ordre de la moitié-tandis que ce soit le même ordre de grandeur pour les enchères annuelles.

La méthodologie consiste à régresser séparément le prix et d'autres variables endogènes caractérisant la dynamique de l'enchère sur notre estimation de la valeur ex post du bien, la quantité mis en vente, la volatilité de l'actif sous-jacent, les décisions de participation des enchérisseurs -variables endogènes mais que l'on contrôle bien par les décisions passées de participation- et des variables contrôlant des influences saisonnières. Enfin nous séparons aussi le temps en deux périodes: l'année 2002 et la période à partir de l'année 2003. Nous savons qu'il y a eu des changements concernant les règles de clôture autour de cette période, règles qui du point de vue théorique sont susceptibles d'avoir une influence considérable.[10] Nous trouvons que ce changement de règle a une influence considérable sur le mécanisme de formation des prix. On trouve que la valeur ex post a un pouvoir explicatif du prix final beaucoup plus grand pour la règle de clôture avec une date fixe qu'avec une date aléatoire. Dans le premier cas, les enchérisseurs peuvent être incité à enchérir juste avant la date de clôture de telle sorte que l'enchère est semblable à une enchère discriminatoire sous pli-scellés. En revanche, dans le second cas, l'enchère conserve entièrement son caractère dynamique. Les résultats vont dans le sens de l'hypothèse d'équilibres plus collusifs dans les enchères dynamiques: dans ces dernières, les enchérisseurs captent une plus grosse part du profit sous-jacent. L'hétérogénéité des enchérisseurs explique de manière peu significative les variations dans le temps de la formation des prix. De fortes variations restent encore largement inexpliquées et pourraient être rationalisées par des variations entre des équilibres plus ou moins collusifs.

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Chapitre 1: The Shill Bidding Effect versus the Linkage Principle

The Shill Bidding Effect versus the Linkage Principle*

Laurent Lamy†

Abstract

The analysis of second price auctions with externalities is utterly modified if the seller is unable to commit not to participate in the mechanism. For the General Symmetric Model (Milgrom and Weber [30]) and standard auction formats, we characterize the full set of separating equilibria that are symmetric among buyers and with a strategic seller being able to bid in the same way as any buyer through a so-called shill bidding activity. The revenue ranking between the first and second price auctions is different from the one arising in [30]: the benefits from the highlighted Linkage Principle are counterbalanced by the 'Shill Bidding Effect'.

Keywords: Auctions, externalities, linkage principle, shill bidding
JEL classification: D44, D80, D82

1 Introduction

In their General Symmetric Model where private signals are positively correlated through affiliation and where a single item is auctioned, Milgrom and Weber [30] (hereafter MW) derived the so-called 'Linkage Principle', one of the most influential results in the auction literature. A first aspect of this principle is the benefit for the seller ex ante to commit to a policy of publicly revealing her signal. A second aspect is that, due to their relative ability to convey information, the English auction[1] raises a higher revenue than the second price auction which outperforms the first price auction. Ausubel [2] extends MW's results in a multi-unit framework with flat

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†Laboratoire d'Economie Industrielle, CREST-INSEE, 28 rue des Saints-Pères 75007 Paris. e-mail: laurent.lamy@ensae.fr

[1] More precisely the English button auction introduced by MW as a model of the traditional English open auction used in auction rooms but which could be a poor description of real-life auctions without any activity rules.

multi-unit demands[2]: Ausubel's dynamic auction for homogenous objects outperforms the (static) Vickrey auction.

However, the Linkage Principle is based on an assumption which goes without saying in the auction and more generally mechanism design literature: the seller (or the designer) is able to commit not to participate secretly, under a false name bid for example, in the mechanism. This assumption may be less plausible in some contexts, notably in online electronic auctions as emphasized by Dobrzynski [10], even if shill bidding is prohibited as on eBay.[3] Shill bidding is a pervasive phenomenon in such auctions and is very difficult to detect in practice. How is it possible to prevent the formation of rings of sellers which have no formal acquaintance and whose objective is to shill bid under each other sales? Dobrzynski tells how a fraudulent seller manages to sell a daub, attempting to copy the style of some Diebenkorn's masterpieces, for over 135,000 $ without pretending any certification. Her investigation brings her to 'a list of 33 Internet names that repeatedly bid on one another's offerings' and that is suspected to have formed a ring that raises bids in order to make potential real buyers believe that it was a masterpiece. These last were unaware of the extent of the shill bidding activity involving so many different identities who were supposed to be art experts by eBay's reputation mechanism.

The aim of the present paper is to delimit the degree of validity of the aforementioned revenue ranking in the light of the ability for the auctioneer to commit not to participate in the mechanism. Various formats are not altered in the same way by the shill bidding activity. On the one hand, first price auctions are immune to shill bidding provided that the reserve price is higher than the seller's reservation value: the seller does not find profitable to raise a shill bid since it can only lower her payoff by lowering the probability of sale without modifying the payment of the winner. On the other hand, in the second price auction, to submit a shill bid can possibly raise the revenue of the seller insofar as a shill bid can set the winning price. Furthermore, we show that in this format and with strict interdependent values, any equilibrium contains a shill bidding activity in mixed strategy. Such an equilibrium is shown to raise a smaller revenue than the one without shill bids and the reserve price being fixed to the lower bound of the support of the above mixed strategy: if the seller can commit to this reserve price, she induces the same set of participants which are also bidding more aggressively since they are not fearing to pay a second highest bid coming from the seller. Combining the above observations, we obtain what we call the 'shill bidding effect': a countervailing force to the Linkage Principle in favour of first price auctions.

[2]Perry and Reny [32] display an example where the first aspect of the principle fails in a multi-unit auction without flat demand.

[3]Family members, roomates and employees of the seller are enclosed in this prohibition (for more details see http://pages.ebay.com/help/policies/seller-shill-bidding.html).

In MW's framework, we derive the whole set of buyer-symmetric separating equilibria in the second price auction when the commitment ability not to use shill bids is relaxed. In general, the characterization of an equilibrium of such a Bayesian game between the seller and the buyers is not tractable. That is the reason why Vincent [35] and Chakraborty and Kosmopoulou [7], the only two papers that analyse shill bidding with interdependent valuations to the best of our knowledge, respectively analyse an example with a specific distribution of valuations and the pure common value case with binary symmetric signals.[4] We solve the tractability issue by restricting our analysis to the case of an uninformed seller and by adding a suitable quasi-concavity assumption which generalizes Myerson's regularity assumption on the virtual utility functions. From a technical perspective, the way we solve the two overlapped differential equations coming from the optimization programs of an informed and uninformed agent is, to the best of our knowledge, new to auction theory and could, perhaps, be useful in other applications as well. The analysis of the English auction is slightly more complex and involves the use of multiple shill bids. It is differed to section 6 and if not explicitly mentioned the second price auction is the format that is considered.

A crucial step in the analysis is the 'no-gap' lemma which states that the lowest shill bid and the lowest possible bid of an active buyer (i.e. a buyer who has a positive probability to win) must coincide. This lemma should be compared with the opposite property which characterizes second price auctions without shill bidding activity and with externalities. Either for informational externalities with affiliation as in MW or for negative allocative externalities as in Jehiel and Moldovanu [19], the lowest bid of an active buyer is strictly higher than the reserve price. In the general symmetric model with affiliation, MW note that in the second price auction *at equilibrium there will be no bids in a neighborhood of r [the reserve price].* As a corollary, if such a gap exists, the seller would strictly raise her revenue if she could secretly 'shill bid' above r and below the lowest equilibrium bid of an active buyer: it never changes the allocation and strictly raises the price in the event where only one buyer is participating. This incentive to raise secretly the effective reserve price with a shill bid suggests that shill bidding will reduce the level of trade. On the contrary, if the seller can commit to the announced reserve price and not to use shill bids, then she can commit to any level of trade and so to the one that maximizes her revenue. The equilibria derived in the previous literature without shill bids are not candidate equilibria of the modified auction with the seller's shill bidding activity: the seller must use a mixed shill bidding strategy in any equilibrium.[5] Moreover, we derive the

[4]The revenue comparisons we make between standard formats do not make sense in [7] since the revenue equivalence theorem does not hold with binary signals. Consistent with our results, [7] shows that the shill bidding activity makes sellers and buyers worse off and reduces the probability of trade.

[5]In MW's framework, Vincent writes 'If the seller's use value s were common knowledge,

optimal equilibrium with shill bids and, as the previous intuition suggests, it does involve a lower level of trade and a lower revenue than the optimal equilibrium without shill bids.

In general revenue and welfare comparisons between the optimal first and second price auctions with shill bidding are undetermined. But if signals are not correlated, then only the 'shill bidding effect' matters: the first price auction with an optimal reserve price still implements the optimal auction design with commitment not to use shill bids and thus unambiguously outperforms the optimal equilibrium of the second price auction both in term of revenue and welfare. Moreover, to shed some light on what drives this shill bidding effect, we derive a comparative static result about the loss due to the unability to commit not to participate in the mechanism. The comparison is made across different environments according to a partial order that captures the degree of the interdependence of preferences. We show that the differences in term of revenue and welfare between the optimal second price auction with and without commitment increase with the degree of interdependence.

1.1 Related Literature

The first contributions on shill bidding, also qualified as phantom bids or lift-lining, analyze the English auction and perceive this activity as an additional flexibility that raises the revenue. In the asymmetric pure private value model, Graham et al [15] state that shill bidding can raise the revenue of the English auction since it is an opportunity for the seller to fix a reserve price that depends on the whole history of the auction.[6] In a similar vein, Lopomo [26] analyses the English auction in MW's framework when the auctioneer can be active in the mechanism as any buyer but in a non-anonymous way. Contrary to a shill bidding activity, the auctioneer's activity is thus transparent such that she could not fool the market. [26] then establishes that the English auction with a strategic seller is optimal among a class of robust mechanisms due to what could be referred to as a 'flexibility effect'. In general, the impact of shill bids in the English auction is ambiguous due to this 'flexibility effect' that can possibly outweigh the 'shill bidding effect'. However, if the setup is symmetric and if signals are independent, then shill bids unambiguously deteriorate the welfare and the revenue of the English

then whether or not a reserve price was announced would make no difference in a [second price] auction - buyers would simply compute the seller's optimal [reserve price] and behave as if it were announced', [35] p 579. Thus he misses the issue that, in the event where the seller has a reservation value which is common knowledge, the standard equilibrium of the second price auction with the optimal reserve price is not implementable with a secret reserve price due to the gap.

[6] e.g. on the identity and the time where potential buyers exit the auction. With the use of such history-dependent shill bidding strategy, Izmalkov [18] implements the optimal mechanism of Myerson [31] with a standard English auction.

auction as shown in section 6.

By means of lab experiments, Levin et al [23] question the relevance of the revenue ranking between the English and the first price auctions. The difference is statistically positive only for super-experienced bidders. However, they omit the shill bidding issue from the seller as well as from the buyers. Katkar and Reiley [21] have run on eBay a field experiment to test the benefit of using a secret reserve price for Pokémon cards. An eBay's secret reserve price is equivalent to a shill bid if eBay's standard auction fits the second price auction model. They show that a public reserve price raises more revenue than setting an equivalent secret reserve price -a result that is consistent with our results.

A distinction should be made between shill bidding from buyers or from the seller. In the second price auction, shill bidding from buyers is never profitable. Nevertheless, in the English auction, shill bidding from a buyer could be a way to distort profitably the Bayesian updating of his opponents. A main contribution in this vein is Yokoo et al [36]: they do not consider shill bidding from the mechanism designer's point of view but from the buyers who could use false-name bids by using multiple identifiers. They establish a sufficient condition on buyers' preferences to make the Vickrey-Clarke-Groves mechanism robust to shill bidding. Ausubel [2] and Ausubel and Milgrom [3] also investigate manipulations with multiple identities in multi-unit auctions. Nevertheless, though [2] contains an interdependent value model, the possibility to exit the auction very early under a false-name bid to manipulate the other bidders' priors is absent.

Finally, this paper is also related to a growing strand of the mechanism design literature which relaxes the commitment ability of the designer. In McAfee and Vincent [28], the designer cannot commit never to attempt to resell the good if he fails to sell it. They characterize the optimal reserve price strategy of the seller for standard mechanisms. Skreta [33] extends their analysis to fully general mechanisms but with only one buyer. Zheng [37] analyzes a complementarity commitment failure: the designer cannot ban resale market. He asks whether Myerson's optimal auction can be implemented. Vartiainen [34] considers auction design when parties cannot commit to any action in the mechanism. Dequiedt and Martimort [9] relax the assumption of *public* communication between the principal and her agents and are thus introducing non-manipulability constraints.

This paper is organized as follows: Section 2 introduces the model and the notation. Section 3 briefly recalls the equilibrium derivation with the commitment ability and introduces the quasi-concavity assumptions on which our analysis with shill bids relies heavily. Section 4, the core of the paper, derives the whole set of equilibria of the second price auction when the seller can use shill bids in mixed strategy. Revenue and welfare comparisons between first and second price auctions and comparative statics results are presented in section 5. Section 6 analyzes the English button auction. Extensions are gathered in Section 7: allocative externalities, sequential auctions, the Amsterdam and Anglo-Dutch auctions, a binding reservation value for the seller, entry fees, endogenous entry and the auctioneer's fees policy are considered. The proofs are all relegated in the Appendix.

2 The Model

We consider the General Symmetric Model introduced by MW, i.e. an auction in which $n > 1$ symmetric buyers compete for the possession of a single object. Each buyer receives a one-dimensional signal X_i such that $X_1, \cdots, X_n$ are affiliated and distributed according to a continuous density f which is assumed to be strictly positive on $[\underline{x}, \overline{x}]^n$.[7] Our subsequent notation follows MW. The actual value of the object for buyer i depends not solely on his own signal X_i but also on the entire vector of signal X: this value is denoted $V_i = u_i(X)$ and is assumed to be non negative. On the contrary, if a buyer does not acquire the object, his payoff is normalized to zero. Furthermore, we consider that the signals of his opponents strictly influence one's valuation for the object in a monotonic way.

Assumption 1 (Strict Interdependent Values) *The valuation function u_i is strictly increasing in all variables.*

The model is symmetric: signals are distributed symmetrically (i.e. the density function f is exchangeable) and a buyer's valuation is a symmetric function of the other buyers' signals. Denote by $f^{(k:n)}$ the density of the k^{th} order statistic of $X_1, \cdots, X_n$ ($F^{(k:n)}$ the corresponding cumulative distribution function (CDF)) and $f_{-i,\theta}^{(k:n)}$ the density function of the k^{th} order statistic of $X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_n$ conditional on $X_i = \theta$ ($F_{-i,\theta}^{(k:n)}$ the corresponding CDF). Due to symmetry, the index $-i$ is dropped in the following analysis.

Let us define the function $v : [\underline{x}, \overline{x}]^2 \rightarrow \mathbb{R}$ (respectively $w : [\underline{x}, \overline{x}]^2 \rightarrow \mathbb{R}$) by $v(x, y) = E[V_1 | X_1 = x, Y_1 = y]$ (respectively $w(x, y) = E[V_1 | X_1 =$

[7] Our model is slightly simpler than MW's: we do not consider that the seller receives a signal that is affiliated with the buyers signals. An informed seller modifies considerably the analysis, e.g. the equilibrium concept should rely on Bayesian updating after that the seller announces the chosen mechanism (even if she can commit ex ante to some standard mechanism, e.g. a first price auction, she may not be able to commit ex ante to a given reserve price). On the one hand, if the private information of the seller is verifiable, then the first part of the so-called linkage principle suggests that the seller has interest to commit ex-ante to reveal this information. On the other hand, if the seller's information is both relevant to the buyers' valuations and to the seller's reservation value and is also not verifiable, then this corresponds to a lemon problem which lies outside the scope of our analysis. Jullien and Mariotti [20] and Cai, Riley and Ye [5] have analysed such a signalling game, which is discussed later in section 7.

$x, Y_1 \leq y]$), where Y_1 denotes the first order statistic of the signals received by buyer 1's opponents and $E[V_1|A]$ denotes the expectation of V_1 conditional on the event A. Due to the strict monotonicity of u_i, v and w are strictly increasing in both arguments. With a slight abuse of notation, we denote $v(x) := v(x,x)$ and $w(x) := w(x,x)$. Furthermore, assumption (1) also implies that $v(x) > w(x)$ for $x > \underline{x}$. Most of the analysis considers that the object is valueless for the seller. The analysis extends (as done in section 7) easily if the seller has a binding reservation value as long as it is common knowledge. Only subsection [5.1] considers a framework, as in [35]'s note, where the seller is privately informed about her reservation value.

The following main example will be used throughout the paper to illustrate our insights and the equilibrium construction.

Example 1 *The signals X_i are independent and uniformly distributed on the interval $[0,1]$. The value depends linearly on the signals: $u_i(x_i, x_{-i}) = \alpha \cdot x_i + (1-\alpha)\frac{\sum_{j \neq i} x_j}{n-1}$, where $\alpha \in [\frac{1}{n}, 1)$. The example depends on two parameters: the number of buyers n and the parameter α which represents the strength of the influence of one's own signal relative to the opponents' signals. The limit $\alpha = \frac{1}{n}$ corresponds to the pure common value case whereas $\alpha = 1$ corresponds to the pure private value case. We can easily compute the functions v and w: $w(x) = \frac{1+\alpha}{2} \cdot x$ and $v(x) = (\frac{1+\alpha}{2} + \frac{1-\alpha}{2\cdot(n-1)}) \cdot x$. The difference $v(x) - w(x) = \frac{1-\alpha}{2\cdot(n-1)} \cdot x$ represents for a buyer with a signal x and conditional on having the highest signal the shift in the expected value of the object when he learns that the second highest buyer also receives the signal x.*

For any given standard auction, the timing of the game is as follows. First each agent is privately informed about his signal. Second the seller announces a reserve price.[8] Then, the auction mechanism is played in such a way that the seller can submit shill bids, i.e. she has the ability to forge false names in order to submit anonymous bids. Finally, the object is allocated according to the auction mechanism, resale is banned and the seller cannot re-auction it.

The new step relative to the previous literature is that we consider the seller being unable to commit not to use shill bids in the auction mechanism. Indeed, except for the English auction (see section 6), submitting more than two bids is never strictly better than the best response with only one bid. Implicitly, this step is a departure from the mechanism design literature which considers non-anonymous mechanisms where bidders can be identified and where consequently shill bidding is not an issue if the seller is able to commit to a given mechanism.

Closely related is the possibility for the seller to cancel the final allocation after all bids have been submitted and both the winning bidder and the

[8] In the "supplementary material", we show that the shill bidding effect is strengthened if the seller can use a stochastic reserve price policy.

winning price have been set. This has been studied by Horstmann and LaCasse [17] under the terminology 'secret reserve price', whereas some work, as Vincent [35], uses this terminology for what we call shill bidding from now on. Contrary to [17]'s 'secret reserve prices', shill bidding is a way for the seller to manipulate directly the winning price. Moreover, shill bidding should not be confused with the forgery of new bids *after* observing the initial bids as in the literature about corruption in auctions.[9] It sticks to the conventional definition of the auction except that the seller can play the auction as other bidders.

In MW's analysis with commitment not to participate in the mechanism, the symmetric separating equilibrium of standard auctions is characterized by a function, denoted by $b(\cdot)$, mapping a buyer's own signal X_i into a bid $b_i = b(X_i)$. Hereafter, MW's symmetric equilibrium will be referred to as the equilibrium with commitment or as the equilibrium without shill bids. If the seller cannot commit not to use shill bids, the auction is a Bayesian game between the seller and the buyers. The bidding strategy of the seller must be added in the equilibrium concept.

Definition 1 *In the first and second price auctions with shill bids, a buyer-symmetric separating strategy profile is a couple (b,G) where $b : [\underline{x}, \overline{x}] \to \mathbb{R}^+$ is the nondecreasing function mapping a signal into a positive bid (the null bid is equivalent to non participation), G represents the CDF according to which the seller sets her reserve price (possibly using a shill bid) and such that any bid that has a strictly positive probability to win corresponds to at most one type. A buyer-symmetric separating equilibrium with shill bids (also shortly referred to as equilibrium with shill bids or equilibrium without commitment.) then is a buyer-symmetric separating strategy profile (b,G) such that both the buyers and the seller play their best response strategy.*

Throughout this work, we restrict ourselves to buyer-symmetric equilibria in which buyers use the same strategy. Since $b(\cdot)$ is monotonic, by Lebesgue's Theorem, it is differentiable almost everywhere and the first derivative of b, when it is properly defined, is then denoted by b'.

In the same way, a density, denoted by g, related to the CDF G can be defined almost everywhere. Thus we do not exclude a priori any atom in the shill bidding activity. Nevertheless, we establish later that in equilibrium the shill bidding activity involves no atom except possibly at the lower bound of the shill bidding activity. Denote by $\underline{r}^{shill}$ (respectively $\overline{r}^{shill}$) the lowest (resp. highest) possible shill bid, i.e. $\underline{r}^{shill} = \max [x|G(x) = 0]$ (resp. $\overline{r}^{shill} = \min [x|G(x) = 1]$). Bids strictly below $\underline{r}^{shill}$ are inactive insofar as their probability to win is null. Buyers who bid above this cut off point are called active buyers.

[9] See Burguet and Perry [4] and Compte et al [8].

3 Equilibria with Commitment not to use Shill Bids

In this section, we recall first the results without shill bidding. The equilibrium of the second price auction without shill bidding is characterized by a gap between the reserve price and the lowest equilibrium bid: at equilibrium there will be no bids in a neighborhood of r [the reserve price] as originally noted by MW. More generally (e.g. also for the first price auction), for any buyer i and conditional on any signal X_i, the function mapping the highest opposing bid (the reserve price being included as a bid) to the expected value of the object is discontinuous at r. This distinctive feature of the equilibrium has been recently mentioned in the empirical auction literature as a way to test common value models against private value models when the reserve price is binding. Hendricks, Pinkse and Porter [16] and Athey and Haile [1] have considered this idea although no formal test has been yet developed.

Proposition 3.1 (Milgrom, Weber) *The equilibrium with commitment of the second price auction with a reserve price r is characterized by a threshold $\widetilde{x}$, such that $r = w(\widetilde{x})$, below which buyers do not participate (or equivalently raises a bid below the reserve price) and above which the equilibrium bid is given by:*

$$b_{SP}(x) = v(x), \quad if \quad x \geq \widetilde{x}. \tag{1}$$

The probability that the object is sold at equilibrium equals to $1 - F^{(1:n)}(\widetilde{x})$ and is also referred to as the 'level of trade'. A low $\widetilde{x}$ reflects a strong participation or equivalently a high level of trade. Figure [1] illustrates the gap between the reserve price and the lowest bid of a participating buyer: with strict interdependent values and for $\widetilde{x} > \underline{x}$, we have $r = w(\widetilde{x}) < v(\widetilde{x}) = b_{SP}(\widetilde{x})$. This gap implies that the seller would find strictly profitable to raise secretly the reserve price at least up to $v(\widetilde{x})$. If $v(\widetilde{x})$ is strictly inferior to the optimal reserve price of the equilibrium with commitment, then she would find profitable to raise a secret reserve that is even strictly greater than $v(\widetilde{x})$. Then the equilibrium analysis with commitment is no more valid and the intuition is that the seller will not be able to set the optimal reserve price but rather that the equilibrium reserve prices will be too high in equilibrium. On the contrary, the equilibrium of the first price auction with commitment to the optimal reserve price can be implemented without commitment because raising secretly the reserve price with a shill bid unambiguously raises a lower revenue.

Insert Figure [1]

Proposition [3.1] characterizes the equilibria with a pure reserve price policy to which the seller is committed. Then we can easily derive the expected revenue of the seller as a function of the participation threshold x induced by the reserve price $r = w(x)$, denoted by $U^{com}(x)$.

$$U^{com}(x) = (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty v(u) f^{(2:n)}(u) du.$$

The first term corresponds to the event where the second highest bid is the reserve price, whereas the second term corresponds to the event where the second highest bid comes from an active buyer. The value x^{**}_{SP} (which could also be characterized by the optimal reserve price r^{**}_{SP}) that maximizes the above expression will be referred to as the optimal threshold of the second price auction with commitment.

Example 2 *In our main example, we consider independent signals that are uniformly distributed on* $[0, 1]$. *Then the probability that the reserve price* $r = w(x)$ *is larger than the second highest bid and less than the highest bid equals to* $(F^{(2:n)}(x) - F^{(1:n)}(x)) = n \cdot x^{n-1} \cdot (1 - x)$ *whereas the density of the second highest bid equals to* $f^{(2:n)}(x) = n(n-1) \cdot x^{n-2} \cdot (1 - x)$. *Then we obtain the optimal threshold by maximizing* $U^{com}(x)$*:* $x^{**}_{SP} = \frac{2\alpha}{1+3\alpha}$. *The corresponding optimal reserve price is* $r^{**}_{SP} = \frac{1+\alpha}{1+3\alpha}$. *Note that coincidently as in the pure private value case, the optimal reserve price is independent of the number of buyers.*[10] *The optimal threshold increases with* α*, i.e. decreases with the strength of the common value component. The intuition is that a high reserve price reduces more participation when the common value is intense and is thus less valuable.*

In the rest of this section we introduce some notation and assumptions that will be useful in the analysis with shill bids. Relative to the General Symmetric Model, we make an additional assumption that is not crucial but allows us a simple characterization of the whole set of equilibria with shill bids. In the independent private value framework where $w = v$, this assumption corresponds to Myerson's regularity assumption, i.e. the virtual surplus $x - \frac{1-F(x)}{f(x)}$ is assumed to be quasi-monotone. This assumption is also equivalent to the expected revenue with commitment being a unimodal function of the reserve price. We assume that two given maps, which corresponds to so-called 'quasi-revenues' as the discussion below emphasizes, are unimodal.

Assumption 2 (Unimodality of the quasi-revenues) *The following maps qualified as quasi-revenues are strictly unimodal (or strictly quasi-concave)*

[10] It does not merely result from the independence of the signals but also from the specific form of the valuations such that the expression $(v(x) - w(x)) \cdot (n - 1)$ is independent of n.

functions.

$$x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty w(u) f^{(2:n)}(u) du, \tag{2a}$$

$$x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot v(x) + \int_x^\infty v(u) f^{(2:n)}(u) du. \tag{2b}$$

Denote by x^* the associated mode of the quasi-revenue (2a) and $r^* = w(x^*)$ the corresponding reserve price. With a slight abuse of terminology, both x^* and r^* are qualified as the mode of the quasi-revenue.

The maps (2a) and (2b) are closely related to the revenue of the seller as a function of the level of trade x. On the one hand, the map (2a) equals to the revenue $U^{com}(x)$ minus the positive term $\int_x^\infty (v(u) - w(u)) f^{(2:n)}(u) du$ which is decreasing in x. On the other hand, the map (2b) equals to the revenue $U^{com}(x)$ plus the positive term $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot (v(x) - w(x))$. Those additional terms (relative to the revenue) would be equal to zero in a pure private framework. To this extent, we qualify those maps as quasi-revenues. Then assumption (2) could be interpreted as follows: the quasi-revenues are unimodal functions of the reserve price. A marginal increase of the reserve price has two effects: first it reduces the level of trade which reduces revenue and second it increases the revenue in the event when there is only one active buyer. The unimodality assumption, which is satisfied in all standard examples, states that the second marginal effect is dominant below a cut off point r^* whereas the first one is dominant above r^*.[11]

Whereas the unimodality of the map (2a) is used heavily throughout the paper in particular to derive necessary conditions on the set of possible shill bids in equilibrium, the corresponding assumption on the map (2b) is used only in Proposition [4.5] where it guarantees that the remaining possible equilibria actually are suitable candidates.

The following innocuous assumption states that the mode, x^*, of the quasi-revenue is strictly higher than the lowest possible signal $\underline{x}$. This assumption is satisfied if for example the optimal equilibrium without shill bids involves a binding reserve price. Its aim is to avoid the case where the optima with and without shill bids involves no binding reserve price and are thus equivalent.

Assumption 3 *The optimum of the quasi-revenue (2a) involves a reduction of the level of trade: $x^* > \underline{x}$.*

Example 3 *In our main example, the mode of the quasi-revenue is independent of n and α and equals to $x^* = \frac{1}{2}$. At the optimum, only half of the buyers are active on average. It corresponds to the reserve price $r^* = \frac{1+\alpha}{4}$*

[11] See lemma 1 in [5] for standard sufficient conditions such that the map (2b) is strictly concave.

4 Equilibria with Shill Bids

In this section the analysis focuses first on the second price auction with shill bids when the announced reserve price is initially set to zero in the first stage of the game. For expositional purposes, in our previous definition of an equilibrium with shill bids, we have restricted the analysis to separating strategies. Indeed, as in Lizzeri and Persico [25], our characterization of the full set of equilibria corresponds to the class of equilibria in *nondecreasing behavioural strategies* except for the case of independence of signals where it is fully general. It is left to the reader that a regularity analysis such as the one developed in [25] can be undertaken.

We characterize the whole set of equilibria in propositions [4.4] and [4.5], our main technical contribution. This result leads us to give a key property of the equilibria: only shill bids above the mode of the quasi-revenue (2a) are sustainable in equilibrium, i.e. $\underline{r}^{shill} \geq r^*$. An equilibrium (b, G) should satisfy two overlapped differential equations. The one coming from the buyers' optimization program is a first order linear differential equation relative to function G but this equation also depends on the bidding function b. The one coming from the seller's optimisation program is a first order linear differential equation relative to the function b and does not depend on the CDF G. Nevertheless, the problem is not standard since the range on which this second differential equation is valid depends on the shill bidding activity and thus on G.[12]

The characterization of the set of equilibria proceeds in three main lemmata which give necessary conditions on candidate equilibria. The no-gap lemma establishes an initial condition for the bidding function b: the lowest possible bid of an active bidder must be equal to the lowest possible reserve price. Added to the two differential equations resulting from the profit-maximizing behavior of the buyers and the seller, it is shown that an equilibrium is uniquely characterized by its lowest possible shill bid $\underline{r}^{shill}$. Lemma [4.3] states that only equilibria with $\underline{r}^{shill} \geq r^*$ are potential candidates. Proposition [4.5] concludes by verifying that the remaining candidates gradually selected by our necessary conditions are actually suitable equilibria.

[12] Engelbrecht-Wiggans et al. [12] have considered the sale of a common-value object in the first price auction when one bidder has private information and the others have access only to public information. Garratt and Tröger [13] consider the sale of an object to two kinds of bidders: speculators who are commonly known to have no use value for the object and independent private-value bidders. In those two papers, the equilibria are characterized by a similar system of two differential equations. Nevertheless, the tricky part of our analysis- the characterization of the suitable supports for the bidding activity of the uninformed bidder- is circumvented there since the lower bound of the support of the bidding activity is shown to be zero and the upper bound is common to all kinds of bidders.

The preceding section has mentioned that MW's equilibria are no more valid with shill bidding: no reserve price in pure strategy is sustainable by an equilibrium with shill bids and where the seller does not use shill bids, except the symmetric equilibria where the seller submits a shill bid superior to $w(\overline{x})$ and where the object remains in the seller's hand with probability one. The argument was that, in such a case, the seller could profitably exploit the gap between the reserve price and the lowest possible bid of an active bidder. Indeed this argument implies more generally that any equilibrium with shill bids must have no gap between the lowest possible shill bid of the seller and the lowest possible bid of active buyers. Denote by $\underline{x}^{shill}$, the cut off point for participation, i.e. $\underline{x}^{shill}$ is defined such that $\underline{r}^{shill} = w(\underline{x}^{shill})$ as in MW.

Lemma 4.1 (The No-gap Lemma) *If the equilibrium contains some trade, i.e. $\underline{x}^{shill} < \overline{x}$, then the equilibrium bid of a buyer with the type $\underline{x}^{shill}$ equals to the lowest possible shill bid $\underline{r}^{shill}$:*

$$b(\underline{x}^{shill}) = w(\underline{x}^{shill}) = \underline{r}^{shill}. \tag{3}$$

The second no-gap lemma is technical and states that $b(\cdot)$ must be continuous above $\underline{x}^{shill}$.

Lemma 4.2 (The Second No-gap Lemma) *The map $b(\cdot)$ is continuous on the range of signals of active bidders, i.e. on the interval $[\underline{x}^{shill}, \overline{x}]$*

The no-gap lemma implies that $\underline{r}^{shill} \geq b(\underline{x}^{shill})$. Moreover, we have $\overline{r}^{shill} < b(\overline{x})$ since the expected revenue of the seller is strictly positive in any equilibrium with a positive probability of sale and thus a shill bid involving a null expected revenue can not be part of an equilibrium. Finally, from the continuity of $b(\cdot)$, we obtain that any shill bid $r \in [\underline{r}^{shill}, \overline{r}^{shill}]$ corresponds to the bid of an active buyer: there exists a type $x \in [\underline{x}^{shill}, \overline{x}]$ such that $b(x) = r$. The seller's randomization over bids corresponds to a randomization over types and then to convert the type into a shill bid according to the bid equilibrium mapping b. Let $G^* = G \circ b$ the corresponding CDF that represents the shill bidding activity of the seller as a randomization over types and g^* the corresponding density with the convex hull of its support denoted by $[\underline{x}^{shill}, \overline{x}^{shill}]$.

Assuming that his opponents bid according to a (common) strategy $\beta(\cdot)$ which is a monotonically strictly increasing and differentiable function of his type and that the seller shill bids according to g^*, the maximization problem of a buyer given that he has type x is:

$$\max_{y \in [\underline{x}, \overline{x}]} \left(\int_{\underline{x}^{shill}}^{y} \left[(w(x,u) - \beta(u)) \cdot F_x^{(1:n-1)}(u) + \int_u^y (v(x,s) - \beta(s)) \, f_x^{(1:n-1)}(s) ds \right] g^*(u) du \right). \tag{4}$$

The first term in the integral corresponds to the event where the highest competing bid is from the seller whereas the second term represents the payoff when the highest competing bid is from a buyer.

Then, for any point which is not an atom of the shill bidding strategy, the first order condition implies :

$$b(x) = \alpha(x) \cdot w(x) + (1 - \alpha(x)) \cdot v(x) \tag{5}$$

where $\alpha(x) = \frac{g^*(x) \cdot F_x^{(1:n-1)}(x)}{g^*(x) \cdot F_x^{(1:n-1)}(x) + G^*(x) \cdot f_x^{(1:n-1)}(x)}$.

The map $b(x)$ is a weighted sum of $w(x)$ and $v(x)$. If buyer 1's type is x and the maximum opposing bid is also $b(x)$, then the expected value for the object for buyer 1 is the sum of $w(x)$ weighted with the probability that the highest bid is a shill bid of the seller and $v(x)$ weighted with the probability that the highest bid is from one of his opponents. For signals above $\overline{x}^{shill}$, equilibrium bids are such that $b(x) = v(x)$ since the probability to be in tie with the seller is null. In the same way, at an atom of the shill bidding strategy, the optimization program implies that $b(x) = w(x)$.

As in MW, the bidding strategy of an active buyer is the expected value of the item conditional on the event that he is in tie with another bidder. The bidding function of an active buyer lies between two bounds: the lower bound $w(x)$ which corresponds to the bidder expected value conditional on his signal being x and the tie-bidder being the seller and the upper-bound which corresponds to the bidder expected value conditional on his signal being x and the tie-bidder being one of his opponent bidder.

The necessary conditions derived so far are illustrated in Figure [2] where a typical equilibrium is depicted.

Insert Figure [2]

We now turn to the seller's equilibrium condition. Denote by $U_S^b(x)$ the seller's expected revenue if the buyers bid according to b and if she submits a shill bid corresponding to the bid of a buyer with a type x, i.e. corresponding to a reserve price of $b(x)$. The seller's expected revenue is:

$$U_S^b(x) = (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot b(x) + \int_x^{\overline{x}} b(s) f^{(2:n)}(s) ds. \tag{6}$$

This expression should be put in parallel with the one of the quasi-revenue introduced in assumption (2a). The same comments are relevant. The similarity between those expressions will be used in the next lemma.

If the equilibrium shill bidding strategy is g^*, then the seller is indifferent between any bid $b(x)$ such that $g^*(x) > 0$. Differentiating this expression with respect to x we obtain the following differential equation for the bid function in the corresponding range where $g^*(x) > 0$:

$$(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot b'(x) - f^{(1:n)}(x) \cdot b(x) = 0. \tag{7}$$

For expositional purposes, we assume now that the CDF G^* contains no atom except possibly at $\underline{x}^{shill}$ and that the support of its related density is an interval. Those points are proved independently in lemma (D.1) as a preliminary to the proof of Proposition [4.5]. Thus the differential equation (7) is satisfied on the range $[\underline{x}^{shill}, \overline{x}^{shill}]$.

Coupled with assumption (2a), the first-order condition (7) rules out reserve prices that are below r^* as established by the following lemma. Otherwise, under an initial condition with $\underline{r}^{shill} < r^*$, the solution of the differential equation (7) would be strictly lower than w in the right neighborhood of $\underline{x}^{shill}$.

Lemma 4.3 *A necessary condition on $\underline{r}^{shill}$ is: $\underline{r}^{shill} \geq r^*$.*

This lemma formalizes one of the key insights of this paper: low reserve prices are not sustainable in equilibrium. It is not sufficient that $x \rightarrow (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty w(u) f^{(2:n)}(u)du$ is locally decreasing at $\underline{x}^{shill}$ to guarantee the existence of such an equilibrium. There may be solutions of the first-order differential equation (7) with the initial condition (3) which are hitting the lower bound before reaching the upper bound and are therefore not suitable solutions. Indeed, due to the unimodality assumption, the quasi-revenue (2a) is decreasing for $x > x^*$ and a solution of (7) with an initial condition such that $\underline{r}^{shill} \geq r^*$ can not hit again the lower bound w.

The necessary conditions derived so far are summed up in the following proposition.

Proposition 4.4 (Characterization: necessary part) *In the second price auction without a reserve price, a buyer-symmetric separating strategy profile (b, G) where the item is sold with a strictly positive probability is an equilibrium with shill bids only if:*

- $\underline{x}^{shill} \in [x^*, \overline{x})$.

The strategy of an active buyer is such that $b(x) = bb(x)$, for $x \in [\underline{x}^{shill}, \overline{x}^{shill}]$ and $b(x) = v(x)$, for $x > \overline{x}^{shill}$ where bb and $\overline{x}^{shill}$ are characterized by:

- *The initial condition: $bb(\underline{x}^{shill}) = w(\underline{x}^{shill})$*
- *The differential equation (7) on the range $[\underline{x}^{shill}, \overline{x}]$: $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot bb'(x) - f^{(1:n)}(x) \cdot bb(x) = 0$*
- $bb(\overline{x}^{shill}) = v(\overline{x}^{shill})$.

The seller's shill bidding strategy $G = G^* \circ b^{-1}$ is fully characterized by:

- *The initial conditions: $G^*(\overline{x}^{shill}) = 1$ and $G^*(\underline{x}^{shill}) = 0$*
- *The differential equation on the range $[\underline{x}^{shill}, \overline{x}^{shill}]$: $g^*(x) \cdot F_x^{(1:n-1)}(x) \cdot (b(x) - w(x)) + G^*(x) \cdot f_x^{(1:n-1)}(x) \cdot (b(x) - v(x)) = 0$.*

  The strategy of an non-active buyer is unconstrained provided that the seller does not find it profitable to raise a shill bid lower than $\underline{r}^{shill}$, i.e. $U_S^b(x) \leq U_S^b(\underline{x}^{shill})$ if $x < \underline{x}^{shill}$. This condition is always satisfied if non active buyers bid zero.

For instance, we have not checked whether those candidates to be equilibrium do not contain any profitable (global) deviation. This is the object of the following proposition which states that those necessary conditions characterize an essentially unique equilibrium for each lower bound $\underline{x}^{shill} \in [x^*, \overline{x})$.

Proposition 4.5 (Characterization: sufficiency part) *For any given lower bound of the shill bidding activity $\underline{x}^{shill} \in [x^*, w(\overline{x}))$, an equilibrium satisfying the necessary conditions derived in proposition [4.4] exists and is essentially unique insofar as the strategies of active buyers and the seller and thus also the expected payoffs of the agents are uniquely determined.*

The seller's most preferred equilibrium corresponds to the solution with $\underline{x}^{shill} = x^*$.

Remark 4.1 *The results immediately extend to the general case where the auction contains an initial announced reserve price $r = w(\widehat{x})$. In this case, we should truncate the set of equilibria that we derived such that the lowest possible shill bid must be greater than the announced reserve price, i.e. the line "$\underline{x}^{shill} \in [x^*, \overline{x})$" must be replaced by "$\underline{x}^{shill} \in [\max\{x^*, \widehat{x}\}, \overline{x})$" in propositions [4.4] and [4.5].*

Remark 4.2 *The equilibria in MW are qualified as robust*[13] *because the equilibrium strategy of a potential buyer is still a best response if the highest bid of his opponents and the corresponding identity were disclosed. Without commitment, the equilibrium strategy of a potential buyer is still a best response if only the highest bid of his opponents were disclosed but not his identity: if he learns that it is the seller, then the winner could possibly regret his bid.*

Consider an equilibrium with shill bids. From proposition [4.4], we have that the seller uses a strictly mixed strategy on a given support $[\underline{r}, \overline{r}]$. Then

[13]This is a weaker robustness property than the ex-post Nash Equilibrium concept which applies for the English button auction with commitment where the equilibrium strategy of a potential buyer is still a best response if the strategies of all his opponents were disclosed.

consider the equilibrium without shill bids and the reserve price $\underline{r}$. First, both equilibria are inducing the same participation threshold $\underline{x}$ such that $\underline{r} = w(\underline{x})$. Second, bidders are less aggressive in the former equilibrium since they are bidding according to b such that $b(x) < v(x)$ on the range $[\underline{r}, \overline{r})$ and that $b(x) = v(x)$ for bids above $\overline{r}$ whereas all participants are bidding according to v in the equilibrium with commitment. Thus we obtain the following corollary that is generalized in subsection 5.1 in a framework with an informed seller.

Corollary 4.6 *For the second price auction, any equilibrium with shill bids is strictly outperformed in term of revenue by an equilibrium with commitment.*

Let us illustrate our equilibrium construction by giving the form of the equilibria with shill bids in the special case where signals are statistically independent and distributed according to the CDF F (density f).

In this case, closed form solutions can be easily derived and reserve price supports are geometrically characterized. The independence of the signals implies that $(F^{(2:n)}(x) - F^{(1:n)}(x))) = n(1-F(x)) \cdot F^{n-1}(x)$ and $f^{(1:n)}(x) = n \cdot F^{n-1}(x) \cdot f(x)$. Then assumption (2a) is equivalent to $x \to (1-F(x)) \cdot w(x)$ being strictly unimodal and x^* equals to its mode. Similarly, assumption (2b) is equivalent to $x \to (1-F(x)) \cdot v(x)$ being strictly unimodal. Equation (7) reduces to $(1-F(x)) \cdot b(x)$ being constant. For any $\underline{x}^{shill} \geq x^*$, $\overline{x}^{shill}$ is then uniquely defined by $(1-F(\underline{x}^{shill})) \cdot w(\underline{x}^{shill}) = (1-F(\overline{x}^{shill})) \cdot v(\overline{x}^{shill})$ where $\overline{x}^{shill} > \underline{x}^{shill}$ (which is geometrically characterized in Figure [3]).

Insert Figure [3]

Example 4 *In our main example, we can compute that:*

$$\overline{x}^{shill} = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\frac{1+\alpha}{2}}{\frac{1+\alpha}{2} + \frac{1-\alpha}{2(n-1)}} \cdot \underline{x}^{shill}(1 - \underline{x}^{shill})}.$$

As expected, the above expression is increasing in $\underline{x}^{shill}$. More interesting is the point that $\overline{x}^{shill}$ is increasing with α and decreasing with n. The intuition is that the seller uses higher shill bids when the winner's curse is stronger because the incentive to use shill bids that convey a better information is then stronger, a point that we formalize in section 5.

Then the closed form of the buyers' strategy b as a function of $\underline{x}^{shill}$ the lowest type mimicked by the seller is derived from the first order condition (7):

$$b(x) = \frac{1 - F(\underline{x}^{shill})}{1 - F(x)} \cdot w(\underline{x}^{shill}), for \quad \underline{x}^{shill} \leq x \leq \overline{x}^{shill}$$

$$b(x) = v(x), for \;\; x > \overline{x}^{shill}.$$

Finally, the CDF of the seller's shill bids is given by:

$$G^*(x) = \exp\left(-\int_x^{\overline{x}^{shill}} \frac{v(u) - b(u)}{b(u) - w(u)} \cdot (n-1)\frac{f(u)}{F(u)} \cdot du\right).$$

Figure [3] depicts the set of equilibria in the case of statistically independent signals on a finite support. The dotted interval $[\underline{x}^{shill}, \overline{x}^{shill}]$ represents a possible support for the shill bidding activity. The dotted interval $[\underline{x}^{shill-opt}, \overline{x}^{shill-opt}]$ represents the particular candidate where $\underline{x}^{shill-opt} = x^*$, i.e. the seller's preferred equilibrium. In our main example and in this preferred equilibrium, the seller mimics the types in the interval $[\frac{1}{2}, \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\frac{1+\alpha}{8}}{\frac{1+\alpha}{2} + \frac{1-\alpha}{2(n-1)}}}]$. This equilibrium is now the reference that will be used to compare the second price auction with and without commitment.

5 Revenue Comparisons - The 'Shill Bidding Effect'

In the independent private values environment, the well-known revenue equivalence theorem establishes that the first price and the second price auctions raise the same revenue for any given reserve price. This result has been established by Myerson [31], but it is more general and applies in MW's General Symmetric Model provided that signals are statistically independent (Theorem 3.5 in Milgrom [29]). This equivalence result holds without shill bids. The second price auction's performance is strictly deteriorated by the shill bidding activity as stated in corollary [4.6]. On the contrary, the first price auction's equilibria are immune to shill bidding. Finally, we obtain that the first price auction raises a strictly higher revenue than the second price auction in the framework with independent signals and with shill bids.

Moreover, the level of trade is unambiguously reduced with shill bidding in the second price auction compared to the first price auction with an optimal reserve price. Thanks to the unimodality assumption on the quasi-revenue (2a), the cut off point r^* nicely separates equilibrium reserve prices of the second price auction with and without commitment. The first half of the statement results from proposition [4.4] which states that the seller submits shill bids that are higher than r^* with probability one in an equilibrium without commitment. We prove the second half in the following lemma where it is shown that the optimal reserve price with commitment is less than r^*.

Lemma 5.1 *The optimal reserve price policy of the second price auction without shill bids is to use a reserve price r_{SP}^{**} such that:*

$$r_{SP}^{**} < r^*.$$

The following proposition, our main economic contribution, gathers those insights.

Proposition 5.2 (The Shill Bidding Effect) *Under the additional assumption of independence of signals and without commitment, the optimal first price auction raises strictly more revenue than any equilibrium in the second price auction. Moreover, we have:*

$$r_{SP}^{**} = r_{FP}^{**} < r^* = \underline{r}^{shill-opt} \leq \underline{r}^{shill},$$

where r_{FP}^{**} equals to the optimal reserve price of the first price auction (with or without commitment) and $\underline{r}^{shill-opt}$ is the minimum shill bid of the seller in her most preferred equilibrium in the second price auction without commitment. As a corollary, the probability of sale and thus the welfare is higher in the optimal first price than in the second price auction.

In general, with strictly affiliated values, the revenue and welfare comparisons between the first and second price auctions with shill bidding are undetermined.[14] Nevertheless, the informational linkage between the price paid and the valuation of the item is then reduced in the second price auction with shill bids. Without shill bids, the price paid (given that it is higher than the reserve price) gives the highest signals of his opponents. On the contrary, with shill bidding, the price paid is a 'blurred' signal: it could either reflect the highest signal of his opponents or the shill bidding activity of the seller. Hence, shill bidding also reduces the benefit of the Linkage Principle itself.

The proposition is silent about the extent of the difference in term of revenue between the first price and the second price auction without commitment. In particular is there any factor which drives this difference? The same question is legitimate for the probability that the object is sold which also corresponds to a welfare perspective. First we examine numerically the corresponding comparative statics relative to α and n in our main example. Then we will formalize the intuitions developed by our numerical results.

Example 5 *In Table 1, we compare, in our main example, the revenue of the optimal equilibrium with commitment with the seller's most preferred equilibrium without commitment varying the parameters α and n which capture the strength of the influence of the highest opponent's signal in one's valuation. The row 'shill bidding' (respectively 'commitment') corresponds to the revenue without (with) commitment whereas the last row 'difference %' of the table corresponds to the revenue gain in percentage of the commitment ability.*

[14] In particular, if signals are strictly affiliated and if the interdependency of the valuations vanishes, i.e. in the neighborhood of the pure private value case such that $w = v$, then only the Linkage Principle matters.

We also give the optimal level of trade x^{**} and the support of the types mimicked by the seller's shill bidding activity $[\underline{x}^{shill-opt}, \overline{x}^{shill-opt}]$ for those equilibria with and without commitment. It illustrates that apart from the limits $n = \infty$ or $\alpha = 1$ which are corresponding to pure private value cases, we have: $x^{**} < \underline{x}^{shill-opt} < \overline{x}^{shill-opt}$. We also give the probability that the object is sold to a 'real' bidder. The row 'prob sale commit' represents the probability that the item is sold (to a 'real' potential buyer) in the optimal auction with commitment. In general, this is equal to $1 - F^{(1:n)}(x^{**})$, which equals to $1 - x^{**n}$ in this application. Similarly, the row 'prob sale shill' represents the probability that the item is sold in the optimal auction without commitment. The general expression equals to $\int_{\underline{x}^{shill-opt}}^{\overline{x}^{shill-opt}} (1 - F^{(1:n)}(u))g^*(u)du$.[15]

We can observe two main trends in the numerical results we obtain. First, the difference in the revenues with and without shill bids is decreasing in α and increasing in n. In a nutshell, the greatest the common value, the greatest the incentive to shill bid and the greatest the seller is penalized with regard to her expected revenue. Second, holding the number of bidders constant, if the common value component is greater, then the object is sold more often with commitment but less often without commitment in the optimal auction. This follows the same intuition as the original 'Linkage Principle' though signals are independent. If the common value is important, the seller prefers to commit to a low reserve price because a high reserve price penalizes more participation. On the contrary, if the seller can not commit, then her incentive to shill bid is greater if the common value is important since the second highest bid conveys more information. Thus the seller can not refrain from submitting high shill bids.

From a numerical point of view, it is worthwhile to note that the support of the shill bidding activity remains very significant when the winner's curse is reduced whereas the corresponding differences in revenue become quickly negligible. Similarly, the difference between the probability of sale with and without commitment converges to zero at a much slower pace (in percentage) than the difference of revenue. On the whole, it suggests that shill bidding may be more detrimental to the welfare than to the revenue.

The next proposition sheds some light on the welfare and revenue differences between the optimal second price auction with and without commitment. Those differences exactly equal the respective differences between the first price and the second price auction without commitment in the framework with statistically independent signals.

Proposition 5.3 (Welfare and Revenue Differences and the Degree of Interdependence) *Consider two environments $1, 2$ with the same distribution of signals and*

[15] This is the only computation which requires to calculate the strategy of the seller g^*. It is not needed for the computation of the revenue.

α	1/2	2/3	3/4	1	1/2				
n	2				3	4	5	10	∞
x^{**}	0.4	0.44	0.46	0.5	0.4				
prob sale commit	0.84	0.81	0.79	0.75	0.94	0.97	0.99	1.00	1
$\underline{x}^{shill-opt}$	0.5								
$\overline{x}^{shill-opt}$	0.75	0.70	0.68	0.5	0.69	0.67	0.64	0.59	0.5
prob sale shill	0.65	0.68	0.69	0.75	0.80	0.89	0.94	1.00	1
shill bidding	0.333	0.363	0.378	0.42	0.428	0.492	0.536	0.636	0.75
commitment	0.360	0.377	0.386	0.42	0.446	0.503	0.543	0.636	0.75
difference %	8.0	3.9	2.1	0	4.1	2.2	1.2	$5 \cdot 10^{-4}$	0

Table 1: Participation threshold, probability of sale and revenue with and without commitment varying α and n

where the functions (w_1, v_1) *and* (w_2, v_2) *are such that* $w_1 = w_2$ *and* $v_1(x) > v_2(x)$*, for* $x > \underline{x}$*. Then, if we consider the optimal second price auction with commitment and if we restrict ourselves to the seller's most preferred equilibrium in the case without commitment, we have:*

1. *The probability that the object is sold in the second price auction with commitment (respectively without commitment) is strictly bigger (lower) in environment* 1 *than in environment* 2.

2. *The difference between the welfare of the second price auction with and without commitment is strictly bigger in environment* 1 *than in environment* 2.

3. *The difference between the revenue of the second price auction with and without commitment is strictly bigger in environment* 1 *than in environment* 2.

The proof relies in particular on the fact that $\overline{x}_1^{shill} > \overline{x}_2^{shill}$. For the same distribution of signals, the similarity of w and $\underline{x}^{shill}$ in both environment implies that the equilibrium bid functions $b_1(x)$ and $b_2(x)$ are equal until it reaches the bound $\min_i v_i(x)$. Because $v_2 < v_1$, the bound v_2 is the first to be reached. This point is easily seen in figure [3] where an increase of the degree of interdependence implies an increase of $(1 - F(x)) \cdot v(x)$ everything else staying unchanged, which pushes $\overline{x}^{shill}$ on the right.

Proposition [5.3] can be interpreted as stating that the value of commitment both in term of revenue and welfare is increasing with the degree of interdependence. Let us firstly explain why environment 1 can be viewed as suffering from a greater degree of interdependence than environment 2.

Consider the bidder with the highest signal and consider that in both environments he values the object identically conditional on his signal x and on having the highest signal. Then due to the strict interdependent values assumption, the expected value of the object raises with the signal of his highest opponents. The shift of the expected value if the highest opponent's signal is also x equals to $v_i(x) - w_i(x) > 0$. This shift is bigger in environment 1, i.e. a buyer cares more about his highest opponent's signal, than in environment 2.

Remark 5.1 *It imposes a very partial ordering on different environments insofar as the distribution of types must be the same. The assumption that the functions w_i are the same is somehow a normalization in order to make the level of welfare comparable in the two different environments: we mean that there is no loss of generality to fix the function w_i to any strictly increasing function, e.g. $w_i(x) = x$. If we have $w_i(x) = g_i(x)$ in the initial framework, then it is sufficient to reparametrize the signals according to $x := g_i^{-1}(x)$. After such a reparametrization, to apply Proposition [5.3] we need that $v_1(g_1^{-1}(x)) > v_2(g_2^{-1}(x))$, for $x > \underline{x}$. Note however that this is not an innocuous renormalization of the functions w since both w and F are hold fixed in both environments.*

In the general case without commitment, two effects are at work to compare the first and second price auctions. The original 'linkage principle' highlighted by MW, which relies on the correlation between types, is at work. But another strength is also at work: the 'shill bidding effect'. Whereas the 'linkage principle' is beneficial to formats which convey more information, it is not a surprise that those formats are exactly those that are more vulnerable to shill bidding: the incentive to shill bid increases with the ability of the format to convey information. The main insight is that the channels of the linkage principle and the 'shill bidding effect' do not coincide exactly and that finally the global effect remains undetermined if signals are not independent.

5.1 Generalization: the value of commitment with an informed seller

This subsection establishes that the inability to commit not to use shill bids strictly lowers the revenue of the second price auction in a very general way. This result goes beyond our model and remains true when the seller is informed. Any (suitable) equilibrium without commitment is strictly dominated by an equilibrium with commitment to a pure reserve price strategy for some seller's types. It illustrates a standard intuition in mechanism design: the principal is better off if she has a greater commitment power.[16] The

[16] In a framework where the seller is not privately informed about her reservation value and with the possibility to use a stochastic reserve price policy, the result would have

following proposition formalizes in a broader framework the gain of commitment. For this proposition, the seller may be privately informed of a multi-dimensional signal S and possibly have a reservation value which depends on her information but also on the whole set of signals, $u_0(S, X_1, \cdots, X_n)$. In the same way, the buyers' valuations $V_i = u_i(S, X)$ may depend on S. Denote by b_0 the seller's shill bid which depends on S.

Proposition 5.4 *Consider a buyer-symmetric separating equilibrium with shill bids* (b, G) *and where the support of the types mimicked by the seller is* $\mathrm{I} = [\underline{x}^{shill}, \overline{x}^{shill}]$. *Suppose in addition that*

$$E[V_1|X_1 = x, b_0 = b(x), Y_1 \leq x] \leq E[V_1|X_1 = x, b_0 \leq b(x), Y_1 = x] \qquad (8)$$

for any $x \in \mathrm{I}$ *and that the inequality is strict on a positive measure of* I, *and suppose also that*

$$E[V_1|X_1 = x, b_0 \leq b(\widetilde{x}), Y_1 \leq x] \leq E[V_1|X_1 = x, Y_1 \leq x] \qquad (9)$$

for any $x, \widetilde{x} \in \mathrm{I}$,

then this equilibrium is strictly outperformed by an equilibrium without shill bids and a pure reserve price policy for some seller's types.[17]

The proposition relies on two additional conditions: equation (8) means that a tie with the seller is bad news in term of a bidder's valuation compared with a tie with another buyer. Equation (9) means that a low shill bid is bad news in term of a bidder's valuation. The proof establishes that the seller's types that are setting the lowest reserve prices are better off by publicly committing to those reserve prices: the set of active bidders does not shrink[18] whereas they are bidding more aggressively because they are not fearing to be in tie with the seller.

In a framework where the seller's signal affects only her reservation value and not the valuation of the buyers and where her signal is not correlated with the buyers' signals, then the level of the shill bid conveys no information for the buyers' valuations and equations (8) and (9) are both satisfied. We obtain the following corollary.

been immediate insofar as the outcome of any equilibrium without commitment can be duplicated as the outcome of an equilibrium with commitment if the reserve price policy corresponding to the shill bidding activity of the previous equilibrium is used. Such an argument is the standard manner in mechanism design to prove that commitment makes the principal (weakly) better off.

[17] In the proof, I consider that the buyers do not update their priors as a function of the announced reserve price in the case where the seller chooses to commitment not to use shill bids. Corollary [5.5] does not rely on this assumption since it is assumed that there is no link between the seller's information and the bidders' valuations.

[18] This point is satisfied because we focus on the seller's types that use the lowest reserve prices. In general, the set of active bidders will shrink making the whole effect of such a commitment undetermined.

Corollary 5.5 *If S is drawn independently of $(X_1, \cdots, X_n)$ and if S does not influence buyers' expected valuations, then any equilibrium of the second price auction without commitment is strictly outperformed by some equilibria with commitment and a pure reserve price strategy* for some seller's types.

This point may seem inconsistent with Vincent [35]'s note which exhibits a numerical example, in a common value second price auction, where the seller's ex ante expected revenue without commitment outperforms her expected revenue in the second price auction with commitment when the optimal pure reserve price policy is announced. The agents' valuations in his framework fit with the previous corollary. However, his example relies on the fact that the seller chooses to commit before being privately informed. The timing of our model is slightly different: first the seller is privately informed, second he chooses to commit to be or not to be able to use shill bids. Thus we exclude the kind of 'cross-subsidies' between the different seller's types that appears in [35].[19]

6 The English (Button) Auction

Our previous analysis is restricted to the first and second price auctions where shill bids are only valuable for the seller who is interested to submit at most one shill bid. In the English auction, incentives to raise shill bids are much stronger: the seller may be willing to submit any number of shill bids. Similarly, potential buyers may be willing to use multiple identities to quit early the auction in order to convey an information making the object less valuable to other bidders or to fool the seller's shill bidding strategy. Contrary to Chakraborty and Kosmopoulou [7]'s analysis of the English auction, we consider that the seller is able to submit numerous shill bids as suggested by the empirical evidence in [10].

Let us discuss the case where only the seller is able to submit shill bids. Consider that the announced reserve price is strictly binding, i.e. $r > w(\underline{x})$.[20] Then, the game has no equilibrium. This is due to the fact that in any equilibrium with shill bids, it can be proved that bidders revise their beliefs about the number of 'real' participants in such a way that a supplementary participant corresponds to a positive updating about the expected number of 'real' participant. Consequently, the seller makes a profitable deviation

[19]Corollary 5.5 does not stand in line with Horstmann and LaCasse [17] who examine how the seller can communicate her private information with a secret reserve price. This signaling effect is specific to a dynamic framework where a seller with a good signal prefers to reauction the item later insofar the bidders will refine their assessment of their valuation from their new signals and the information contained in the fact that the seller has preferred to cancel the first auction's allocation.

[20]The following argument will hold with a non-binding reserve price if we add some uncertainty about the number of potential buyers as in [7].

by submitting a higher number of shill bids in order to convey a better information to the bidders: it consists in submitting a supplementary shill bid that exits the auction immediately after the announced reserve price. The no-equilibrium result is then reminiscent of the Integer Game of Maskin [27].

The natural way to avoid this problem is to add a supplementary action to the seller: the possibility of an invasion of shill bids such that the active buyers are unable to observe the dynamic of the exits in the button auction.[21] If the invasion action is used at the beginning of the button auction, then the seller can either continue to use this action, or irrevocably use a finite number of shill bids, or finally quit definitively the auction. In equilibrium, the seller will chose the invasion action until there are at least two active bidders. For the same reason as above, to use a positive finite number of shill bids can never be a best response. Therefore, the bidders' beliefs are such that a finite number of shill bids is bad news in term of a bidder's valuation relative to the invasion state. Finally, the dilemma for the seller is to exit or not to exit given the timing of the exits of the bidders which she is supposed to observe perfectly. Moreover, it is always optimal to remain active if there are at least two bidders since the probability of a simultaneous exit is null.

In a nutshell, when the invasion strategy is sustained as it is always the case in the equilibrium path,[22] the strategy of a given potential buyer, which is an increasing function by standard arguments, corresponds simply to a threshold that is simply a function of his signal and that we denote by $b(x)$. On the other hand, the strategy of the seller is a function of all signals except the first order statistic. This set of information is denoted by Y_{-1} and the shill bidding strategy of the seller by $shill(y_{-1})$ where y_{-1} is a given realization of Y_{-1}. Next we derive the first order conditions that link those two strategies.

As usual, a bidder is active until he is indifferent between winning at that price or losing the item, which corresponds here to receive a null payoff. Winning at the price $b(x)$ means that the seller's shill bid $shill(y_{-1})$ is equal to $b(x)$ and thus conveys some information about his opponents signals. Therefore, the bidders' equilibrium strategy satisfies the equation:

$$b(x) = E[V_1|X_1 = x, shill(y_{-1}) = b(x)]. \tag{10}$$

The strategy of the seller is equivalent to the one that maximizes her reserve price above $b(y_2)$ (the bid of the second highest bidder) given that the

[21] Another way is to assume a mild bounded rationality assumption: bidders are unable to distinguish exits of bidders if there are too many participants.

[22] Out of equilibrium beliefs and optimal responses should be specified to properly define an equilibrium. For example, the belief that all remaining shill bids are from the seller leads to a suitable specification if the invasion strategy is not sustained.

distribution of the signal of the remaining bidder is $F^{(1:n)}(x|Y_{-1} = y_{-1})$. The strategy of the seller is the solution of the following maximization program:

$$shill(y_{-1}) = Arg \max_{s \in [b(y_2), b(\overline{x})]} s \cdot (1 - F^{(1:n)}(b^{-1}(s)|Y_{-1} = y_{-1})). \tag{11}$$

In general, the characterization of a solution of those first order conditions is not tractable. It is the reason why we have restricted the analysis of the second price auction to an uninformed seller. In the English auction, even if she is not informed at the beginning of the game, she becomes informed after having observed some bidders' exits: the seller has received an information that modifies her strategy. Then the bidders must update their beliefs about the other signals according to this strategy.

Nevertheless, if signals are independent, then $F^{(1:n)}(x|Y_{-1} = y_{-1}) = F^{(1:n)}(x)$ and the strategy of the seller is not affected by the timing of the previous exits that are unobservable to the bidders due to the invasion strategy. More precisely, the seller exits the auction either at a maximizer of equation (11) or immediately after the exit of the second highest bidder. Finally, the price paid by the winner is either the second highest bid of a real buyer or a shill bid of the seller that conveys no information about the signals of his opponents. Equation (10) is thus formally equivalent to equation (5). In the same manner, the seller's maximization program (11) is equivalent to the maximization the seller's revenue in the second price auction as in equation (6). Finally, the English auction becomes strategically equivalent to the second price auction and the seller should then use the mixed strategies that we characterize in Proposition [4.4]. In the equilibria of the English auction, the seller picks a shill bid b as in one equilibrium of the second price auction and then uses the invasion strategy until $\max\{b, b(y_2)\}$.

To summarize, the English auction with shill bidding is equivalent to the second price auction if signals are independent.

Note that if both the seller and the bidders can shill bid and use the invasion of shill bids action, then there is an equilibrium such that any active bidders use this action when he is active. Such an equilibrium of the English auction corresponds to one of the second price auction.

7 Extensions

7.1 Negative allocative externalities

Jehiel and Moldovanu [19] consider a single-unit auction with two potential buyers such that a loser suffers from a negative externality: a non-purchaser prefers that the good remains unsold. Then the valuation for the good is not properly defined and depends on whether the best competing offer is the seller (by means of the reserve price or a shill bid in our framework) or the other buyer. Then with a reserve price r, the function mapping the expected 'valuation' of the object to the highest opposing bid is discontinuous at r and the same gap is present.

Jehiel and Moldovanu [19] establish that, with commitment, the first price and the second price auctions are equivalent. This is a consequence of the revenue equivalence theorem which relies on the symmetry and the independence of types. Our whole analysis with shill bidding could be translated in their framework. Each buyer receives a private signal x_i such that x_i represents the profit when he acquires the object and that $g(x_i, x_{-i}) < 0$ represents agent i's utility when his opponent acquires the item, utilities being normalized to zero when the object remains in the seller's hand. Then the function w is such that $w(x) = x$ and the function v such that $v(x) = x - g(x, x) > w(x)$.

Now we can apply our previous results (provided that the suitable quasi-concavity assumptions are made). From Proposition [5.2] we obtain that the seller is better off with the first price auction. From Proposition [5.3], we obtain that if the negative externalities are greater in environment 1 than environment 2, i.e. $|g^1(x, x)| > |g^2(x, x)|$ for any x, then the revenue and welfare differences between the first and second price auctions are greater in environment 1 than in 2.

7.2 Sequential auctions

In the pure private value framework, McAfee and Vincent [28] characterize the optimal reserve price path for a sequence of first or second price auctions when the seller can not commit not to re-auction an unsold item after a delay. They establish that *the well-known revenue equivalence theorem for one-shot auctions with independent private value extends to the dynamic auction environment*. Nevertheless, this equivalence theorem does not hold anymore if the seller is unable to commit not to use shill bid insofar as a similar gap is present. For a given reserve price R_t, bidders with valuations in the neighborhood of R_t prefer not to participate in the auction in order to obtain the item for a strictly lower price at a following period given that the seller will strictly lower the reserve price, i.e. $R_{t+1} < R_t$. On the other hand, for bidders above the participation threshold, the unique weakly undominated strategy is to bid his own valuation.[23]

7.3 The Amsterdam and Anglo-Dutch auctions

Klemperer [22] advocates for the Anglo-Dutch auction: a two stage auction where the auctioneer begins by running an ascending auction in which price is raised continuously until all but two bidders have dropped out and

[23] In a two-unit framework, Caillaud and Mezzetti [6] derive a similar gap for a sequence of two ascending auctions where the seller choses strategically the reserve price of the second auction as a function of the information revealed in the first auction.

where, in the second stage, the two remaining bidders play a first price auction where they should outbid the stopping price X of the first stage. Goeree and Offerman [14] consider the Amsterdam auction where a premium is added relative to the Anglo-Dutch auction: bidders in the second stage receive a percentage of the difference between the lowest bid in the second stage and X. Those formats are not robust to shill bidding in MW's framework and also in the pure private (possibly asymmetric) independent value framework. In an equilibrium with commitment of the Anglo-Dutch auction, the seller would find it profitable to participate and never exit in the first stage and to bid X in the second stage *for any realization of the signals*. It comes from the fact that: first, the strategy of a bidder in the second stage is an increasing function of X and thus the last remaining bidder is more aggressive ; second, a losing bidder in the first stage would have never submitted a bid greater than the amount X (respectively the bid in the second stage of the single "real" winning bidder of the first stage) in the second stage in the pure private value case (resp. in MW's framework). With a premium, the above incentive to shill bid is strengthened since a shill bid enables the seller to avoid to pay a costly premium.

7.4 A binding reservation value

Suppose that the seller's valuation for the object is Π_S which is common knowledge. The preceding analysis of the expected profit of the seller should be modified by adding the term $F^{(1:n)}(x) \cdot \Pi_S$ to $U_S^b(x)$ the previous expression of the expected revenue (6). Then for the second price auction as well as for the first price auction, the lowest equilibrium reserve price should be greater than Π_S.

A surprising insight of the literature on auctions with externalities is that it may be optimal for the seller to fix a reserve price that is lower than her reservation value. In Jehiel and Moldovanu [19], such a low reserve price pushes buyers to bid more aggressively because they are fearing that the item goes in their competitor's hand instead of staying in the seller's hand. Such an insight fails if the seller cannot commit not to use shill bids. This new constraint on the reserve price is binding even for the optimal first price auction if the optimal reserve price policy with commitment involves a reserve price lower than Π_S. Thus, in the general case, the 'shill bidding effect' may damage both auction formats. But as our analysis emphasizes those constraints are always more restricting for the second price than for the first price auction.

Note that if the reservation value is greater than $w(\overline{x})$, then any buyer-symmetric equilibrium without commitment involves no trade. Thus bilateral negociations should be unambiguously preferred to auctions.

7.5 Entry fees

As suggested by MW's Theorem 19, in order to raise her revenue, the seller may better use entry fees and a null reserve price rather than only a reserve price either for the first price or for the second price auction. In the first price auction such a policy is still feasible with shill bids. On the other hand, with the second price auction, such a policy cannot be used to raise the revenue because of the binding constraint on the feasible reserve price policy. Consequently, the possibility to use entry fees may increase the discrepancy between the first price and the second price auction.

7.6 Endogenous Entry of Bidders

There is a couple of papers in the auction literature that endogenize the number of bidders participating in the mechanism (due to the costly activity to get informed about their valuations for example), e.g. Engelbrecht-Wiggans [11] and Levin and Smith [24]. Those papers express a severe critic of the traditional optimal design insights as the discrepancy between the seller and the social planner objectives. Nevertheless, those papers rely on a strong commitment ability of the designer: the seller must be able to commit to an auction mechanism with a given reserve price before the agents decide to incur the costs for participation. Even if she can commit in advance to the selling mechanism, she may not be able to commit not to use shill bids. Then the results of the literature with endogenous entry are modified even in the pure private value framework.

Levin and Smith [24] highlight that MW's revenue ranking between the first and second price auctions is still valid with endogenous entry. We argue that it may not be longer true even in private value due to the 'shill bidding effect': with a first price auction, the commitment to a null reserve price is credible whereas in the second price or English auction the seller will deviate with a positive reserve price, a point that will be anticipated in equilibrium in the entry process. Indeed in the symmetric independent private value framework, for the second price or English auction with shill bidding, Myerson's reserve price is still the optimum with endogenous entry whereas for the first price auction, the optimum is attained approximately when the seller charges neither entry fees nor reservation price.

7.7 Auctioneer's fees

So far, we have considered that the seller is the auctioneer and that he consequently receives the entire share of the winning price. In practice, however, it is generally not exactly the case: either because the seller should pay a tax on the final price or because the auction is run by an intermediary. For example, on eBay, according to the range of the winning price, the seller must pay a final value fee: between 0.01 \$ and 25.00\$, the fee represents

5.25% of the winning price, then this fee decreases to 1.50%. However, if the item is not sold and thus remains “officially” in the seller’s hand, then no fee is charged. Thus the impossibility of a pure reserve price without shill bids in equilibrium and more generally the no-gap lemma are both relying on the assumption that the shill bidding activity involves no cost. In general, for a given announced price $r = w(x)$, submitting a shill bid at $v(x)$ may not be a profitable deviation.

Denote by τ the fee on the final price, such that the seller receives only $(1-\tau)\cdot p$. Then the impact in the seller’s surplus of such a deviation would equal to:

$$(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot (v(x) - w(x)) \cdot (1-\tau) - F^{(1:n)}(x) \cdot \tau \cdot w(x). \quad (12)$$

The first term corresponds to the gain of such a shill bid by raising the price paid by the buyer from $w(x)$ to $v(x)$ in the event where only one buyer is participating. The second term corresponds to the fee paid by the seller when she should buy the item by means of the shill bid: it reflects the financial cost of the shill bidding activity due to the fee. If expression (12) is negative, the profitability to raise shill bids is no more guaranteed. However, the fact that this term is negative is not sufficient to guarantee that not to use shill bids is an equilibrium for this announced reserve price. Shill bids above $v(x)$ may be profitable, especially if the seller assigns a high valuation for the item or equivalently if the initial reserve price r is too low. On the whole, an increase of τ makes shill bids less attractive and is thus a way for the seller not to use shill bids.

eBay’s fees are mostly of two kinds: first an insertion fee which is an increasing function of the reserve price and second a final value fee which represents the percentage of the winning price that eBay gets. As an example, under the actual fees in US, a seller who chooses the reserve price 30$ and obtains a winning price of 35$ is charged typically 2.8$: 1.2$ is charged as an insertion fee and about 1.6$ as a final value fee. This current paper has a strong policy recommendation for auctions’ houses as eBay from a welfare perspective: in order to limit the shill bidding activity, they should charge more by means of a final fee than an insertion fee. Whereas we have argued that an increase in the final value fee reduces shill bidding, insertion fees are also not neutral with regard to shill bidding. A steep insertion fee function gives an incentive for the seller to choose a null announced reserve price and to use shill bids as an effective reserve price, a point that is also true in a pure private framework. Thus there will be a double dividend in putting a null insertion fee for any announced reserve price and to raise instead the final value fee. But although eBay officially prohibits shill bidding, it is not clear whether the auctioneer really suffers from this activity, as formerly emphasized by [7] and whether is really inclined to make a switch in its fees’ policy.

This policy recommendation should be compared with the optimal broker's mechanism proposed by Jullien and Mariotti [20] with a menu of transaction fees contingent on the reserve price. In their model, which is reminiscent of the lemon problem, the seller is privately informed about the quality of the object which corresponds to her valuation and also enters additively in the buyers' valuations. If the seller runs herself the auction, she has an incentive to raise a high reserve price (running the risk of no trade) in order to make believe that the object is of high quality and this leads to an equilibrium with fewer trade compared to the situation where the seller's type would be common knowledge (this is the lemon effect). On the other hand, if the seller faces the broker's optimal mechanism with a menu of insertion fees contingent on the reserve price, then the lemon effect is reduced.

We conclude that eBay fee's policy may be a trade-off between the 'lemon problem' that calls for insertion fees that are contingent on the reserve price and the shill bidding issue that calls for a final value fee.

Appendix

A Proof of the No-gap Lemma [4.1]

Suppose on the contrary that $b(\underline{x}^{shill}) > \underline{r}^{shill}$, then the seller can strictly raise her revenue by secretly raising strictly the reserve price and staying below $b(\underline{x}^{shill})$. It does not change the probability of selling the object whereas it strictly raises its price in the case where the shill bid corresponds to the second order statistic of all bids, an event which occurs with a strictly positive probability provided that $\underline{x}^{shill} < \overline{x}$.

B Proof of the Second No-gap Lemma [4.2]

Suppose on the contrary that there is a point x such that $b(x^-) < b(x^+)$ where $b(x^-)$ (respectively $b(x^+)$) denotes the left (right) limit at x which are well defined since $b(\cdot)$ is monotone. Two events may happen depending on the fact that shill bids may occur in the left neighborhood of x.

First, no shill bids occurs in the left neighborhood of x, then the buyers optimazation program[24] implies that locally $b(x)$ is equal to $v(x)$ and thus $b(x^-) = v(x)$. Moreover, still from the buyers optimization program, $b(x)$ belongs to the interval $[w(x), v(x)]$ very generally and thus $b(x^+) \leq v(x)$. Finally, $b(x^-) = b(x^+)$ since $b(\cdot)$ is monotone and a contradiction has been raised.

Second, shill bids are used in the left neighborhood of x. Then, similarly to the proof of the first 'no-gap' lemma, a shill bid of $b(x^+)$ raises unam-

[24]The buyers optimization program is presented further in the analysis of section 4. But the properties we use here can be proved independently.

biguously a strictly higher revenue than a shill bid about $b(x^-)$ in the left neighborhood of x, which raises a contradiction with the seller using a best response strategy. Thus we have proved the second no-gap lemma.

C Proof of Lemma [4.3]

Suppose that $\underline{x}^{shill} < x^*$. Locally in the right neighborhood of x, the map b is uniquely charaterized by the initial condition (3) and the differential equation (7).

The map $x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty w(u) f^{(2:n)}(u)du$ is strictly increasing for $x < x^*$ due to assumption (2a). Then in $x = \underline{x}^{shill}$, we have: $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w'(x) - f^{(1:n)}(x) \cdot w(x) \geq 0 = (F^{(2:n)}(x) - F^{(1:n)}(x) \cdot b'(x) - f^{(1:n)}(x) \cdot b(x)$ where $b(x) = w(x)$. We conclude that $b'(x) \leq w'(x)$ in the right neighborhood of x. As a consequence, b is strictly lower than w in the right neighborhood of x which raises a contradiction.

D Proof of Proposition [4.5]

Lemma D.1 *The CDF G^* contains no atom in the range $(\underline{x}^{shill}, \overline{x}^{shill}]$. The set of signals such that $g^*(x) = 0$ is of measure null in the range $[\underline{x}^{shill}, \overline{x}^{shill}]$. Then the support of G^* is the interval $[\underline{x}^{shill}, \overline{x}^{shill}]$.*

Proof 1 *Consider that G^* has an atom at $\widetilde{x} > \underline{x}^{shill}$. Then we have $b(\widetilde{x}) = w(\widetilde{x})$. Since, $\widetilde{x} > \underline{x}^{shill}$, the unimodality assumption implies that $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w'(x) - f^{(1:n)}(x) \cdot w(x) \geq 0$ in the left neighborhood of $\widetilde{x}$. Furthermore, the bidding function b satisfies the equation $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot b'(x) - f^{(1:n)}(x) \cdot b(x) = 0$ in the left neighborhood of $\widetilde{x}$. As a consequence, $b(x) < w(x)$ in the left neighborhood of $\widetilde{x}$, which raises a contradiction.*

Consider that the density g^* is null on $(\widetilde{x}_1, \widetilde{x}_2) \subset (\underline{x}^{shill}, \overline{x}^{shill})$ and that it is strictly positive in the right neighborhood of $\widetilde{x}_2$. Then we have $b(\widetilde{x}_2) = v(\widetilde{x}_2)$. Since b crosses v from below at $\widetilde{x}_1$, we have $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot v'(x) - f^{(1:n)}(x) \cdot v(x) \leq 0$ at $\widetilde{x}_1$. As a consequence, the unimodality assumption implies that $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot v'(x) - f^{(1:n)}(x) \cdot v(x) \geq 0$ in the right neighborhood of $\widetilde{x}_2$. Furthermore, the bidding function b satisfies the equation $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot b'(x) - f^{(1:n)}(x) \cdot b(x) = 0$ in the right neighborhood of $\widetilde{x}_2$. As a consequence, $b(x) > v(x)$ in the right neighborhood of $\widetilde{x}_2$, which raises a contradiction.

D.1 Uniqueness

From the Fundamental Theorem of Differential Equations, the functions bb and G^* are uniquely defined as the solution of a differential equation and an initial condition since the regularity condition is satisfied at their respective initial points. The only non standard point we have to check for uniqueness is that there exists no solution bb of the differential equation 7 that hits the upper bound $v(\cdot)$ at x and then goes strictly below $v(\cdot)$ to reach again the upper bound $v(\cdot)$ at $x' > x$. Note that if bb goes strictly below $v(\cdot)$, it will hit the bound before $\overline{x}$ since the seller can not be indifferent between selling the item with a strictly positive probability with the event with a null probability of sale. If such a solution exists, a multiplicity of equilibria would potentially arise, one with $\overline{x}^{shill} = x$ and the other with $\overline{x}^{shill} = x'$. Denote by $\widetilde{x}$ the smallest solution of the equation $bb(\widetilde{x}) = v(\widetilde{x})$ for $\widetilde{x} > \underline{x}^{shill}$. Then, we have $bb'(\widetilde{x}) \geq v'(\widetilde{x})$. Otherwise, it would raise a contradiction with the assumption that $\widetilde{x}$ is a minimal solution. Furthermore, from the unimodality assumption (2b), since $(F^{(2:n)}(\widetilde{x}) - F^{(1:n)}(\widetilde{x})) \cdot v'(\widetilde{x}) - f^{(1:n)}(\widetilde{x}) \cdot v(\widetilde{x}) \leq 0$, we obtain that for any $x > \widetilde{x}$, we have $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot v'(x) - f^{(1:n)}(x) \cdot v(x) \leq 0$. It means that above $\widetilde{x}$, any solution bb should cross the bound $v(\cdot)$ from below. In particular, for $\widetilde{x}$, the solution can not stay below the bound $v(\cdot)$.

D.2 Sufficiency

The preceding analysis has established that the buyers are bidding according to their best response strategy provided that $b(\cdot)$ is actually strictly increasing, which is immediately true from equation (7). The point that is not immediate is that we have to check that the seller would not find profitable to deviate either by raising a shill bid lower than $\underline{x}^{shill}$ or by raising a shill bid higher than $\overline{x}^{shill}$. On the one hand, we have directly assumed in the proposition [4.4] that $U_S^b(x) \leq U_S^b(\underline{x}^{shill})$ if $x < \underline{x}^{shill}$ such that the first deviation is never profitable. On the other hand, we have $b(x) \leq v(x)$ in the left neighborhood of $\overline{x}^{shill}$ and $b(\overline{x}^{shill}) = v(\overline{x}^{shill})$, thus $b'(\overline{x}^{shill}) \geq v'(\overline{x}^{shill})$. Since equation (7) is satisfied at $\overline{x}^{shill}$, we obtain finally that

$$(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot v'(x) - f^{(1:n)}(x) \cdot v(x) \leq 0$$

for $x = \overline{x}^{shill}$. Then due to the unimodality assumption (2b), this implies that this inequality is satisfied for any $x \geq \overline{x}^{shill}$. Since the above expression corresponds to the derivative of the seller's expected revenue above $\overline{x}^{shill}$, we conclude that the seller does not find profitable to shill bid above $\overline{x}^{shill}$.

The remaining point to check is that those equilibria are well defined: more precisely it remains to show that the shill bidding strategy g, which is implicitly defined in the proposition, is actually a density function or equivalently that it is a feasible strategy. From equation (5) and the initial condition $G^*(\overline{x}^{shill}) = 1$, we obtain that:

$$G^*(x) = \exp\left(-\int_x^{\overline{x}^{shill}} \frac{v(u) - b(u)}{b(u) - w(u)} \cdot \frac{f_u^{(1:n-1)}(u)}{F_u^{(1:n-1)}(u)} \cdot du\right), \quad x > \underline{x}^{shill} \qquad (13)$$

Immediately, we have $lim_{x \to \underline{x}^{shill}} G^*(x) \in [0,1)$. With a standard assumption as $\frac{f_u^{(1:n-1)}(u)}{F_u^{(1:n-1)}(u)} = O(1)$, then $lim_{x \to \underline{x}^{shill}} G^*(x) = 0$ and the shill bidding strategy is a mixed strategy without any atom. It is also immediately checked that $g > 0$ for $x > \underline{x}^{shill}$ since b lies between the two bounds w and v what has been obtained by the restriction $\underline{x}^{shill} \geq x^*$ (therefore b can not hit the lower bound w again) and the suitable choice of $\overline{x}^{shill}$ which ensures that b remains under the upper bound v. Remark that if $lim_{x \to \underline{x}^{shill}} G^*(x) \in (0,1)$, then the strategy of the seller involves an atom at $\underline{x}^{shill}$. Anyway, the solution is a feasible strategy.

D.3 The most preferred equilibrium

Consider two equilibria characterized by $\underline{x}_1^{shill}$ and $\underline{x}_2^{shill}$ where $\underline{x}_1^{shill} < \underline{x}_2^{shill}$. Then we have $\overline{x}_1^{shill} < \overline{x}_2^{shill}$. Otherwise, the two solutions bb_1 and bb_2 will cross which raises a contradiction since both functions are satisfying the same differential equation (7) and are not equal. The respective expected revenue of the seller in those two equilibria are corresponding to the quasi-revenue (2b) respectively in $\overline{x}_1^{shill}$ and $\overline{x}_2^{shill}$. In this range, we are in the decreasing part of the quasi-revenue as shown above. Finally, the revenue is higher for $\underline{x}_1^{shill}$. As a corollary, the seller's most preferred equilibrium corresponds to the one with $\underline{x}^{shill} = x^*$

E Proof of Lemma [5.1]

Note first that the sum of a continuous strictly unimodal function with a mode m and a strictly decreasing function attains his optimum at $m' < m$. The expression of the revenue without shill bids as a function of $x = w^{-1}(r)$, the cut off point x corresponding to the reserve price r, is equal to $x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty b(u) f^{(2:n)}(u)du$, where $b(u) = v(u)$. Thus it is equal to the sum of the quasi-revenue (2a) and $\int_x^\infty (w(u) - b(u)) f^{(2:n)}(u)du$, which is strictly decreasing since $b(u) > w(u)$. We conclude with the assumption that the quasi-revenue (2a) is strictly unimodal.

F Proof of Proposition [5.3]

Denote by $U_i^{com}(x)$ (respectively $W_i^{com}(x)$) the revenue (resp. the welfare) of the second price auction in environment i with commitment and with the participation threshold being equal to x or equivalently with the reserve price $r = w(x)$. Denote by $U_i^{shill}(x)$ (respectively $W_i^{shill}(x)$) the revenue (resp. the welfare) of the second price auction in environment i with shill bids and with the participation threshold x or equivalently with $r = w(x)$ as the lower bound of the support of the shill bidding strategy. We assume that environment 1 suffers from a greater winner's curse that environment 2, i.e $v_1(x) > v_2(x)$ for $x > \underline{x}$.

F.1 Proof of the welfare comparison in Proposition [5.3]

The difference $U_1^{com}(x) - U_2^{com}(x)$ equals to

$$\int_x^{\overline{x}} (v_1(u) - v_2(u))f^{(2:n)}(u)du. \tag{14}$$

This last expression is a strictly decreasing function of x since $(v_1(u) - v_2(u)) > 0$. Consequently the mode of U_1^{com} is strictly inferior to the mode of U_2^{com}. Finally, the probability to sell the object in the optimal second auction in environment 1 is strictly greater than in the optimal second auction in environment 2. The difference between the welfare of the second price auction with commitment in environment 1 and 2 is then equal to:

$$W_1^{com}(x_1^{**}) - W_2^{com}(x_2^{**}) = \int_{x_1^{**}}^{x_2^{**}} w(u)f^{(1:n)}(u)du > 0.$$

The welfare is greater in environment 1 since the seller has a smaller incentive to raise the reserve price.

Now we consider the case without commitment. The probability to sell the object in environment i equals to:

$$\int_{\underline{x}_i^{shill-opt}}^{\overline{x}_i^{shill-opt}} (1 - F_i^{(1:n)}(u))g_i^*(u)du. \tag{15}$$

Indeed $(1 - F_i^{(1:n)}(u))$ is independent of i, strictly positive if $u < \overline{x}$ and is also a decreasing function. Then we prove that the distribution of the types mimicked by the seller in environment 1 is strictly greater than the corresponding distribution in environment 2 in the sense of first order stochastic dominance. More precisely, it is sufficient to prove that $G_1^*(x) \leq G_2^*(x)$ and that the inequality is strict on a positive measure.

From equation (13), the stochastic dominance is equivalent to:

$$\int_x^{\overline{x}_1^{shill-opt}} \frac{v_1(u) - b(u)}{b(u) - w(u)} \cdot \frac{f_u^{(1:n-1)}(u)}{F_u^{(1:n-1)}(u)} \cdot du \geq \int_x^{\overline{x}_2^{shill-opt}} \frac{v_2(u) - b(u)}{b(u) - w(u)} \cdot \frac{f_u^{(1:n-1)}(u)}{F_u^{(1:n-1)}(u)} \cdot du. \tag{16}$$

All terms in the integrand are identical except v_i. So the integrand of the first term is greater than the integrand of the second.

Furthermore, the highest possible type mimicked by the seller is greater in environment 1 than in 2: $\overline{x}_1^{shill-opt} > \overline{x}_2^{shill-opt}$. It comes from the fact that we have $v_2(\overline{x}_1^{shill-opt}) < v_1(\overline{x}_1^{shill-opt}) = b(\overline{x}_1^{shill-opt})$, where b is the solution of equation (7). Thus the map b should cross v_2 for a type strictly smaller than $\overline{x}_1^{shill-opt}$.

Finally, we have proved that equation (16) is true.

Then, without commitment, the probability to sell the object is greater in environment 2 than in 1. The difference between the welfare of the second price auction without commitment in environment 1 and 2 is then equal to:

$$W_1^{shill}(\underline{x}^{shill-opt}) - W_2^{shill}(\underline{x}^{shill-opt}) = \int_{\underline{x}^{shill-opt}}^{\overline{x}_1^{shill-opt}} H(u) \cdot (g_1^*(u) - g_2^*(u))du,$$

where $H(u)$ equals to $\int_u^{\overline{x}} w(s)f^{(1:n)}(s)ds$. Since H is decreasing (the welfare is strictly increasing with the probability of sale) and from the first order stochastic dominance, we obtain that the above difference is positive or equivalently that the welfare is greater in environment 2 than in 1.

After combining the two differences above, we obtain the first two point of Proposition [5.3].

F.2 Proof the revenue comparison in Proposition [5.3]

In order to prove for $x \geq x^*$ that

$$U_1^{com}(x_1^{**}) - U_1^{shill}(x) > U_2^{com}(x_2^{**}) - U_2^{shill}(x), \tag{17}$$

the left-hand (respectively right-hand) term representing the gain of commitment in environment 1 (resp. 2) if the equilibrium with the participation threshold x is played in equilibrium without commitment, it is sufficient to prove the inequality:

$$U_1^{com}(x_2^{**}) - U_1^{shill}(x) > U_2^{com}(x_2^{**}) - U_2^{shill}(\widehat{x}), \tag{18}$$

where $\widehat{x}$ equals to the lower bound of the shill bidding activity of the equilibrium in environment 2 with the initial condition such that the upper bound of the shill bidding activity is equal to the one that will arise in environment 1 with the participation threshold x. In particular, $\widehat{x} > x$.

The inequality (18) immediately implies (17) since:

- $U_1^{com}(x_2^{**}) < U_1^{com}(x_1^{**})$ (from the definition of x_1^{**})
- $\widehat{x} > x$ and $x \to U_2^{shill}(x)$ is decreasing in x for $x > x^*$. It results from the fact that $U_2^{shill}(x)$ can be viewed as the sum of two decreasing function in the range $[x^*, \overline{x}]$: $x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^{\infty} w(u)f^{(2:n)}(u)du$ (due to assumption (2a)) and $\int_x^{\infty} (v(u) - w(u))f^{(2:n)}(u)du$ because $v(u) - w(u) > 0$.

Finally, equation (18) is equivalent to:

$$\int_{x_2^{**}}^{\widehat{x}} (v_1(u) - v_2(u))f^{(2:n)}(u)du > 0 \tag{19}$$

which is true because $v_1 > v_2$ and $\widehat{x} > x_2^{**}$

We conclude by noting that equation (17) with $x = x^*$ corresponds to the third point of Proposition [5.3].

G Proof of Proposition [5.4]

We show that a seller's type which uses the lowest reserve price $\underline{r}^{shill}$ will be better off by committing not to use shill bids and announce the reserve price $r = \underline{r}^{shill}$. In the equilibrium with shill bids, the participation threshold $\underline{x}^{shill}$ is characterized by the equation: $\underline{r}^{shill} = E[V_1|X_1 = x, b_0 = \underline{r}^{shill}, Y_1 \leq x]$. With commitment, the participation threshold $\underline{x}^{shill}_{com}$ is characterized by the equation: $\underline{r}^{shill} = E[V_1|X_1 = x, Y_1 \leq x]$. Assumption (9) for $x = \widetilde{x} = \underline{x}^{shill}$ and the monotonicity of $x \to E[V_1|X_1 = x, Y_1 \leq x]$ implies that $\underline{x}^{shill} \geq \underline{x}^{shill}_{com}$: with commitment, the set of participants does not shrink.

In equilibrium without commitment, the revenue of the seller corresponds to an auction where bidders below signal $\underline{x}^{shill}$ are not participating whereas bidders above $\underline{x}^{shill}$ are bidding according to b such that $b(x)$ is a strict weighted sum of $E[V_1|X_1 = x, b_0 = b(x), Y_1 \leq x]$, the expected value of the item conditional on winning after a tie with the seller and $E[V_1|X_1 = x, b_0 \leq b(x), Y_1 = x]$, the expected value of the item conditional on winning after a tie with another bidder. Due to our assumption in equation (8), this weighted sum is strictly below $E[V_1|X_1 = x, b_0 \leq b(x), Y_1 = x]$. Moreover, from assumption (9) we have $E[V_1|X_1 = x, b_0 \leq b(x), Y_1 = x] \leq E[V_1|X_1 = x, Y_1 = x]$, where $E[V_1|X_1 = x, Y_1 = x]$ corresponds to the strategy of a bidder under commitment. Therefore the seller could have raised a higher revenue by committing to the reserve price $\underline{r}^{shill}$, which would have induced a lower participation threshold and with the commitment pushing them to bid strictly more aggressively, which strictly raises the revenue in the event (occurring with positive probability) where there are two active bidders above $\underline{r}^{shill}$.

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A Supplementary Material : Stochastic reserve price policy

This supplementary material is devoted to the generalization of the previous analysis when the seller is able to use a stochastic reserve price policy.[25] Without shill bids, the derivation of the optimal reserve price policy is untractable. It is an open question whether a mixed reserve price policy, on a support $[\underline{r}, \overline{r}]$, may outperform what we call the optimal auction (with a pure reserve price). On the one hand, according to the same argument as for Proposition [5.4], the seller would have preferred to commit to the reserve price $\underline{r}$ if the reserve price that is chosen by the committed randomization is $\underline{r}$. On the other hand, if the randomization leads to the reserve price $\overline{r}$, then the seller obtains a higher revenue than if she would have committed to the pure reserve price $\overline{r}$ and possibly a higher revenue than any pure reserve price policy. Under a mixed reserve price policy, all types x such that $w(x) \geq \underline{r}$ are participating and bidding according to v. On the contrary, if she commits to the reserve price $\overline{r}$, then only the types x such that $w(x) \geq \overline{r}$ are participating and bidding according to v. Consequently, such an optimal mixed reserve price policy would rely exactly on the same kind of cross subsidizations that occurs in [35]: a low reserve price induces a low revenue but raises the participation which benefits for the high realizations of the reserve price.

Surprisingly, the derivation of the optimal reserve price policy becomes tractable with shill bids. To make the analysis tractable, I assume that the seller is informed about the draw of the stochastic policy before to submit a shill bid. We show that the optimal equilibrium with a stochastic reserve price and shill bids corresponds to the seller's preferred equilibrium derived in proposition [4.5]. Thus this additional instrument does not raise the revenue of the seller.

First note that any shill bidding activity can be replicated by a stochastic reserve price policy. Then, without any loss of generality, we restrict ourselves to equilibria where there is no shill bidding activity. We use the same notation for the stochastic reserve price policy as for the shill bidding activity in our previous analysis. The unique difference concerns the seller's equilibrium conditions which are softened. As before, $U_S^b(x)$ is required to be smaller on $[\underline{x}, \underline{x}^{shill}]$ and $[\overline{x}^{shill}, \overline{x}]$ than on any point on the range $[\underline{x}^{shill}, \overline{x}^{shill}]$. Then $U_S^b(x)$ is only required to be non-increasing on $[\underline{x}^{shill}, \overline{x}^{shill}]$. Otherwise, if there are some types x and x' in $[\underline{x}^{shill}, \overline{x}^{shill}]$ such that $x < x'$ and $U_S^b(x) < U_S^b(x')$, then the seller would find profitable to raise the shill bid $b(x')$ in the event where the reserve price $b(x)$ is drawn. Then at a point where b is differentiable, we have:

[25] I thank Olivier Compte for suggesting the following investigation. Note that in real life auctions, such policies have never been used to the best of my knowledge.

$$(F^{(2:n)}(x) - F^{(1:n)}(x) \cdot b'(x) - f^{(1:n)}(x) \cdot b(x) \leq 0. \qquad (20)$$

The monotonicity of $U_S^b(x)$ also precludes any discontinuity such that $b(x^+) > b(x^-)$ on the range $[\underline{x}^{shill}, \overline{x}^{shill}]$. The monotonicity of b precludes discontinuities such that $b(x^+) < b(x^-)$. Finally, we obtain the Second No-gap lemma. Similarly, the No-gap lemma is still valid in this environment.

An important step in the previous analysis was that only reserve price above r^* are sustainable in equilibrium with shill bids. Lemma (4.3) remains valid.

Lemma A.1 *A necessary condition on $\underline{r}^{shill}$ is: $\underline{r}^{shill} \geq r^*$.*

The proof is the same noting that only (20) matters.

Proof 2 *Suppose that $\underline{x}^{shill} < x^*$. Locally in the right neighborhood of x, the map b is uniquely charaterized by the initial condition (3) and the differential equation (7).*

The map $x \rightarrow (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty w(u) f^{(2:n)}(u)du$ is strictly increasing for $x < x^*$ due to assumption (2a). Then in $x = \underline{x}^{shill}$, we have: $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w'(x) - f^{(1:n)}(x) \cdot w(x) \geq 0 \geq (F^{(2:n)}(x) - F^{(1:n)}(x) \cdot b'(x) - f^{(1:n)}(x) \cdot b(x)$ where $b(x) = w(x)$. We conclude that $b'(x) \leq w'(x)$ in the right neighborhood of x. As a consequence, b is strictly lower than w in the right neighborhood of x which raises a contradiction.

The lemma above shows that stochastic reserve prices do not alleviate the impossibility of a level of trade below x^*. The following result shows that the optimal stochastic reserve price can be implemented with a pure announced reserve price policy in the second price auction with shill bids. In a nutshell, the possibility to use a stochastic reserve price policy strengthens our main result: the loss in term of revenue due to the non-commitment ability can only be greater in an environment with stochastic reserve prices than in the standard case with pure reserve prices.

Proposition A.2 *The optimal equilibrium with a stochastic reserve price policy and shill bids corresponds to the seller's preferred equilibrium among the equilibria with shill bids.*

Proof 3 *With a pure reserve price policy, the lower bound of the support of the shill bidding activity $\underline{x}^{shill}$ characterizes a unique equilibrium with shill bids. Beside, stochastic reserve price policies enlarge the equilibrium set for a given $\underline{x}^{shill}$. However, we show that among those equilibria, the optimal one corresponds to the one that can be implemented with a pure reserve price.*

Denote by b (respectively $U_S^b(x)$ and U_S^b) the bidding strategy (resp. the seller's revenue as a function of the type mimicked by the shill bidding activity *and the expected seller's revenue) in the equilibrium with shill bids and the announced reserve price* $w(\underline{x}^{shill})$*. Denote by* B *(respectively* $U_S^B(x)$ *and* U_S^B*) the bidding strategy (resp. the seller's revenue as a function of the type mimicked by the stochastic reserve price and the expected seller's revenue) in an equilibrium with a stochastic reserve price and with shill bids such that no-shill bids are submitted at equilibrium and that the lowest possible reserve price is* $w(\underline{x}^{shill})$*.*

First, we have $b(x) \geq B(x)$ *for any* $x \geq \underline{x}^{shill}$*. Suppose on the contrary that there exists some* x *such that* $b(x) < B(x)$ *and take* $u = \min_{x \geq \underline{x}^{shill}} \{x | b(x) < B(x)\}$*. Since* b *and* B *are continuous (the second no-gap lemma still holds), we have* $b(u) = B(u)$*. Then since* b *and* B *are satisfying respectively the differential equations (7) and (20), we obtain that* $B'(x) \leq b'(x)$ *in the left neighborhood of* u *which raises a contradiction with the definition of* u*.*

As a consequence, we obtain that $U_S^b(\underline{x}^{shill}) \geq U_S^B(\underline{x}^{shill})$*. Finally, we obtain that* $U_S^b \geq U_S^B$*, because* $U_S^b = U_S^b(\underline{x}^{shill})$ *and* $U_S^B \leq U_S^B(\underline{x}^{shill})$ *since* $U_S^B(\cdot)$ *is non-increasing.*

Figure 1 : Equilibrium with commitment not to use shill bids

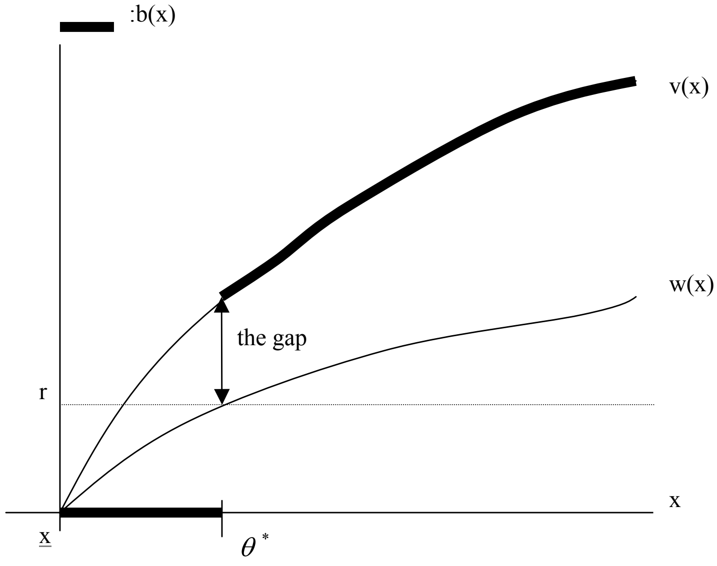
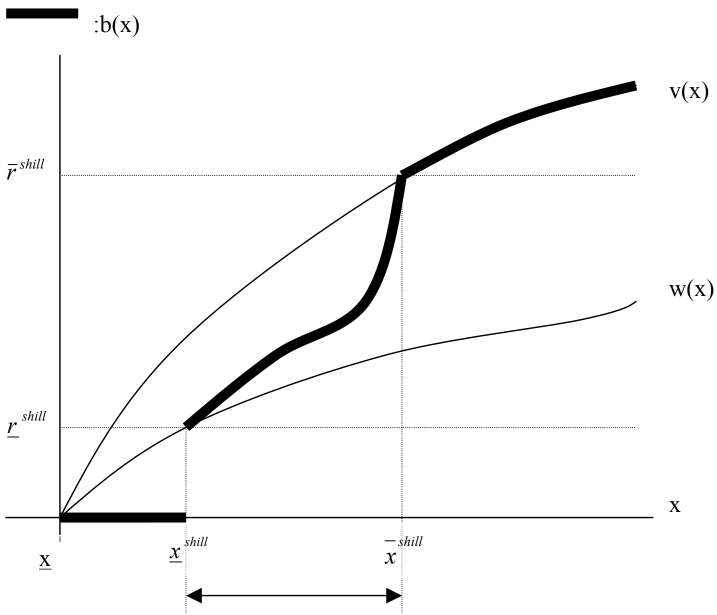
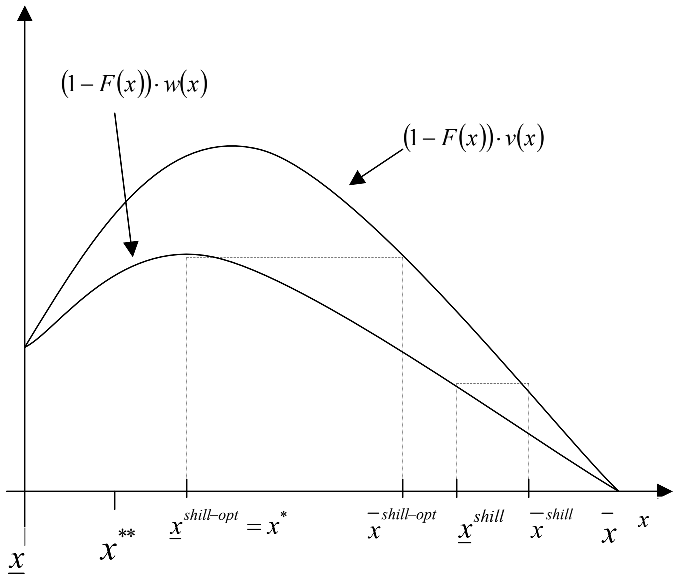


Figure 2 : Equilibrium with shill bidding



Support of the shill bids distributed according to G*.

Figure 3 : Geometrical Resolution of the set of equilibria



Chapitre 2: The Ausubel-Milgrom Proxy Auction with Final Discounts

The Ausubel-Milgrom Proxy Auction with Final Discounts*

Laurent Lamy†

Abstract

We slightly modify the Ausubel-Milgrom [3] Proxy Auction by adding a final stage which possibly induces some discounts relative to the final accepted bids of the 'original' auction. The proxy auction with final discounts is such that the outcome is a point in the bidder optimal frontier of the Core. Then truthful reporting is an equilibrium if and only if the Vickrey outcome is in the Core, a condition that is necessary but not sufficient in the original version of the proxy auction as illustrated by an example.

Keywords: Auctions, multi-unit auctions, Core, Vickrey implementation
JEL classification: D44, D45

1 Introduction

Ausubel and Milgrom [3] (A&M henceforth) introduce an ascending proxy auction which is supposed to combine the advantages from Vickrey insofar as the efficient allocation is implemented in dominant strategy and from first price 'menu' auctions insofar as it is robust to shill bidding and losers' deviation. On the one hand, robustness to losers' deviation is satisfied very generally and follows from the fact that the final outcome of the proxy auction belongs to the Core relative to the reported preferences. On the other hand, truthful reporting is a dominant strategy only if the final outcome is the Vickrey outcome. Since the outcome of the proxy auction lies always in the Core, this condition is satisfied only if the Vickrey outcome lies in the Core. However, A&M derives a stronger sufficient condition: implementation of the efficient allocation in dominant strategy is obtained under a buyer-submodularity condition, which is equivalent to the condition that the

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†Laboratoire d'Economie Industrielle, CREST-INSEE, 28 rue des Saints-Pères 75007 Paris. e-mail: laurent.lamy@ensae.fr

Vickrey outcome lies in the Core for any set of bidders. Generically, it corresponds to the condition that goods are substitutes in assignment problems without allocative externalities.

A natural question raised by A&M's analysis is then to characterize for the ascending proxy auction the full set of preferences that implements the efficient outcome in truthful dominant strategy. The natural candidate are preferences such that the Vickrey outcome is in the Core. However, A&M do not clarify whether this condition is sufficient or not.

First, this note provides an example such that the Vickrey outcome is in the Core but where the Ausubel-Milgrom proxy auction does not lead to the Vickrey outcome. Second the main contribution of this note is to propose a slight modification of the auction by adding a final stage, where the seller awards some discounts to the final prices, such that the mechanism always leads to a bidder-optimal frontier outcome. Those final discounts are a kind of 'Vickreyfication' of the proxy auction giving better incentives for truthful reporting. For this modified auction, and more generally for the class of auctions leading to the bidder-optimal frontier, the set of preferences such that truthful reporting is a Nash Equilibrium is perfectly characterized: the Vickrey outcome must be in the Core. Without ambiguity, the final discount stage is an improvement from A&M perspective: it enlarges the set of bidders' preferences such that truthful reporting is an equilibrium whereas the modified auction keeps the desirable properties resulting from the 'Core membership' of the final outcome. Moreover, the truthful equilibrium of the proxy auction with final discounts can be implemented by a dynamic mechanism leaving some privacy about bidders' valuations. Finally, in the background of our analysis, we clarify the status of the final outcome of the 'original' Ausubel-Milgrom proxy auction: it implements a payoff in the weak bidder-optimal frontier relative to the reported preferences, i.e. some -but not all- bidders' payoff may be raised such that the outcome remains in the Core.

Another strand of the auction literature uses a seemingly different approach to reach dynamically the Vickrey allocation: Clock Auctions [8, 10, 6, 1, 2] are mimicking a Walrasian tâtonnement. Demange, Gale and Sotomayor [8] and Gul and Stacchetti [10] propose dynamic auction mechanisms with differentiated commodities which are converging to the smallest Walrasian prices and truthful reporting is hence an equilibrium if and only if those Walrasian prices coincide with Vickrey's. The Bikhchandani and Ostroy [5]'s linear programming formulation of the assignment model allows de Vries et al [7] to link the two approaches by interpreting the aforementioned clock auctions as a primal-dual algorithm and A&M's auction as a subgradient algorithm.

Ausubel's clock auctions ([1, 2]) are implementing the Vickrey outcome by means of an auctioneer whose announced prices are converging to a Walrasian equilibrium price vector. However, contrary to [8, 10], a 'clinching' rule disconnects the prices that are paid with the closing prices of the auctioneer making truthful reporting an equilibrium for a larger set of preferences. For general valuations and for a larger class of ascending price auctions, Mishra and Parkes [14] generalize the idea of using Walrasian prices to reveal preferences and then to implement the Vickrey payoffs via price discounts. Our final discount stage presents a similarity with such 'clinching' rules: the pricing rule gets closer to Vickrey's. We express our idea in the perspective of A&M's package auction. Nevertheless, as in [14], our discount stage applies more generally, e.g. also for auctions corresponding to primal-dual algorithms.

This note is organized as follows. Section 2 introduces the assignment problem and the related Core concepts. Section 3 defines the algorithm which concisely characterizes the Ausubel-Milgrom proxy auction. The algorithm is the iteration of a mapping whose fixed points are Core outcomes. Section 4 gives an example where the final outcome is not in the bidder-optimal frontier of the Core. Section 5 concludes by proposing the final discount stage modification and characterizes the set of preferences that renders the truthful strategy an equilibrium.

2 Model and notation

There are N buyers (With a slight abuse of notation, N will represent the set as well as the number of buyers) indexed by $l = 1, \ldots, N$ and a seller designated by $l = 0$. For any set of buyers $S \subset N$, denote by S^* the set $S \cup \{0\}$. Denote by M the finite set of indivisible items to be auctioned. We define an allocation as an assignment of the items, denoted by $\mathcal{A} = (\mathcal{A}_0, \mathcal{A}_1, \cdots, \mathcal{A}_N)$ where $\mathcal{A}_l \in M$ specifies the items acquired by agent l. Denote by $\mathbf{A}$ the set of feasible allocations, i.e. such that $\bigcup_{l \in N^*} \mathcal{A}_l = M$ and $\mathcal{A}_l \cap \mathcal{A}_k = \emptyset$. Each buyer l has a valuation vector $\Pi_l = (\Pi_l(\mathcal{A}), \mathcal{A} \in \mathbf{A})$, where $\Pi_l(\mathcal{A}) \geq 0$ specifies the value of allocation $\mathcal{A}$ to bidder l. Denote by $\Pi := (\Pi_1, \ldots, \Pi_N)$ the vector of all buyers' types. As in A&M, we limit and simplify the set of preferences by the following assumptions.

First, we consider a private values framework where each agent is privately informed about his preferences and where one's valuation depends solely on his private signal and not on his opponents' signals. Hence, we exclude any informational externality. Second, we exclude any allocative externality: an agent's valuation on an assignment depends solely on the set of items that he acquires. Third, we consider that the seller is indifferent to the final allocation.[1] Fourth, we consider that a buyer obtains his lowest

[1] Indeed, all the analysis can be extended to a framework with allocative externalities provided that non-purchasers are indifferent to the final assignment, i.e. they can 'escape

valuation when he acquires no item[2]. Fifth, we consider that agents are risk neutral.

To summarize, if bidder $l \in N$ pays a bid $b_l(\mathcal{A})$ such that the allocation $\mathcal{A}$ is chosen, then he earns a net payoff of $\Pi_l(\mathcal{A}) - b_l(\mathcal{A})$, where $\Pi_l(\mathcal{A})$ depends only on $\mathcal{A}_l$. On the other hand the seller's payoff is her revenue $\sum_{l \in N} b_l(\mathcal{A})$. The payoff of a bidder who acquires nothing and pays nothing is thus normalized to zero.

We define a (feasible) outcome as an $N+1$-uple $(\mathcal{A}, (b_l)_{l \in N})$ where $\mathcal{A}$ is the allocation chosen and b_l the price paid by buyer l to the seller. Equivalently, a vector of net payoffs $(\pi_l)_{l \in N^*}$, such that there exits an allocation $\mathcal{A}$ such that $\sum_{l \in N^*} \pi_l = \sum_{l \in N^*} \Pi_l(\mathcal{A})$,will be referred to as a (feasible) outcome. Given a set of players and of preferences, we face an allocation problem $\{N, (\Pi_l)_{l \in N}\}$. Our perspective is to implement the efficient allocation denoted $\mathcal{A}^*$, which maximizes the total welfare, i.e. $\mathcal{A}^* \in Arg\max_{\mathcal{A} \in \mathbf{A}} \{\sum_{l \in N} \Pi_l(\mathcal{A})\}$.

As a useful tool for the following analysis, we first characterize the coalitional form game (N^*, w) associated with the allocation problem $\{N, (\Pi_l)_{l \in N}\}$, where N^* is the set of players and w is the coalitional value function. For any coalition of buyers $S \subset N$, w is defined by the following expression:

$$w(S^*) = \max_{\mathcal{A} \in \mathrm{A}} \sum_{l \in S} \Pi_l(\mathcal{A}) \quad ; \quad w(S) = 0$$

In particular, it means that if the seller is not a member of the coalition, then the coalition obtains no items.

Then we define the set of core outcomes, denoted by $Core(N^*, w)$, related to this coalitional value function w:

$$Core(N^*, w) = \left\{ (\pi_l)_{l \in N^*} \mid (a) : \sum_{l \in N^*} \pi_l = w(N^*); \ (b) : \ (\forall S \subset N^*) \ w(S) \leq \sum_{l \in S} \pi_l \right\}$$

(a) is the feasibility condition meaning that a Core outcome implements the efficient allocation, whereas inequalities (b) mean that the payoffs are not blocked by any coalition S.

Remark 2.1 *The outcome resulting from a transfer of payoffs from a given buyer l to the seller remains in the Core if the initial outcome is in the Core and provided that π_l remains nonnegative. This comes from the fact that inequalities (b) when $0, l \in S$ are not altered, whereas such inequalities with $0 \notin S$ are always satisfied provided that $\pi_l \geq 0$, i.e. the individual*

to the moon' in Jehiel and Moldavonu's [12] terminology, and a non-neutral seller provided that she is not strategic. The generalization of the proxy auction with allocative externalities has been investigated by Ranger [15].

[2] This is a weaker form of the 'free disposal' assumption made in A&M and Ranger [15]. It is indeed sufficient in their analysis.

rationality constraint is satisfied, and that inequalities (b) with $l \notin S$ are only strengthened. In particular, the outcome such that $\pi_0 = w(N^*)$ and $\pi_l = 0$ for all $l = 1 \dots N$ belongs to the Core which is thus non empty.

Another specific outcome is the Vickrey outcome, denoted by $\pi^V := (\pi_l^V)_{l \in N^*}$, such that buyer l's payoff π_l^V equals to $w(N^*) - w(N^* \setminus \{l\})$ and the seller receives the revenue $\pi_0^V = w(N^*) - \sum_{l \in N} \pi_l^V$. A main issue in the analysis of the Ausubel-Milgrom proxy auction is whether the Vickrey outcome is a Core outcome. This is equivalent to the fact that the set of Pareto-optima from the perspective of the buyers is a singleton (A&M theorem 6). This set will be qualified as the bidder-optimal frontier of the core.

Definition 1 *The bidder-optimal frontier (respectively the weak bidder-optimal frontier) of the core is the set containing the elements $(\pi_l)_{0 \leq l \leq N^*} \in Core(N^*, w)$ such that there exists no outcome $(\pi_l')_{0 \leq l \leq N^*} \in Core(N^*, w)$ with $\pi_l' \geq \pi_l$ for all $l = 1 \dots N$ and such that at least one inequality is strict (respectively with $\pi_l' > \pi_l$ for all $l = 1 \dots N$).*

On the one hand, the bidder-optimal frontier is a standard concept in the literature. In particular, Berheim and Whinston [4] have established that the outcomes of the coalitional-proof equilibria of the first price 'menu' auction coincide with the bidder-optimal frontier. On the other hand, we are not aware of any previous work in multi-unit auctions that focuses on the weak bidder-optimal frontier. Nevertheless, we show in next section that, in general, the Ausubel-Milgrom proxy auction ends in the weak bidder-optimal frontier.

A&M has introduced a condition that makes truthful reporting a dominant strategy in the ascending proxy auction: buyer-submodularity.

Definition 2 (Buyer submodularity) *The coalitional value function w is buyer-submodular if for any $l \in N$ and any coalitions S and S' satisfying $l \in S \subset S' \subset N$, we have*

$$w(S^*) - w(S^* \setminus \{l\}) \geq w(S'^*) - w(S'^* \setminus \{l\})$$

The term $w(S^*) - w(S^* \setminus \{l\})$ represents the surplus associated with the presence of bidder l in the coalition S^*. It is a kind of 'substitutes' condition: the bidders should be viewed as substitutes insofar as the surplus associated with the presence of a bidder is non-increasing with the set of competitors.

3 The Ausubel-Milgrom proxy auction

We first define the ϵ-Ausubel-Milgrom proxy auctions ($\epsilon > 0$). The Ausubel-Milgrom proxy auction is the mechanism defined by taking the limit

$\epsilon \to 0$.[3] As in A&M, those sealed bid mechanisms can be viewed as the outcome of the ascending package auction where each bidder l has instructed a 'proxy agent' that bids on his behalf according to a straightforward bidding strategy parameterized by ϵ which reflects the increment used by the bidders to reduce their target profit. Indeed what is below referred to as the Ausubel-Milgrom proxy auction is a specific version of the family of proxy auctions defined by A&M. Considering the whole family would not modify the insights but introduce cumbersome notation. A&M slackens their analysis by considering a family of straightforward bidding strategies. Here we give more structure to the definition of the straightforward strategies used by proxy bidders. In A&M, the parameter ϵ corresponds to the upper bound on the bid increments. Here it reflects the increment used for target profit reductions. Furthermore, A&M restricts their analysis to the limit $\epsilon \to 0$, whereas all our results are valid for any increment ϵ.[4]

Definition 3 *The ϵ-Ausubel-Milgrom proxy auction (A^ϵ, b^ϵ) is the function mapping Π, the vector of reported valuations, into an outcome in $\mathbf{A} \times \mathbb{R}^N$ according to the following algorithm:*

Initialization $a := (\emptyset, \ldots, \emptyset)$; $\widehat{\pi}_l = \max_{\mathcal{A} \in \mathbf{A}} \Pi_l(\mathcal{A})$; $y := 0$
While $y = 0$ **do**
    $y := 1$
    for $l = 1$ *to* N **do**
        if $\Pi_l(a) < \widehat{\pi}_l$
            then $\widehat{\pi}_l := \max\{0, \widehat{\pi}_l - \epsilon\}$; $y := 0$
        $b_l(\mathcal{A}) = \max\{0, \Pi_l(\mathcal{A}) - \widehat{\pi}_l\}$ $(\forall l \in N, \forall \mathcal{A} \in \mathbf{A})$
    [5] $a := Arg\max_{\mathcal{A} \in \mathbf{A}} \left(\sum_{l=1}^{N} b_l(\mathcal{A})\right)$

[3] The outcome of ϵ-generalized ascending proxy auctions lies in the compact set of feasible and individually rational payoffs. So there exists a sequence $(\epsilon_n)_{n \in \mathbb{N}}$ such that the limit exists.

[4] Note also that the ascending proxy auction in A&M is not fully characterized: the final outcome still remains ambiguous (except under the buyer-submodularity condition). In their definition of the proxy auction, they consider the limit where the upper bound of bid increments are negligibly small $\epsilon \to 0$. For example, each bidder could use in its straightforward strategy a different increment ϵ_l such that $\epsilon_l = \lambda_l \cdot \epsilon$. Up to a normalization, each specific choice of $(\lambda_l)_{l=1,\cdots,N}$ defines a specific mechanism at the limit $\epsilon \to 0$. Implicitly, our analysis is restricted to a uniform increment across bidders, i.e. $\lambda_l = 1$. To convince himself that the Ausubel-Milgrom proxy auction is sensitive to the choice of the increments, it is left to the reader to check that, in our example in section 4, if $(\lambda_l)_{l=1,\cdots,N}$ is chosen such that λ_1 is sufficiently smaller than $\lambda_2 = \lambda_3 = \lambda_4$, then the Ausubel-Milgrom proxy auction (without final discounts) would converge to the Vickrey outcome unlike the case with uniform increments.

[5] In the event of a tie between different allocations, we assume at least that the efficient allocation is chosen if available. This tie-breaking rule, that is not made in A&M, guarantees that the algorithm stops exactly when it reaches the Core. This is just a technical trick that guarantees that the weak bidder-optimal frontier are fixed points of the

9 $A^{\epsilon}(\Pi) := a; \quad b_l^{\epsilon}(\Pi) := b_l(a)$

The function mapping $(a, (\widehat{\pi}_l)_{l\in N}) \in \mathbf{A} \times \mathbb{R}^{+N}$ *into itself according to the preceding loop 'while' (lines 2-8) is qualified as the* $T^{\epsilon} - mapping$*. The numbers* $\widehat{\pi}_l$ *are referred to as* target profits*. For a bidder* l*, if* $b_l(\mathcal{A}) = \Pi_l(\mathcal{A}) - \widehat{\pi}_l$*, then allocation* $\mathcal{A}$ *is referred to as a* target allocation *relative to bidder* l*.*

Note that the $N+1$-uples $(a, (\widehat{\pi}_l)_{l\in N})$ are not necessary (feasible) outcomes. That is the reason why we use the 'hatted' $\widehat{\pi}$ for target profits instead of π , which is the notation used for outcomes.

As in A&M, this algorithm could be interpreted as a dynamic auction with the original buyers represented by proxy bidders whose strategies are entirely determined by the reports Π_l to the proxy. We comment then the algorithm in this perspective. At each round of this relating dynamic auction algorithm (i.e. for each iteration of the loop 'while') and for each bidder l, there are two kind of allocations. First, there are the target allocations for which the profit equals to the target profit $\widehat{\pi}_l$ if this allocation is chosen by the seller. The target profit $\widehat{\pi}_l$ also corresponds to the highest profit that is conceivable at the current round for agent l (for any possible continuation of the algorithm). Second, there are the allocations for which the profit is less than the target profit and for which the bids are still null (line 7).

Given the current bids of the proxies, the algorithm chooses a current allocation which maximizes the auctioneer's payoff, i.e. that maximizes the sum of all agents bids (line 8).

The tricky part of the algorithm is the dynamic revision of the bids by the proxies (line 4-7). Two events may arise for bidder l when his is asked to revise his bids. First, the current allocation chosen is a target allocation (if $\Pi_l(a) \geq \widehat{\pi}_l$), then he does not change his bids. Second, if the current allocation is not a target allocation (if $\Pi_l(a) < \widehat{\pi}_l$, line 5), then the buyer reduces his target profit by ϵ (line 6) (provided that his target profit remains positive, else he stops at his null target profit), which roughly corresponds to outbid ϵ on target allocations (line 7). When he reduces his target profit, it possibly corresponds to bid on new allocations: the target allocations sets are increasing sets as the algorithm goes along. For the other allocations the bids are still sticking to zero. Such a strategy is referred to as a *straightforward bidding strategy* in A&M. The algorithm stops when no agent outbids his previous bids (in the algorithm, the binary variable y reflects this information: $y = 1$ meaning that all agents have been inactive in the previous round.).

The rest of this section establishes the link between the final outcome of the proxy auction with Core outcomes. A&M has proved that the proxy

$T^{\epsilon} - mapping$. Otherwise, the algorithm would stop anyway when it reaches the interior of the Core.

auction terminates at a Core outcome which is roughly equivalent to the fact that fixed point of T^{ϵ} are in the Core. Hatfield and Milgrom [11] show that the Ausubel-Milgrom proxy auction is a cumulative offer processes that approaches the Core from above in term of bidders payoffs and ends at the bidder-optimal frontier under a 'substituability' condition. This condition enables them to apply fixed point theorems in lattices: the Core has then a lattice structure implying that the bidder-optimal frontier is a singleton and thus that the Vickrey outcome is in the Core. Nevertheless, the intuition of the tâtonnement from above is not restricted to such 'substitutes' preferences as will be stated in proposition [3.2] where we show that the final outcome always converges to the weak bidder optimal frontier relative to the reported preferences. Before, the following proposition characterizes the Core as the set of fixed points of the map T^{ϵ}, an approach which is similar to Echenique and Oviedo [9].

Proposition 3.1 *The set of fixed points of the $T^{\epsilon}-$ mapping are the Core outcomes.*

Proof 1 *The following lemma provides another expression for the revenue of the seller after the application of the $T^{\epsilon}-$ mapping.*

Lemma 3.1 *The seller maximum revenue at the current allocation a that maximizes her revenue when the vector of target profit is $(\widehat{\pi}_l)_{l \in N}$ is given by:*

$$\widehat{\pi}_0 = \max_{a \in \mathbf{A}} \sum_{l=1}^{N} b_l(a) = \max_{S \subset N} \left\{ w(S^*) - \sum_{l \in S} \widehat{\pi}_l \right\} \tag{1}$$

Proof 2 *This is a careful rewriting by reversing the order of the 'max' operator. For more details, refer to A&M pp. 20. The seller's maximization program can be viewed as two stages: first choose the set of buyers S that will pay a strictly positive amount and obtain consequently a target allocation, second given this set of buyers choose the allocation that maximizes their contribution. For the target allocations of buyers in S there is no distortion between the bids and the preferences (they are all equal up to a constant) and the second stage then consists in choosing the allocation they jointly prefer, i.e. corresponding to the surplus $w(S^*)$.*

Suppose that $(a, (\widehat{\pi}_l)_{l \in N})$ is a fixed point of the $T^{\epsilon}-$ mapping. For any fixed point, each bidder obtains his target profit, i.e. $b_l(a) = \Pi_l(a) - \widehat{\pi}_l$. Hence, for any other allocation a', we have:

$$\sum_{l \in N} \Pi_l(a) - \sum_{l \in N} \widehat{\pi}_l = \sum_{l \in N} b_l(a) \geq \sum_{l \in N} b_l(a') \geq \sum_{l \in N} \Pi_l(a') - \sum_{l \in N} \widehat{\pi}_l$$

The first inequality uses the definition of allocation a which maximizes the seller's revenue. The second inequality follows from the bidding strategy that *corresponds to the target profits. The inequality between the two extreme implies that allocation a is the efficient allocation and the feasibility constraint is then satisfied. At a fixed point, target profit equals realized profit and equation (1) is satisfied with $\widehat{\pi}_l = \pi_l$ and the blocking constraints of the Core are thus fulfilled. Finally we have proved that fixed points of T^ϵ are Core outcomes.*

Now suppose that $(a, (\widehat{\pi}_l)_{l\in N})$ is a Core outcome. Then we have both: $\sum_{l\in N} b_l(a) = w(N^*) - \sum_{l\in N} \widehat{\pi}_l$ and $\sum_{l\in N} b_l(a) \geq w(S^*) - \sum_{l\in S} \widehat{\pi}_l$. Finally, the maximum in equation (1) is reached for the coalition N, i.e. the entire set of bidders receive their target profit. Hence, due to our tie-breaking rule restriction (see footnote 5), the efficient allocation is chosen and each agent obtains his target profit. Finally, it means that no agent reduces his target profit and then that the outcome is a fixed point

The ϵ-Ausubel-Milgrom proxy auction corresponds to the iteration of the mapping T^ϵ with the initial target profits being equal to $\max_{\mathcal{A}} \Pi_l(\mathcal{A})$ for each bidder l (line 1). For any increment ϵ, the target profit of each bidder is decreasing along the path of the algorithm and is also decreasing from less than ϵ at each step. Therefore, when the algorithm stops, target profits are distant of at most ϵ from the weak bidder-optimal frontier. We say that the final outcome $\epsilon -$ *approximates* the weak bidder-optimal frontier according to the following definition.

Definition 4 *An outcome $(a, (\pi_l)_{l\in N})$ is said to $\epsilon -$ approximate a set K if there is an outcome $(a', (\pi'_l)_{l\in N})$ in K such that $a = a'$ and $|\pi_l - \pi'_l| \leq \epsilon$ for any $l \in N$.*

Proposition 3.2 *In the ϵ-Ausubel-Milgrom proxy auction, given the reported preferences, the final outcome $\epsilon-$approximates the weak bidder optimal frontier of the Core. Consequently, the Ausubel-Milgrom proxy auction ends in the weak bidder-optimal frontier.*

Proof 3 *Suppose that the final outcome $(a, (\pi_l)_{l\in N})$ does not $\epsilon-$approximates the weak bidder optimal frontier of the Core. Suppose additionally that the outcome $(a, (\pi_l + \epsilon)_{l\in N})$ is not in the Core. From standard convex analysis (see in Rockafellar [16]), the Core is a polyhedral convex set and there exists a hyperplane separating the Core and the singleton $(a, (\pi_l + \epsilon)_{l\in N})$. Thus there is a point in the interval $[(\pi_l)_{l\in N^*}, (\pi_l + \epsilon)_{l\in N^*}]$ which belongs to the bidder optimal frontier raising a contradiction with $(a, (\pi_l)_{l\in N})$ $\epsilon-$approximating the weak bidder optimal frontier. Finally we have proved that the outcome $(a, (\pi_l + \epsilon)_{l\in N})$ is in the Core. Then as pointed by remark [2.1], the whole cube $\{(a, (x_l)_{l\in N}) | \pi_l \leq x_l \leq \pi_l + \epsilon\}$ is included in the Core. It means that, in the previous round of the ϵ-Ausubel-Milgrom proxy auction, the state of the algorithm was necessary in this cube, which raises a contradiction with proposition [3.1] which states that the algorithm stops when it reaches a Core outcome.*

4 An Illustrative Example

The following example with 4 bidders and 2 identical items for sale without externalities has several aims.[6] First, it illustrates the dynamics of the algorithm. Second, it gives an example where the final outcome is not in the bidder-optimal frontier but only in the weak bidder-optimal frontier of the Core. Third, it illustrates our proposal to add a final discount stage to the Ausubel-Milgrom proxy auction. Thanks to the modification, truthful reporting is an equilibrium which leads to the single-valued bidder-optimal frontier, hence the Vickrey outcome. On the contrary, thruthful reporting is not an equilibrium in the 'original' proxy auction in this example.

Bidder 1 is valuing 100 the first item and 0 an additional item, bidder 2 and 3 are identical and are valuing any additional item 60. For the moment, the bidders have substitutes preferences. Let us introduce an additional bidder 4 who has complement preferences: he values the bundle of the two items 100, but values 0 a single item. Bidder 4 suffers from complementarity. Note that bidder 4 is neutral from a Vickrey implementation point of view: he does not change the efficient allocation which is to assign the items either to the couple $\{1,2\}$ or to $\{1,3\}$ and to make both purchasers pay the amount of 60. Bidder 4 is also neutral from a Core allocation point of view: he does not modify the structure of the Core.

$$Core(\{1,2,3,4\}) = \left\{ (\widehat{\pi}_l)_{0\leq l\leq N} \mid \sum_{l=0}^{N} \widehat{\pi}_l = 160; \quad \pi_1 \in [0,40] \ ; \ \pi_2 = \pi_3 = \pi_4 = 0 \right\}$$

If bidder 4 were absent, then we could apply A&M's results since the buyer-submodularity would be satisfied. Consequently, the Vickrey outcome would be in the Core and truthful reporting would be a Nash equilibrium in the proxy auction. Nevertheless, the mere presence of bidder 4 disturbs the dynamics of the auction. The final outcome is no longer the Vickrey outcome.

Let us detail a little the dynamic of the auction which could be decomposed into three distinct stages which correspond to modifications in the target allocation set for some agents. Just to fix ideas, consider that the bid increment ϵ equals to 1 (but any smaller increment does not modify the insights). Moreover, in case of ties (line 8 of the algorithm), say that the

[6] Four is the minimal number of bidders such that Vickrey is in the Core (relative to the whole coalition) and w is not buyer-submodular. The buyer submodularity condition is equivalent to Vickrey being in the Core for each possible coalition (A&M theorem 7). Then for less than two bidders, the equivalence between buyer-submodularity and Vickrey is the Core is tautologic. For three bidders, Vickrey in the Core implies $w(N^*) - w(N^* \setminus \{i\}) \leq w(N^* \setminus \{j\}) - w(N^* \setminus \{i,j\})$ (otherwise $N^* \setminus \{i,j\}$ would be a blocking coalition). Given that $w(\{0\}) = 0$, the remaining inequalities to obtain the buyer-submodularity are of the kind: $w(\{i,j\}^*) \leq w(\{i\}^*) + w(\{j\}^*)$. Those inequalities are always satisfied without externalities. Thus, for three bidders, Vickrey in the Core implies w buyer-submodular. Obviously, two is also the minimal number of items for w not being buyer-submodular without externalities.

allocations which favour the bidders with the smaller indexes are chosen. To simplify the presentation, we just focus on five possible assignments: the optimal assignments $\{1,2\}$ and $\{1,3\}$, and the assignments $\{2\}$, $\{3\}$, $\{4\}$ which give both items to the related single bidder. Indeed those are the only relevant assignments in the auction algorithm. Other assignments, such as $\{1\}$ or $\{2,3\}$, are omitted in the following analysis of the target allocation and received bids.

Stage 1: from round 1 to round 80 Initially, the target allocations of bidder 1 are $\{1,2\}$ and $\{1,3\}$. The target allocation of bidder 2 (respectively 3 and 4) is $\{2\}$ (respectively $\{3\}$ and $\{4\}$). Each bidder adds the bid increment 1 on his target allocations except when the chosen allocation is one of those target allocations, i.e. three times out of four he actively bids whereas one time out of four his target allocation is chosen. After round 80, the submitted bids are those shown in table 1.

	{1,2}	{1,3}	{2}	{3}	{4}
1	60	60	0	0	0
2	0	0	60	0	0
3	0	0	0	60	0
4	0	0	0	0	60

Table 1: Bids after round 80

Stage 2: from round 81 to round 130 Now there is a change in the target allocation set of bidders 2 (respectively 3): he is willing to bid for assignment $\{1,2\}$ (respectively $\{1,3\}$). Note that at the beginning of that stage, bidder 1 has reached his Vickrey payoff and that consequently, if he were now strategic, he should better not raise any incremental bid. Nevertheless his proxy bidder will do so. The reason is that he outbids when $\{4\}$ is the selected assignment. The precise dynamic of the selected allocation follows a five period-cycle. Table 2 follows the five first rounds of this second stage. For each round and for each (relevant) assignment, the first row reports the set of agents that is raising an incremental bid on this assignment. For example, at round 83, bidder 1 is raising his submitted bids for assignment $\{1,2\}$ and $\{1,3\}$, whereas bidder 2 (respectively 3) is raising his bid on allocation $\{1,2\}$ (respectively $\{1,3\}$). The second row reports for each assignment the sum of submitted bids and the last column the revenue maximizing assignment that is selected. For example, at round 82, the allocation $\{4\}$ is the single revenue maximizing allocation and is therefore selected.

Then this stage ends at round 130 where the corresponding bids are reported in table 3. The striking point is that bidder 1 has continued to participate actively in the mechanism and has overbidden 10 above the Vickrey

Round	{1,2}	{1,3}	{4}	Selected
81		3	4	
	60	61	61	{1,3}
82	2		4	
	61	61	62	{4}
83	1,2	1,3		
	63	63	62	{1,2}
84		3	4	
	63	64	63	{1,3}
84	2		4	
	64	64	64	{1,2}

Table 2: Incremental Bids for rounds 81-85

outcome bid. He has overbidden as if bidder 4 were a 'serious' opponent against which he should fight.

	{1,2}	{1,3}	{2}	{3}	{4}
1	70	70	0	0	0
2	30	0	90	0	0
3	0	30	0	90	0
4	0	0	0	0	100

Table 3: Bids after round 130

Stage 3: from round 131 to round 170 Now bidder 4's target profit is null and he quits the auction. One of bidder 1's target allocation is always selected such that from now on, bidder 1 does not raise bids anymore. The auction terminate in a 'duel' between agent 2 and 3. The final assignment is either $\{1, 2\}$ or $\{1, 3\}$. The final revenue is 130. All bids are reported in table 4. Note that if bidder 1 were able to reduce slightly his submitted bids, he would do so without modifying the payoffs of the other bidders but only reducing the revenue of the seller.

	{1,2}	{1,3}	{2}	{3}	{4}
1	70	70	0	0	0
2	60	0	120	0	0
3	0	60	0	120	0
4	0	0	0	0	100

Table 4: Bids after round 170

This suggest to add a stage to the generalized proxy auction where the auctioneer reduce incrementally the bid of some bidders such that the remaining allocation still stay in the Core. This stage will be referred to as the final discount stage. In our case, it is clear that such a discount for the winner among 2 and 3 is impossible since the payoffs would be driven out of the Core. On the other hand, the final discount stage will imply a reduction in bidder 1's price: he will pay only 60. Indeed, 60 corresponds to the amount that he should pay to internalize the externality imposed on his opponents. Somehow clumsily, he has bid above 60 because bidder 1's proxy, at the beginning, bid as if he should internalize the externality imposed only on his opponents $\{4\}$, an externality which is stronger than the one he imposes on the bigger set of opponents $\{2, 3, 4\}$. This is exactly those events that the buyer-submodularity condition avoids.

	{1,2}	{1,3}	{2}	{3}	{4}
1	60^+	$60^{\ +}$	0	0	0
2	60	0	120	0	0
3	0	60	0	120	0
4	0	0	0	0	100

Table 5: Bids after the discount to bidder 1

Indeed, it can be proved that the payoff of bidder l in the Ausubel-Milgrom proxy auction is at least $\min_{S \subset N} w(S^*) - w(S^* \setminus l)$.[7] If $\min_{S \subset N} w(S^*) - w(S^* \setminus l) \geq \pi_l^V$, then truthful reporting leads to the Vickrey payoffs and is thus a best response for bidder l. The buyer-submodularity condition is a stronger one. In our example, we have chosen precisely that $\min_{S \subset N} w(S^*) - w(S^* \setminus l) = w(\{1, 4\}) - w(\{4\}) = 0 < \pi_l^V = 40$.

Next section defines properly the final discount stage. Unambiguously, such an additional stage gives better incentives for truthful reporting.

5 The Ausubel-Milgrom generalized proxy auction with final discounts

The final discount stage that we propose for the ϵ-Ausubel-Milgrom proxy auction intervenes when the previous mechanism has stopped: this stage does not modify the final allocation but only the prices that will be paid at the end. Instead of the final accepted bids for the final allocation, the bidders will pay final discounted bids. Iteratively, for all winning bidders, bids on all allocations are uniformly discounted provided that the final allocation remains the revenue maximizing allocation. It should be emphasized that the

[7]It is left to the reader to check that it is proved implicitly in the proof of A&M's Theorem 8.

final discount stage is defined only from the final allocation and the entire set of submitted bids in the last round. Thus it is defined without any reference to the reported preferences to the proxy bidder. Hence, such a discount stage could be implemented in the dynamic version of the ascending package auction. Actually, the Ausubel-Milgrom generalized proxy auction with final discounts falls within the class of ascending price auctions introduced by Mishra and Parkes [14].

Definition 5 (The Final Discount Stage) *The final discount stage is the function mapping* $(\mathcal{A}^*, (b_l(\mathcal{A}))_{1\leq l\leq N, \mathcal{A}\in\mathbf{A}}$*, i.e. the final allocation* $\mathcal{A}^*$ *and the entire set of submitted bids, into the outcome* $(\mathcal{A}^*, (b_l(\mathcal{A}^*))_{1\leq l\leq N}$ *where the final accepted bids are discounted according to the following algorithm:*

for $l = 1$ *to* N **do**
  if $\mathcal{A}_l^* \neq \emptyset$
    then do $b_l(\mathcal{A}) := \max\{0, b_l(\mathcal{A}) - d\}$
    where d *is the largest number such that*
    $\mathcal{A}^* \in Arg\max_{\mathcal{A}\in\mathbf{A}}\left(\max\{0, b_l(\mathcal{A}) - d\} + \sum_{k=1,k\neq l}^{N} b_k(\mathcal{A})\right)$

Remark 5.1 *The final discount stage has not been presented in a symmetric way: a bidder may find strictly advantageous to be selected at the first iterations of the discount stage. A proper randomization makes the discount stage symmetric.*

The final discount stage could also be interpreted relative to the target profits: the discount d on submitted bids of bidder l is equivalent to the increase in d of the target profit of bidder l. After a discount, the efficient allocation must remain the profit maximizing allocation: in the perspective of lemma [3.1], it means that $w(N^*) - \sum_{l\in N}\pi_l = \max_{S\subset N} w(S^*) - \sum_{l\in S}\pi_l$.

The preceding definition is illustrated in Figure [1a] and [1b]. Those figures show the dynamic of the Ausubel-Milgrom proxy auction in a two dimensional target profit space of two selected bidders [8]. Dashed lines depict those dynamics starting from the initial target profit π^{Max} where each bidder obtains his most preferred allocation with a null bid and stoping at a point in the weak bidder-optimal frontier. In both cases, the final outcome of the Ausubel-Milgrom proxy auction is not in the bidder-optimal frontier and the discount stage is an active stage illustrating how the proposal concretely modify the outcome of the action. Fig [1a] is such that the Vickrey payoff is in the Core. In this case, the 'original' proxy auction does not implement truthfully the efficient assignement whereas the proxy auction with final discounts does.

[8] With only two bidders, w is always buyer-submodular and the discounts have no effect on the outcomes since the Vickrey outcome is directly implemented. From 3 bidders, the discounts can possibly modify the final outcome: consider section 4's example and suppress bidder 3 as an example.

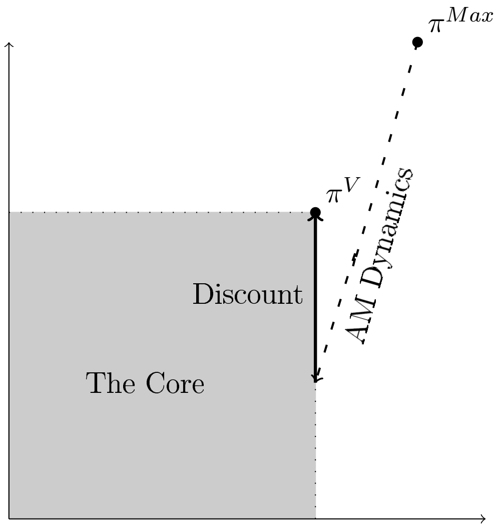

Fig 1a: Vickrey in the Core

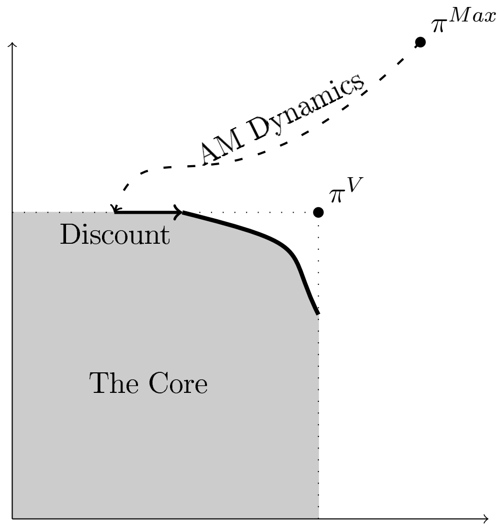

Fig 1b: Vickrey outside the Core

We then prove that, from any point in the Core, the final discount stage leads to a point in the bidder-optimal frontier. This is proved in two steps. First, a bid discount iteration is such that the payoffs remain in the Core. Second, we show that the final payoffs $(\pi_l)_{l \in N}$ do not lie below the bidder-optimal frontier. Otherwise, there is a bidder l such that his bids could be discounted until a target profit $\pi_l^* > \pi_l$ and such that the final allocation is still the revenue maximizing one. Then, when it was bidder l's turn in the final discount stage algorithm, bidder l's target profit should have been raised until π_l^*, which raises a contradiction since it has been raised only until π_l. Since the final allocation of the original ϵ-Ausubel-Milgrom proxy auction is in the Core, the final discount stage applied to the ϵ-Ausubel-Milgrom proxy auction leads to an outcome in the bidder-optimal frontier as stated in the following proposition.

Proposition 5.1 *In the ϵ-Ausubel-Milgrom proxy auction with final discounts, given the reported preferences, the final outcome is in the bidder optimal frontier of the Core.*

Proof 4 *It remains to prove the two aforementioned steps. A bid discount is just a transfer between the seller and one bidder. Both outcomes share the same final allocation $\mathcal{A}^*$. Consequently, condition (a) in the definition of the Core remains unchanged. Moreover the discounts are such that $\mathcal{A}^*$ remains the profit maximizing allocation relative to the target profit, then due to lemma [3.1], inequalities (b) are still satisfied. In a nutshell, the outcome after a discount remains in the Core provided that the initial allocation is in the Core.*
Then it remains to prove that if the final outcome is strictly below the bidder-optimal frontier, or equivalently if the bids of a bidder, say l, could be dis-

counted until a target profit π_l^*, then it should have been discounted previously until that level in the algorithm, which raises a contradiction. Denote by $(\pi_l)_{l\in N}$ (respectively $(\pi_l')_{l\in N}$) the target profits at the end of the final discount stage (respectively just before bidder l's turn in the final discount algorithm). For both target profit vectors, $\mathcal{A}^*$ is the profit maximizing allocation. Moreover, $\pi_l \geq \pi_l'$ since the continuation of the algorithm involves only discounts in bids, or equivalently increases in the target profits. We have assumed above that the target profit vector $(\pi_1, \cdots, \pi_{l-1}, \pi_l^*, \pi_{l+1}, \cdots, \pi_N)$ is such that $\mathcal{A}^*$ is still profit maximizing:[9]

$w(N^*) - \sum_{k\in N, k\neq l} \pi_k - \pi_l^* = \max_{S\subset N}\left\{w(S^*) - \sum_{k\in S,k\neq l}\pi_k - \pi_l^* \cdot I[l\in S]\right\}$.

Since $\pi_l \geq \pi_l'$, we have:

$\max_{S\subset N}\left\{w(S^*) - \sum_{k\in S,k\neq l}\pi_k - \pi_l^* \cdot I[l\in S]\right\} + \sum_{k\in N,k\neq l}\pi_k \geq$
$\max_{S\subset N}\left\{w(S^*) - \sum_{k\in S,k\neq l}\pi_k' - \pi_l^* \cdot I[l\in S]\right\} + \sum_{k\in N,k\neq l}\pi_k'$.

We conclude that:

$w(N^*) - \sum_{k\in N,k\neq l}\pi_k' - \pi_l^* \cdot I[l\in S] \geq \max_{S\subset N}\left\{w(S^*) - \sum_{k\in S,k\neq l}\pi_k' - \pi_l^* \cdot I[l\in S]\right\}$.

Consequently, for the target profit vector $(\pi_1', \cdots, \pi_{l-1}', \pi_l^*, \pi_{l+1}', \cdots, \pi_N')$, $\mathcal{A}^*$ is the profit maximizing allocation. Thus we have raised a contradiction with the course of the final discount stage.

If the Vickrey outcome is in the Core, then the ϵ-Ausubel-Milgrom proxy auction with final discounts implements the Vickrey outcome under truthful reporting since the bidder-optimal frontier is then a singleton coinciding with the Vickrey payoffs (see A&M theorem 6). Hence truthful reporting is a Nash equilibrium strategy profile if the Vickrey outcome is in the Core. The converse statement results from the fact that if his opponents are truthful, a bidder can guarantee himself his Vickrey outcome by one of his best response's report: it corresponds to report the Vickrey profit as a target profit. Thus we have proved the following corollary which is indeed true for any mechanism that implements an outcome in the bidder-optimal frontier relative to the reported preferences.

Corollary 5.2 *Truthful reporting is a Nash equilibrium strategy profile if and only if the Vickrey outcome is in the Core. Then the $\epsilon-$Ausubel-Milgrom proxy auction with final discounts leads to the Vickrey outcome.*

In A&M, the truthful Nash Equilibrium is obtained for the mechanism by taking the limit $\epsilon \to 0$. Otherwise, the mechanism ends generically in the interior of the Core thus not at the Vickrey outcome. On the other hand, note that our corresponding result for the proxy auction with final discounts is true for any increment ϵ.

More generally, even if the Vickrey outcome is not in Core, then the final stage brings the final outcome unambiguously closer to the Vickrey

[9]Denote by $I[l \in S]$ the indicator function equal to 1 if $l \in S$ and else equal to 0.

outcome. According to Milgrom's [13] terminology, the Ausubel-Milgrom proxy auction with final discounts is a core-selecting auction that provides optimal incentives.

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Chapitre 3: Contingent Auctions with Allocative Externalities: Vickrey versus the Ausubel-Milgrom Proxy Auction

Contingent Auctions with Allocative Externalities: Vickrey versus the Ausubel-Milgrom Proxy Auction*

Laurent Lamy†

Abstract

We introduce contingent auction mechanisms, which is a superset of combinatorial auctions, and where bidders submit bids on packages that are contingent on the whole final assignment. Without externalities, the Vickrey and the Ausubel-Milgrom Proxy Auction are both robust if items are perceived as substitutes. Such an equivalence between those formats may not hold with externalities and the analog of the substitute condition is a complex unexplored issue. We analyse those issues in the Negative Group-Dependent Externalities framework, a general structure with allocative externalities between joint-purchasers.

Keywords: Auctions, combinatorial bidding, contingent bidding, allocative externalities, identity-dependent externalities
JEL classification: D44, D45, D62, L90

1 Introduction

For the single item assignment problem in a pure private value framework without externalities, the Vickrey-Clarke-Groves mechanism, also briefly referred to as the Vickrey auction, corresponds to the second price auction. Nevertheless this format is not popular in 'real life' auctions and practitioners prefer the use of the English auction. However, from auction theory's perspective, those formats are roughly equivalent: the efficient allocation, with the price for the purchaser being the highest valuation of his opponents, is implemented in dominant strategy. Several arguments have been developed to explain why the Vickrey auction is so rare, e.g. due to future

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†Laboratoire d'Economie Industrielle, CREST-INSEE, 28 rue des Saints-Pères 75007 Paris. e-mail: laurent.lamy@ensae.fr

interactions, the winner may be reluctant to reveal truthfully his preferences as in Rothkopf et al [36].[1]

Similarly, the Vickrey auction is not used for multi-object assignment problems although a fully general ascending counterpart of the Vickrey auction (which implements the efficient equilibria in dominant strategy) is not available when bidders' valuations go beyond the gross substitutes condition.[2] Nevertheless, auction theorists are more reserved about the relevance of the Vickrey auction in this case insofar as it may fail to be robust against some joint deviations and more specifically against losers' deviations.[3] This failure is related to the fact that, in general, the Vickrey payoffs do not belong to the Core as it is formalized in Proposition 2.4 where the connection between the Core and the robustness against losers' deviations is established. On the contrary, spurred by the success of the Simultaneous Ascending Auction (see Milgrom [28]) used for FCC's spectrum auctions, new ascending multi-object auction formats have been proposed as new tools for practical market design. Among them, many Clock auctions, where the seller mimics a fictitious Walrasian auctioneer, have been proposed ([2], [3], [12], [14], [6]). In those mechanisms, bidders are explicitly restricted to report 'substitutes' preferences, i.e. decreasing marginal utilities if a homogenous good is auctioned.

Ausubel and Milgrom (hereafter AM) have recently proposed an Ascending Auction with Package Bidding, where bidders can report general valuations including complementary preferences, which is converging to a Core outcome according to the reported preferences.[4] A question of primary interest is then to delimit the set of preferences which makes the desired efficient outcome an equilibrium. The Ausubel-Milgrom proxy auction implements the Vickrey payoffs in dominant strategy if the coalitional value function of the related coalitional game is satisfying a so-called bidder-submodularity condition. With this appropriate restriction on the whole set of preferences,

[1]Kagel, Harstad and Levin [15] experimentations also show that unexperienced subjects converge quicker to the (dominant strategy) Nash Equilibrium in the English auction. Compte and Jehiel [8] argues that ascending auctions with a slow pace are beneficial according to an information acquisition perspective.

[2]De Vries et al [11] derive an impossibility result for an ascending auction to implement the Vikrey outcome when at least one buyer's preferences fail the gross substitutes condition. Relaxing the requirement that the final prices in the auction define the payments of the buyers, Mishra and Parkes [31]

[3]Another potential failure of the Vickrey auction, investigated by Yokoo et al. [39], is its lack of robustness against shill bidding: the use of multiple identifiers by a single bidder. In an IO perspective with non-anonymous mechanisms, this failure seems less relevant. In this paper, it is therefore not discussed. Nevertheless, as we characterize the set of preferences such that the bidder-submodularity holds, then Yokoo et al.'s results can be applied.

[4]Closely related are the dynamic matching mechanisms following Gale and Shapley's algorithm, which also have important practical implications. Recently, Hatfield and Milgrom [16] unify those two literatures (without allocative externalities).

Vickrey payoffs are in the Core and correspond to the bidder optimal frontier which is single-valued in that case. Also referred to as "Buyers are substitutes", this condition is central in the combinatorial auction literature, e.g. see the unifying approach of de Vries et al [11] and Mishra and Parkes [31]. However, in the perspective of applications, we are more interested on the restrictions that should be put on bidders' valuations, i.e. the primitives of our assignment model. Without externalities, AM links the abstract bidder-submodularity condition with individual bidders preferences, namely bidder-submodularity is linked with the mutual substitutes condition for goods. With allocative externalities, Ranger [34, 33] introduces a contingent version of the AM-proxy auction and exhibits an Industrial Organisation example, where the bidder-submodularity condition is satisfied: the allocation of capacities before a downstream Cournot market.

On the whole, this literature on multi-unit auctions has not derived a clear-cut argument against the Vickrey auction beside any dynamic substitute because the Vickrey auction is actually robust exactly in the case where those alternative formats implement the efficient allocation in dominant strategy. Without externalities, the Vickrey auction is robust against losers' deviation and shill bidding exactly in the case where items are perceived as substitutes.

The main contribution of this paper is to give somehow the analog of the 'substitutes' condition in a framework with externalities. For example, in the unit-demand case, this condition is straightforward without externalities: the bidder-submodularity condition is always satisfied, hence the substitutes condition always holds. But even in the unit-demand case, the characterization of the preferences such that the bidder-submodularity condition is satisfied is difficult if allocative externalities are allowed. In the so-called Negative Group-Dependent Externalities framework (briefly referred to as the NGDE framework), the set of bidders' preferences such the bidder-submodularity condition holds is characterized. At the same time, in this framework, this paper sheds some light on the differences between the Vickrey and Ausubel-Milgrom proxy auctions: the robustness of the former requires weaker conditions on the primitives of the model but it requires that the designer is able to restrict the set of the reported vectors of joint preferences. This point brings us to a point that has not been emphasized in the previous literature: for the Ausubel-Milgrom proxy auction to be robust, there is no need to restrict the set of possible reported valuations, whereas, in the Vickrey auction, it is a main issue which could undermine it.

The second contribution of this paper is to introduce a general framework with allocative externalities which could be a better description of many assignment problems, e.g. those that have been previously considered in the experimental literature on combinatorial auctions. In a nutshell, the bidders in the NGDE framework are partitioned into groups such that the more joint-purchasers into their group, the less they are valuing items. The terminology 'group-dependent' comes from the fact that the externalities from which a bidder suffers are confined to the final market structure of his own group. The terminology 'negative externalities' comes from the fact that a bidder prefers that few competing bidders of his group acquire items.

The paper is also linked to the auction literature with externalities. One strand, beginning with Milgrom and Weber [30], considers informational externalities. Efficient ex-post robust mechanisms have been derived in Dasgupta and Maskin [10] and Perry and Reny [32]. In generic cases with interdependent valuations and multidimensional signals, Jehiel and Moldovanu [21] and Jehiel et al [18] derive an impossibility result respectively for the Bayes-Nash implementation of the efficient allocation and for the ex post implementation of nontrivial mechanisms. Another strand considers allocative externalities. Identity-dependent externalities in single item environments have been analyzed at first (Jehiel and Moldovanu ([19],[21]), Jehiel, Moldovanu and Stacchetti ([22], [23]), Das Varma [9]). If bidders are not indifferent to the identity of the opponent that has purchased the item, standard auction designs may fail to be efficient and the Core may be empty. In an optimal design perspective with one-dimensional types, Figueroa and Skreta [13] consider a multi-unit assignment issue with both informational and allocative identity-dependent externalities. Here the focus is on dominant strategy implementation in a multi-unit environment with identity-dependent allocative externalities while keeping the private valuations assumption.

This paper is organized as follows: Section 2 introduces the general assignment problem with allocative externalities between joint-purchasers and the related contingent mechanisms. In the same way as standard auctions appear to be inadequate when a bidder values packages of objects and that combinatorial auctions, where bids are made on bundles of objects, may solve inefficiencies, contingent auctions are allowing bidders to express the full dimensions of their valuation vector. As for combinatorial auctions, the issue is the robustness of such mechanisms. We first adapt the results form the literature to our contingent environment. In Section 3, we introduce the NGDE framework and discusses its practical relevance. The main results of the paper appear in sections 4 and 5 which are respectively characterizing the conditions for the Vickrey auction and Ausubel-Milgrom proxy auctions to be robust in the NGDE framework. Section 6 considers the extension where the auctioneer is not only a pure revenue maximizer but also has preferences on the final allocation. Section 7 concludes with a policy perspective. The proofs of our two main results are gathered in the Appendix.

2 Contingent Auctions

2.1 The general assignment problem: definitions and notation

There are N risk neutral bidders (With a slight abuse of notation, N will represent the set as well as the number of bidders) indexed by $l = 1, \ldots, N$ and a seller designated by $l = 0$. There are $M > 1$ indivisible items to be auctioned. We define an allocation, denoted by $\mathcal{A} = \{\mathcal{A}_1, \cdots, \mathcal{A}_M\}$ where $\mathcal{A}_i \in N \cup \{0\}$ denotes the identity of the purchaser of item i, as an assignment of the M items to the bidders (some items could remain in the seller's hands). Denote by $\mathbf{A} := ([1, N] \cup \{0\})^M$ the set of feasible allocations. For a given allocation $\mathcal{A}$, denote by $\mathcal{A}^l$ the set of items assigned to bidder l and $\#\mathcal{A}$ the total number of items that are sold. For any couple of bidders (l, m) and an allocation $\mathcal{A}$, denote by $\mathcal{A}(l \curvearrowright m)$ the allocation resulting from allocation $\mathcal{A}$ by giving all the items assigned previously to bidder l to bidder m.[5]

We make some restrictions on the set of preferences. First we exclude any informational externalities: we consider a private values' framework where one's valuation depends solely on his private signal and not on his opponents' signals. Second, if a bidder does not acquire an item, his utility is normalized to zero. Note that this normalization is not innocuous: it states that a non-purchaser is indifferent to the final allocation. Thus we restrict our analysis to allocative externalities that are somehow orthogonal to the externalities in Jehiel et al papers ([19], [20], [22], [23]). Third, departing from the restricted set of preferences that AM considers, we allow for allocative externalities. The valuation of bidder l for an allocation does not depend solely on the bundle he acquires but also on the bundles his joint purchasers acquire. If bidder l acquires some items, i.e. $\mathcal{A}^l \neq \emptyset$, he derives the payoff $\Pi_l(\mathcal{A})$, where $\mathcal{A}$ is the final allocation. The function Π_l will also be qualified as bidder l's type. On the contrary to AM, we do not make any free disposal assumption. An item can worth less than zero: we may have $\Pi_l(\mathcal{A}) < 0$.[6] Finally, we make an additional assumption that makes sense only in a framework with allocative externalities: for any allocation, the valuation of a purchaser is never reduced if some of his joint-purchasers are replaced by the seller which then keeps the relating items in her hands. This is the following no positive externality assumption.

Assumption 2.1 (No positive externality) *A bidder $l \geq 1$ is said to suffer from no positive externality if his type is such that for any allocation*

[5] Formally, $\mathcal{A}(l \curvearrowright m)$ is defined such that we have $\mathcal{A}(l \curvearrowright m)^k = \mathcal{A}^k$, if $k \neq l, m$; $\mathcal{A}(l \curvearrowright m)^l = \emptyset$; $\mathcal{A}(l \curvearrowright m)^m = \mathcal{A}^m \cup \mathcal{A}^l$.

[6] For example, free disposal can be a quite restrictive assumption for a capacity auction where a purchaser is contractually required to use the capacity.

$\mathcal{A}$ and any bidder $m \neq 0, l$ we have

$$\Pi_l(\mathcal{A}) \leq \Pi_l(\mathcal{A}(m \curvearrowright 0)).$$

Denote by $\Pi := (\Pi_0, \Pi_1, \ldots, \Pi_N)$ the vector of types (With a slight abuse of notation Π will also represent the set of all types).

Eventually, we consider quasi-linear utility. If bidder l pays a bid $b_l(\mathcal{A})$ such that the allocation $\mathcal{A}$ is chosen, then he earns a net payoff of $\Pi_l(\mathcal{A}) - b_l(\mathcal{A})$. The payoff of the seller equals to her revenue $\sum_{l \in N} b_l(\mathcal{A})$ plus her private valuation for the chosen allocation $\Pi_0(\mathcal{A})$, which is assumed to be increasing in $\mathcal{A}^0$.

In such a general framework, bidders do not care solely on their own allocation, i.e. whether they obtain or not some bundles of items, but also on the identity and bundles of their joint purchasers. We will precise later, in section 3, practical examples where such externalities may intervene as first order in the valuations. Standard and also combinatorial auction mechanisms do not allow bidders to express this dimension in their valuations: bidders are not allowed to submit bids that are contingent on their joint-purchasers. Then full efficiency is not guaranteed. On the contrary, in a contingent mechanism, bidders can report contingent bids: the allocation rule and the associated transfers depend on the whole set of contingent bids Π. Thus we define a broader class of auction mechanisms: contingent auctions.

Definition 1 *A contingent auction mechanism $(A, p) : \Pi \to \mathbf{A} \times \mathbb{R}^N$ is a function mapping a vector of types Π into an allocation $A(\Pi)$ and a vector of transfers such that $p_l(\Pi)$ represents the transfer paid by bidder l to the seller.*[7]

Throughout the analysis, our criteria is allocative efficiency. We are looking for auction mechanisms that are ex post efficient. A contingent mechanism is *ex post efficient* if for all Π, $A(\Pi)$, is an efficient allocation relative to the reported preferences, Π, namely:

$$\sum_{l \in \{0\} \cup N} \Pi_l(A(\Pi)) \geq \sum_{l \in \{0\} \cup N} \Pi_l(\mathcal{A}), \quad for\ all\ \mathcal{A} \in \mathbf{A}$$

[7] Contingent auctions should be viewed as a superset of combinatorial auctions. In the special case of unit-demand where a combinatorial auction reduces to a standard auction with a one-dimensional reported type, the dimension of a type could explode with the number of potential joint-purchasers. For example, with $N = 10$ potential bidders and $M = 6$ items, this dimension equals to $\sum_{i=0}^{M-1} \binom{N-1}{i} = 382$. In general, this raises a complexity issue and the computation of the Vickrey allocation is an NP-complete problem. However, in the following NGDE framework where items are homogenous and where externalities presents also a kind of homogeneity, this complexity is considerably reduced and we presume that the computation of the auction mechanism is not a practical issue anymore.

Generically, the efficient allocation is unique and then denoted by $A^*(\Pi)$ or $\mathcal{A}^*$ (if the relative preferences are unambiguous). We mainly consider two kinds of ex post efficient contingent auction mechanisms: first the Vickrey Contingent Auction and then Ausubel-Milgrom contingent auctions that we define hereafter. Of course, ex post efficiency does not guarantee that the resulting allocation is efficient since the reported preferences may not fit with the real ones. The main issues are then the following ones. First what are the incentives for the bidders to report their true preferences? More precisely, we are interested in dominant strategy implementation. Second, how equilibria are robust out of the equilibrium's path? The dominant strategy equilibrium may not be convincing if some simple 'deviations' may be profitable. More precisely, we consider the deviations from coalitions of losing bidders with or without monetary transfers inside the coalition. Let us define formally those two kinds of coalitional deviations.

Definition 2 (Robustness to losers' deviation) *The outcome of a contingent auction is robust against losers' deviation (respectively strongly robust) at a given type vector* Π *if for any reported vector of types* Π^{dev} *such that* l *belongs to the set* S *of deviators, defined by* $S = \{k \in N | \Pi_k^{dev} \neq \Pi_k\}$*, implies that* l *is a loser in the allocation under truthful reporting, i.e.* $A^*(\Pi)^l = \emptyset$*, then this deviation is unprofitable for at least one bidder in* S*:*

$$\exists l \in S \text{ such that } \Pi_l(A^*(\Pi_{dev})) < p_l(\Pi_{dev}).$$

(respectively then this deviation is unprofitable for the whole coalition S*:* $\sum_{l\in S} \Pi_l(A(\Pi_{dev})) < \sum_{l\in S} p_l(\Pi_{dev})$*.)*

The weak version considers coalitions of losers where monetary transfers are not available. This is the definition that will be used for characterizing the robustness of the Vickrey auction against losers' deviations in the NGDE framework. On the other hand, as will be shown in proposition 2.4, the outcome of AM-auctions (more generally of auctions that are leading to Core outcomes) are strongly robust against losers deviations, i.e. even if monetary transfers are allowed between losers. In AM framework without externalities, the subtlety between weak and strong robustness against losers' deviation does not matter as it will be clarified in section 4.

2.2 The coalitional form

A useful tool for the analysis of combinatorial auctions and similarly for contingent auctions is a coalitional form game $(N \cup \{0\}, w)$ that is associated with the assignement problem $\{\mathbf{A}, (\Pi_j)_{j\in N\cup\{0\}}\}$, where w is the coalitional value function. For any coalition of bidders $S \subset N \cup \{0\}$, w is defined by:

$$w(S) = \max_{\mathcal{A}\in \mathrm{A}} \sum_{l\in S} \Pi_l(\mathcal{A}), \quad if \quad 0 \in S \tag{1a}$$

$$w(S) = 0, \quad if \quad 0 \notin S \; [8] \tag{1b}$$

Denote by u_l the net payoff of bidder l. Then we define the set of core payoffs, denoted by $Core(N, w)$, related to this coalitional value function w:

$$Core(N, w) = \left\{ (u_l)_{0 \leq l \leq N} \mid (a): \sum_{l=0}^{N} u_l = w(N);\; (b): \; (\forall S \subset N \cup \{0\})\; w(S) \leq \sum_{l \in S} u_l \right\}$$

(a) is the feasibility condition, whereas (b) means that the payoffs are not blocked by any coalition S.

A subset of the core plays a central role in the analysis of combinatorial auctions and similarly for contingent auctions: those are the Pareto-optima from the perspective of the bidders. This set is qualified as the bidder optimal frontier of the core.

Definition 3 *The bidder optimal frontier of the core is the set containing the elements* $(u_l)_{0 \leq l \leq N} \in Core(N, w)$ *such that there exists no* $(u'_l)_{0 \leq l \leq N} \in Core(N, w)$ *where* $u'_l \geq u_l$ *for all* $l = 1 \ldots N$ *and such that at least one inequality is strict.*

In the analysis of combinatorial auctions, a condition has emerged in the literature which renders truthful reporting an equilibrium of the Ausubel-Milgrom ascending proxy auction and also makes the Vickrey auction robust to shill bidding and against losers' deviations: this is bidder submodularity.

Bidder submodularity is a kind of 'substitutes' condition: the bidders should be viewed as substitutes insofar as the surplus associated with the presence of a bidder is decreasing with the set of competitors.

Definition 4 (Bidder submodularity) *The coalitional value function* w *is bidder-submodular if for any* $l \in N$ *and any coalitions* S *and* S' *satisfying* $0, l \in S \subset S'$*, we have*

$$w(S) - w(S \setminus \{l\}) \geq w(S') - w(S' \setminus \{l\})$$

The term $w(S) - w(S \setminus \{l\})$ represents the surplus associated with the presence of bidder l in the coalition S.

If the bidder-submodularity condition holds, then free-riding issues among bidders are avoided. For example, suppose that two bidders would like to acquire some items and that they have to block other potential bidders in order to win, then the exclusion of those competitors contains no public

[8] If the seller is not a member of the coalition, the coalition obtains no items. Due to our no positive externality assumption, this point is independent of any 'dumping' assumption, i.e. whether the auctioneer can dump objects on bidders who have not bid for them, if the seller is supposed to maximize her revenue among the remaining bidders outside the coalition.

good between these two bidders. In a framework without externalities, AM [Theorem 12] characterizes in some way the set of individual preferences for w being bidder-submodular: items should be regarded as mutual substitutes by the bidders.

2.3 The Vickrey Contingent Auction

Definition 5 *The Vickrey Contingent Auction is the ex post efficient mechanism with the following pivotal prices:*

$$p_l^V(\Pi) = \max_{\mathcal{A}\in\mathbf{A}}\left\{\sum_{k\neq l}\Pi_k(\mathcal{A})\right\} - \sum_{k\neq l}\Pi_k(A^*(\Pi)), \quad \textit{for all } l \in N$$

In the most general framework with allocative externalities, the Vickrey Contingent Auction is such that a non-purchaser could pay and such that a purchaser may be paid to acquire an item that is valuable for him. Indeed, in the preceding restricted framework with externalities, those two peculiarities could not arise.

Proposition 2.1 *For any reported types in* Π*, then for all* l*,* $p_l^V(\Pi) \geq 0$*. If bidder* l *acquires no items, then* $p_l^V(\Pi) = 0$*.*

Proof 1

$$\max_{\mathcal{A}\in\mathbf{A}}\left\{\sum_{k\neq l}\Pi_k(\mathcal{A})\right\} \geq \sum_{k\neq l}\Pi_k(A^*(\Pi)(l \curvearrowright 0)) \geq \sum_{k\neq l}\Pi_k(A^*(\Pi))$$

The first inequality results from the definition of the maximization. The second inequality results from the no positive externality assumption for all $\Pi_k, k \neq l$*. Those inequalities are equalities if bidder* l *acquires no item in* $A^*(\Pi)$*.*

Note that the no positive externality assumption is crucial. In the Vickrey auction, for any reported vector of types for all bidders except two, we can prove easily that the two remaining bidders can report types suffering from positive externalities and then purchase all items and receive an arbitrary high transfer what makes the Vickrey contingent auction unattractive. Each of the two bidders declare that the other as joint purchaser is essential to obtain a very high profit. Thus it should be emphasized that for obvious robustness reasons, the auctioneer should imperatively restrict the set of preferences that can be reported to make it robust against losers' deviation. The no positive externality restriction can be easily implemented given that this restriction concerns each individual report and not the whole set of reports to the auctioneer.

2.4 The Ausubel-Milgrom Contingent Proxy Auction

AM have proposed an 'ascending auction with package bidding'. Indeed, their analysis focuses on the related proxy auction which constitutes a new sealed-bid combinatorial auction. Ranger [34] studies a generalization of this mechanism allowing bidders to express externalities, i.e. a specific contingent auction. Lamy [27] proposes a slight modification of the auction by adding a final stage giving strictly more incentives to report the truth while remaining a Core-selecting auction. The definitions of those specific auctions are not straightforward. However what matters in the following analysis is the property that the final outcome lies in the bidder optimal frontier of the Core given the reported preferences. Our results apply to this class of auctions that we will refer to as Ausubel-Milgrom-auctions (hereafter AM-auctions).[9]

Definition 6 *An AM-auction is a contingent auction mechanism such that the final payoff vector belongs to the bidder optimal frontier of the Core relative to the reported types.*

See Lamy [27] for an algorithm implementing a specific AM-auction. Note that the set of AM-auctions may be empty if the core corresponding to some possible types in Π is empty. Indeed in our framework, the core with respect to the true types is never empty. This is due to the assumption that non-purchasers suffer from no externalities. This crucial assumption is discussed in the supplementary material [26].

AM establishes a link between the structure of the Core and the Vickrey outcome.

Proposition 2.2 (AM Theorem 6) *The bidder optimal frontier is a singleton and then corresponds to the Vickrey outcome if and only if the Vickrey outcome is in the Core.*

If his competing bidders are truthful, we know from the definition of an AM-auction that a bidder can never obtain a payoff greater that his Vickrey payoff: this results from the fact that coalition $N \cup \{0\} \setminus \{l\}$ does not block. Thus if a bidder obtains his Vickrey payoff under truthful reporting, then it is a best response. Coupled with proposition 2.2, we have proved the following corollary.

Corollary 2.3 *If the Vickrey outcome is in the Core, then truthful reporting is a Nash Equilibrium in any AM-auction.*

[9]Indeed, the differences between the mechanisms inside this class are completely unessential insofar all those mechanisms satisfy the same properties that we will recall below. Note that the original proxy auction proposed by AM is not really an AM-auction because the final outcome belongs only to the weak bidder optimal frontier of the Core. Refer to Lamy [27] for more details. Nevertheless, in this format, the bidder-submodularity condition is also a sufficient condition for truthful reporting to be an equilibrium.

Indeed, instead of investigating the class of preferences such that the Vickrey outcome is in the Core for the whole set of bidders N, we investigate preferences satisfying a stronger condition: for any subset of bidders $S \subset N$, the Vickrey outcome is a Core outcome (the Vickrey and Core outcome being defined relative to this subset S). According to Theorem 7 in AM, this is equivalent to the bidder-submodularity of w.

On the other hand, as it is depicted in Figure 2.4, if the Vickrey outcome is not in the Core, then truthful reporting is not an equilibrium: under truthful reporting, an outcome in the bidder-optimal frontier is implemented, then at least one bidder, say bidder 2, does not obtain his Vickrey payoff. If bidder 2 shades off his bids, he is able to reduce the bidder optimal frontier up to the point where the only bidder optima are such that he is guaranteed to obtain his Vickrey payoff. The truncation of the Core resulting from bid-shading is illustrated in Figure 2.4 where the Core is depicted under the shaded area. After the bid reduction Δb, the Core is truncated above by the snaked line. After the optimal bid reduction, Δb^{opt}, the Core becomes the singleton $(0, \pi_2^V)$. The dashed lines departing from π^{Max} the highest payoffs that each bidder can expect with a null transfer and ending in the bidder optimal frontier depicts possible dynamic of the highest expected payoff vector in a dynamic version of an AM-auction.

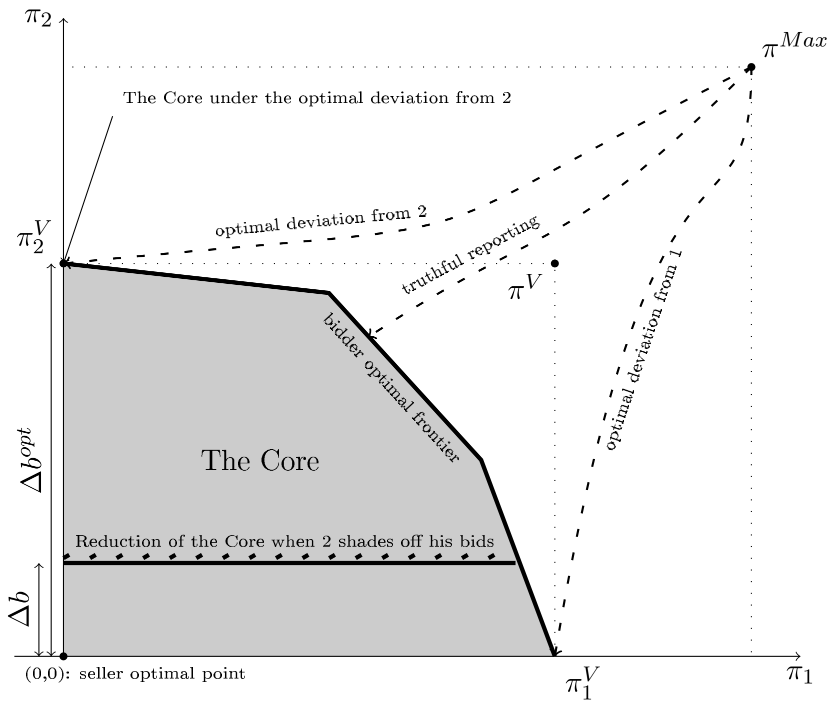

Fig 2.4: AM-auctions dynamics

2.5 Losers' Deviations and Core constraints

Intuitively the robustness against losers' deviations is very closely related to the Core constraints. However, AM do not make any formal link between those two types of constraints in a fully general framework. Indeed, without externalities, they establish a surprising equivalence result: the Vickrey auction is robust against losers' deviation if and only if bidders have 'substitutes' preferences, i.e. when the Vickrey outcome is in the Core. In the following proposition, we show that if a contingent mechanism outcome is in the Core given the reported preferences and also such that for any preferences reported by coalitions of losers the auction outcome remains a Core outcome, then this outcome is strongly robust against losers' deviations. Indeed, only a very restrictive set of constraints among the whole set of (b) constraints in the definition of the Core are used to obtain the result. It gives the intuition that in general, as we will actually show for the NGDE framework, the condition needed for the Vickrey outcome being in the Core is much stronger that the condition needed for the Vickrey outcome being strongly (and a fortiori weakly) robust against losers' deviation. Then it seems that the Vickrey contingent auction should be more robust than AM-auction beside those criteria. However, we should be careful in the interpretation of the result: it does not only require that the underneath true preferences are such that the mechanism outcome is in the Core, it also requires that if some losers jointly deviate, then the mechanism outcome still lies in the Core. As we will also illustrate for the NGDE framework, such conditions are not innocuous and on the whole the relative performance between the Vickrey and AM-auctions may depend on the ability of the auctioneer to restrict the set of joint preferences that may be reported by the bidders.

Proposition 2.4 (Core versus losers' deviation constraints) *Given a set of preferences Π and a contingent auction mechanism (A, p), if any outcome of the contingent mechanism is in the Core, then any truthful outcome is strongly robust against losers' deviation provided that the auctioneer is able to restrict the way losers can deviate such that the reported types always stay in Π.*

Proof 2 *In order to make the allocation switch from $\mathcal{A}^*$ to another allocation $\mathcal{A}'$, the set of losers S should make a total contribution P_S such that $P_S + \sum_{i \notin S} \Pi_i(\mathcal{A}') \geq \sum_{i=1}^{N} \Pi_i(\mathcal{A}^*)$ (otherwise the coalition $N \cup \{0\} \setminus S$, would block. Note that $w(N \cup \{0\} \setminus S) = \sum_{i=1}^{N} \Pi_i(\mathcal{A}^*)$ because bidders in S are losers). Because $\mathcal{A}^*$ maximizes over $\mathcal{A}$ the sum $\sum_{i \in N} \Pi_i(\mathcal{A})$, then it implies that $P_S > \sum_{i \in S} \Pi_i(\mathcal{A}')$ and that consequently the losers do not (jointly) find the deviation profitable.*

Corollary 2.5 *1. AM-auctions are strongly robust against losers' deviation.*

2. *If the set of preferences* Π *is such that the Vickrey outcome is in the Core and provided that the auctioneer is able to restrict the way preferences are reported such that they always stay in* Π*, then the Vickrey contingent auction is strongly robust against losers' deviation.*

An appealing feature of AM-auctions is that they are strongly robust against losers' deviations in a very general manner contrary to the Vickrey contingent auction. However, in the case where it is a dominant strategy to report truthfully its preferences in the AM-auctions, then the Vickrey auction is automatically robust to losers' deviation provided that the auctioneer is able to constraint the deviation of the losers in such a way that they can not jointly report preferences outside the set Π.

Remark 2.1 *Bernheim-Whinston first price menu auction [5] is a contingent auction such that any outcome is in the Core relative to the reported preferences: it selects the seller's optima among the Core outcomes, i.e.* $\pi_0 = w(N),\ \pi_l = 0,\ for\ l \geq 1$*. Then proposition (2.4) implies that it is robust against losers' deviations. On the other hand, Bernheim-Whinston's outcome is the most distant outcome in the Core from the Vickrey outcome and gives then the weakest incentives to report the truth.*

2.6 An Example with allocative externalities

The following example is inspired from Hoppe, Jehiel and Moldovanu [17], where the authors report that 'a major investment bank estimated per-licenses values of Euro 14.75, 15.88, and 17.6 billion for a German UMTS market with 6, 5, and 4 firms, respectively'. It gives a foretaste of the Negative Group Dependent Externalities framework with only one group and unit-demand. It also illustrates our criteria for the performance of a mechanism: incentives to report the true preferences and robustness against losers' deviation. We consider a market for licences such that the more licenses are sold the less it worths for a licensee, whereas a non-licensee remains outside the market and is indifferent to the final market structure.

We consider 3 potential bidders and three identical licences. For each bidder, the value for a licence is supposed to depend only on the number of joint-licensees. We assume unit-demand: each bidder is indifferent between acquiring extra-licences or leaving them unsold in the seller's hands. The three bidders are designated by 1,2,3. Their corresponding valuations are given by the functions $x \to \pi_l(x)$ where x represents the total number of licensees in the final market structure. We discuss the numeric application where the reported preferences are: $\pi_l(1) = 11,\ \pi_l(2) = 7,\ \pi(3)_l = 6$ for each bidder l. The seller, designated by 0, is supposed to be a pure revenue maximizer. The numeric values we have chosen shares the common feature with Hoppe, Jehiel and Moldovanu's report that the functions $x \to \pi_l(x)$ are not concave. We will see later that the concavity of the function π_l will play a central role in our general result about the condition making contingent auctions robust in the NGDE framework. The efficient allocation given the reported preferences is the three-licensees structure $\{1,2,3\}$ and is thus the allocation implemented by the contingent auctions we consider.

According to the reported preferences, the coalitional value function w is not bidder-submodular because $w(N) - w(N \setminus \{l\}) = 4 > 3 = w(N \setminus \{m\}) - w(N \setminus \{l,m\})$ for $l, m \in 1,2,3$. For example, bidders 2 and 3 should be viewed as complement bidders: the additional value to the total surplus provided by bidder 2 grows with the mere presence of bidder 3.

This means that this example is similar to the one depicted in Figure 2.4: the Vickrey outcome lies outside the Core. The Core outcomes $(u_l)_{l=0,\cdots,3}$ are defined by the following constraints:

$$u_0 + u_1 + u_2 + u_3 = 18 \qquad \textit{Feasibility Constraint} \tag{2a}$$

$$u_0 + u_l + u_m \geq 14 \quad , l, m \in \{1,2,3\} \qquad \{0,l,m\} \textit{ do not block} \tag{2b}$$

$$u_0 + u_l \geq 11 \quad , l \in \{1,2,3\} \qquad \{0,l\} \textit{ do not block} \tag{2c}$$

$$u_l \geq 0 \quad , l \in \{0,1,2,3\} \quad \textit{Rationality Constraint} \quad \{l\} \textit{ do not block} \tag{2d}$$

Independently of the true valuations, the reported preferences are never an equilibrium in AM-auctions since bidders can obtain the same allocation at a smaller price by reporting $\pi(1) = 7$, $\pi(2) = 3$, $\pi(3) = 2$, i.e. if he shades his bids by 4.

The Core outcomes can also be rewritten in term of constraints on the vector of prices $(p_l)_{l=1,2,3}$. The feasibility constraint sets that the revenue of the seller is the sum of those prices. The constraint (2b), which could also be qualified as the 'Vickrey constraint', states that each bidder must pay at least the externalities imposed on the remaining bidders. In this example, it corresponds to $p_l \geq 2$, 2 being the price set by the Vickrey contingent auction. The constraints (2c) (respectively (2d)) are equivalent to $p_l + p_m \geq 5$ (respectively $p_l \leq 6$, the individual rationality constraint). Note that for the Vickrey outcome, we have $\pi_2 + \pi_3 = 4 < 5$: that is the only kind of constraints that the Vickrey outcome fails to satisfy in order to be in the Core. It means that, whereas each bidder pays at least the externality that he imposes on the other bidders, a subset of bidders with strictly more than one bidder may possibly pay a total amount that is inferior to the externality they jointly impose. For example, suppose that the true preferences of bidders 2 and 3 are indeed $\pi(x) = 2.25$ for any x and suppose that bidder 1 has reported his true preferences, then the efficient allocation is to attribute one licence to bidder 1. On the other hand, bidder 2 and 3, as losers, find profitable to jointly deviate and report the same preferences as bidder 1 to obtain a strictly positive profit. Finally we have illustrated the losers' deviation issue which relies on the fact that $\pi(\cdot)$ fails to be concave for at least one bidder.

3 The Negative Group-Dependent Externalties Framework (NGDE)

Two preliminary definitions and some notation

Definition 7 *A partition of N is a set of subsets, denoted by $\{G_j\}_{j=1}^{g}$, such that $\bigcup_{j=1}^{g} G_j = N$ and $G_j \bigcap G_k = \emptyset$ for all G_j, G_k. A subset G_j is referred to a group. A partition will also be referred to as a group-dependent structure.*

For bidder l, denote by $G(l)$ his group and denote by $g(l)$ the index corresponding to his group and such that $G_{g(l)} = G(l)$. Denote by $n_l^{\mathcal{A}}$ the number of purchasers in bidder l's group in the allocation $\mathcal{A}$, i.e. the cardinality of the set $G(l) \cap \mathcal{A}$.

Definition 8 *A subpartition of a partition $\{G_j\}_{j=1}^{g}$ of a set N is a partition of N, denoted by $\{B_j\}_{j=1}^{b}$, such that for all $j = 1 \ldots b$, there exists a group G_k, such that $B_j \subset G_k$. A subset $B_j \subset G_k$ is referred to a 'bundle' of bidders inside the group G_k.*

Before the formal definition of the NGDE frameworks, let us present some motivating examples.

3.1 Some motivations for the NGDE framework

If the German UMTS auctions were a starting point because the number of licensed firms was endogenous and that the valuation for a license was a function of this number, firms do not suffer from identity-dependant externalities at first glance: they do not care whether a competing licence is sold to either firm A or firm B. Such a kind of externalities is already present in the literature and especially in Segal [37] and Segal and Whinston [38]. Refer to [37] for a survey of the motivating examples for such 'level of trades' externalities. Note that those works are considering bilateral trading excluding de facto contingent bids. Ranger [33] considers both 'level of trade' externalities through a unique downstream Cournot market that link the bidders and contingent bids in AM proxy auction. In some applications, it is more reasonable to model the downstream competition through several different Cournot markets. In particular, in the problem of allocating airport slots (studied by the pioneering work of Rassenti, Smith and Bulfin [35]), it is clear that some companies do not compete in the same area. The valuation of a North-American company for a slot in London will not depend on the total number of slots that are sold in London's airport but rather on the total number of slots purchased by her direct competitors. She probably prefers that a slot is allocated to a Russian company rather than to one of her most closest competitors, e.g. another North-American company. The NGDE framework will broaden the analysis in this direction and also in a more abstract perspective allowing other forms of downstream interaction between joint-purchasers. Instead of a linkage through the demand function in the downstream market as in Ranger [33], the following example considers the downstream linkage between the producers through their cost functions. In power markets, the final price of electricity is often strongly regulated or/and hedged by forward contracts, whereas the prices of the various inputs may be very volatile: uncertainties about the producers' profits may therefore rely more on the costs than the demand.

Example 3.1 (Capacity in Power Markets) *Investment incentives is a great issue in power markets and a large number of non-market mechanism have been imposed to avoid shortages.*[10] *Either a transmission system operator (ISO) or a Load Serving Entity (LSE) may be willing to run a procurement auction respectively in order to achieve a suitable level of reliability or to meet their capacity obligations.*[11] *Here we consider the case where the inputs (the energy source) is not specified in the procurement. Consider that different kinds of fossil fuel projects are competing. For example, some projects concern generations that use natural gas, whereas some other generators use fuel oil as their primary fuel. Then the production costs suffer from identity-dependent externalities: a generator is valuing differently such a procurement contract according to the nature of the fuel used by the generators that have signed the other contacts. For example, a generator that uses natural gas will prefer that few gas users emerge insofar as a large amount of gas users will induce extra costs, e.g. due to the congestion costs on interconnectors. In such a framework, it seems a reasonable first order approximation to assume that the production costs of a given unit depend on the amount of capacity provided by joint-procurers' units that are using the same kind of fuel, whereas, inside a group, generators differ only beside their output efficiency. The group-structure of the related NGDE framework corresponds here to the partition of the units according to the kinds of inputs. Note that the production costs of a generator are increasing with the number*

[10] See Joskow and Tirole [24] for more details on the inability of competitive market to achieve the optimal level of reliability and the relevant regulatory instruments.

[11] Reliability-Must-Run contracts, long-term contracts for delivered energy at fixed prices, capacity contracts are signed usually under bilateral negotiations. Procurement auctions for capacity are however common for renewable energies.

of joint-procurers inside his group: the value of a contract is then decreasing with the number of joint-procurers inside his group. This point brings us to refine the model in considering negative group-dependent externalities.

Previous theoretical works about allocations with externalities do not fully include the issues raised by those examples. Jehiel, Moldovanu and Stacchetti's externalities are limited to single-item assignements. Pure level of trade externalities as in Segal [37] correspond to a structure with only one group.[12] Consider the allocation of a scarce resource to a given group such that from a welfare analysis, the optimum corresponds to assign all items, then the problem is straightforward and a standard English auction achieve the optimum since it puts the items in the hand to those who value them at most. On the other hand, with indentity-dependent externalities, the allocation problem of a scarce resource such that the optimum corresponds to assign all items is not obvious and a standard English auction does not achieve efficiency.[13] A notable exception in the literature about identity-dependent externalities, which considers positive externalities between joint-purchasers, is Aseff and Chade [1] who characterize the optimal auction when the informational asymmetry is reduced to a one-dimensional parameter.

The rest of this section is devoted to the formal definition of the NGDE framework: first we define the unit demand framework where bidders are interested in at most one item, second we define the multi-unit demand framework where each bidder demand is viewed as a 'bundle' of some unit demand bidders. From now on, except in section 6, we implicitly assume under the NGDE terminology that the seller is indifferent to the final allocation: $\Pi_0(\mathcal{A}) = 0$, for any $\mathcal{A} \in \mathbf{A}$.

3.2 The NGDE framework with unit demand

Definition 9 (NGDE with unit-demand) *An assignment problem* $\{\mathbf{A}, (\Pi_j)_{j \in N \cup \{0\}}\}$ *is said to satisfy the Negative Group Dependent Externalities framework with unit demand if there exists a N+1-uple* $(\{G_j\}_{j=1}^{g}, \pi_1, \cdots, \pi_N)$ *where* $\{G_j\}_{j=1}^{g}$ *is a partition of* N *and where* π_l*,* $l \in N$*, are nonincreasing functions mapping an integer in* $[1, N]$ *into a real number such that for any* $l \in N$ *and for any allocation* $\mathcal{A}$*:*

$$\Pi_l(\mathcal{A}) = \pi_l(n_l^{\mathcal{A}}), \ if \ \mathcal{A}^l \neq \emptyset \tag{3a}$$

$$\Pi_l(\mathcal{A}) = 0, \ if \ \mathcal{A}^l = \emptyset. \tag{3b}$$

[12] Nevertheless, we give up a degree of generality relative to the literature on level of trade externalities by restricting the analysis to 'negative' externalities.

[13] In Lamy [25], various standard auction mechanisms are shown to implement a 'stable' allocation in the NGDE framework.

The N+1-uple $(\{G_j\}_{j=1}^g, \pi_1, \cdots, \pi_N)$ will also be referred as a NGDE framework with unit-demand.

The terminology group-dependent comes from the fact that the valuation of a purchaser depends solely on the assignment of his joint-purchasers inside his group and more specifically on the number of those joint-purchasers, which introduces some symmetry in the model insofar as inside a group there are no identity-dependent externalities. Then we can abusively qualify π_l as bidder's l type. We assume also that π_l is nonincreasing. It means that a purchaser is suffering from negative externalities from the joint-purchasers of his group.

The remaining assumption to complete the NGDE framework is technical and standard in economic analysis.

Assumption 3.1 *Non-Crossing (NC) assumption*
For any group G, for any $i, j \in G$ such that $i < j$, then

$$\pi_i(x) > \pi_j(x)$$

In a nutshell, it says that inside a group, bidders can be unambiguously ranked in term of efficiency. Nevertheless, this assumption is not much restrictive insofar as that in the following multi-unit demand framework where different unit-demand bidders are 'bundled', then for a given number of acquired items and making vary the number of joint-purchasers, then bidders are not ranked unambiguously.

3.3 The NGDE framework with multi-unit demand

Definition 10 (NGDE with multi-unit demand) *A Negative Group Dependent Externalities framework with multi-unit demand (due to bundling) is a N+2-uple $(\{G_j\}_{j=1}^g, \{B_j\}_{j=1}^g, \pi_1, \cdots, \pi_N)$ such that the N+1-uple $(\{G_j\}_{j=1}^g, \pi_1, \cdots, \pi_N)$ is a NGDE framework with unit demand and $\{B_j\}_{j=1}^g$ is a subpartition of $\{G_j\}_{j=1}^g$.*

Let us connect this definition with the previous general assignment problem with externalities and the NGDE framework with unit-demand. Consider the NGDE framework with unit-demand and consider that inside a group, some of the unit-demand identities could be bundled together under a same common bidder that bids under a unique identity and on the behalf of their mutual interests. Those 'bundled' bidders bid in order to maximize the sum of the profit of its identities, who after the first allocation step (the contingent auction) allocate the items to the identities in their bundle who are valuing the items at most. Due to the non crossing condition (3.1) and because we consider that a bundle B of individual entities covers only entities of the same group, then it is a dominant strategy to allocate the acquired items to the most 'efficient' bidders in B, i.e. those with the smallest index in the NGDE framework. A bidder may possibly prefer not to use an acquired item and then obtain the payoff as if he leaves it in the seller's hand. However, such an event will never happen for any best response strategy in the auctions we consider such that there is no loss of generality to exclude those events.

By an appropriate indexation, we can write $B_j := \{j_1, \dots j_{B_j}\}$. Finally, the valuation of the bundle B_j for an allocation $\mathcal{A}$ depends on two parameters: the number of items purchased by the bidder (bundle) j and the total number of items purchased in j's group $n^{\mathcal{A}}_{g(j)}$. This valuation function is then denoted by $\widetilde{\pi}_{B_j}(x, y)$, a function of x the number of items that the bundle purchases and $y \geq x$ the number of acquired items in his group and can be derived from the original valuations $(\pi_k)_{k \in N}$:

$$\widetilde{\pi}_{B_j}(x, y) = \sum_{l=1}^{x} \pi_{j_l}(y).$$

Note that $(x, y) \to \widetilde{\pi}_{B_j}(x, y)$ is concave over x for all y because $l \to \pi_l(y)$ is decreasing for all y. Thus in the NGDE framework with multi-unit demand bidders exhibit diminishing marginal utilities as expected since it corresponds to the 'substitutes' condition of AM in the case on homogeneous items. Conversely, for any function $(x, y) \to \widetilde{\pi}_{B_j}(x, y)$ which is concave over x for all y and such that $(x, y) \to \widetilde{\pi}_{B_j}(x, y) - \widetilde{\pi}_{B_j}(x-1, y)$ is decreasing over y, then it can be written as the sum of valuations satisfying the NGDE-framework. The 'virtual' valuation of the identities composing the bundle can be derived as:

$$\forall l, \pi_{j_l}(y) = \widetilde{\pi}_{B_j}(l, y) - \widetilde{\pi}_{B_j}(l-1, y)$$

The following lemma will allow us to extend our bidder-submodularity characterization from unit-demand to multi-unit demand in the NGDE framework.

Lemma 3.1 *If a NGDE framework with unit-demand* $(\{G_j\}_{j=1}^{g}, \pi_1, \cdots, \pi_N)$ *is such that the related coalitional function* w *is bidder-submodular, then for any subpartition* $\{B_j\}_{j=1}^{g}$ *of* $\{G_j\}_{j=1}^{g}$ *the NGDE framework with multi-unit demand* $(\{G_j\}_{j=1}^{g}, \{B_j\}_{j=1}^{g}, \pi_1, \cdots, \pi_N)$ *is such that the related coalitional function* w *is bidder-submodular.*

Proof 3 *Consider two subsets* B_i, B_j *of* N *and let* $B_j = \{j_1, \cdots, j_{B_j}\}$.

$$w(N) - w(N \setminus B_i) \leq w(N \setminus \{j_1\}) - w(N \setminus \{B_i, j_1\}) \leq \cdots$$
$$\cdots \leq w(N \backslash \{j_1, \cdots, j_{k-1}\}) - w(N \backslash \{B_i, j_1, \cdots, j_{B_j - 1}\}) \leq w(N \backslash B_j) - w(N \backslash \{B_i, B_j\})$$

Each inequality results from the definition of the bidder-submodularity of the unit-demand framework. The resulting inequality between the two extremes implies the bidder-submodularity of the multi-unit demand framework.

The argument is true for any bundle of bidders and so not only for a subpartition of $\{G_j\}_{j=1}^g$. Nevertheless, the NGDE framework with multi-unit demand does not make sense if a bidder is a bundle of some unit-demand bidders belonging to different groups because there is then an ambiguity about the usage of an item by that bidder and even a contingent auction is unable to internalize those externalities.

4 Robustness of the Vickrey Contingent Auction

Our starting example in section 2.6 illustrates the necessity of the condition that the negative externality is concave relative to the number of joint-purchasers in one's group for the Vickrey auction to be robust against losers' deviation. Indeed, it can be shown more generally that, for any number of items, if a bidder's preferences, say 1, fail to be concave, then there exists some preferences for his opponents that are exempt of any allocative externalities and such that bidder 1 wins under truthful reporting but such that this outcome is not (weakly) robust against losers' deviation. The failure for bidder 1 of the concavity means the existence of a number of joint-purchasers $x > 0$ such that: $\pi_1(x) - \pi_1(x+1) > \pi_1(x+1) - \pi_1(x+2)$, which is also equivalent to $\pi_1(x) - \pi_1(x+2) > 2 \cdot (\pi_1(x+1) - \pi_1(x+2))$. Suppose that the participation of two bidders (in bidder 1's group) modifies the number of items assigned in this group from x to $x+2$ and that the presence of a single of those bidders modifies this number of items from x to $x+1$ whereas bidder 1 always acquires one item. In the Vickrey contingent auction, each bidder internalizes through his payment at least the externality that he imposes on bidder 1: each should pay at least $\pi_1(x+1) - \pi_1(x+2)$. On the whole, they both contribute at least $2 \cdot (\pi_1(x+1) - \pi_1(x+2))$ and exactly this amount if bidder 1 is the only bidder suffering from allocative externalities and if the resource is not scarce. But this amount is smaller than the amount of the externality that they both impose on bidder 1: $\pi_1(x) - \pi_1(x+2)$. Actually, those two bidders may obtain a positive profit from their participation though they would be losers under truthful reporting.

On the other hand, next proposition shows that if the reported mappings π_i of the winners are concave, then the outcome of the Vickrey auction is (weakly) robust against losers' deviation. The proof is relegated in the appendix.

Proposition 4.1 *In the NGDE frameworks with unit or multi-unit demand, if the functions $x \to \pi_l(x)$ are concave for any l such that $\mathcal{A}^l \neq \emptyset$, then any joint deviation by losing bidders, such that the whole set of reported types still fits with the NGDE framework*[14]*, is unprofitable for at least one deviator in the Vickrey contingent auction.*

[14]Note that we do not require the maps $x \to \pi_l(x)$ reported by losing bidders to be concave neither the non-crossing assumption.

Remark that contrary to AM-auctions which are strongly robust against losers' deviation, we obtain here only the weak form of the robustness against losers' deviation.[15] This point is illustrated in the following example in the unit-demand framework with two groups.

Example 4.1 *Consider two items and four unit-demand bidders without allocative externalities. Bidders* 1 *and* 2 *are valuing the item* 100*. Bidders* 3 *and* 4 *are valuing the item respectively* 99 *and* 10*. Under truthful reporting, the Vickrey outcome is to give the items to* 1 *and* 2 *and to make each pay* 99*. However, this outcome is not strongly robust against the joint losers' deviation where bidders* 3 *and* 4 *are reporting types according to the group structure* $\{G_1 = \{1, 2, 3\}, G_2 = \{4\}\}$ *and where the reported valuations are* $\pi_3(1) = 200,\ \pi_3(x) = 0,\ x > 1$ *and* $\pi_4(x) = 100,\ x > 0$*. Note that this joint deviation fits with the NGDE framework since bidders* 1 *and* 2 *do not report any externalities. Under this deviation, the items are allocated to* 3 *and* 4*: bidder* 4 *pays nothing, whereas bidder* 3 *pays* 100*. If they can transfer at least a unit of money, then the deviation may be profitable for both deviators.*

Proposition [4.1] should be put in parallel with Theorem 13 in AM. Without externalities, AM characterizes the set of preferences such that losers' deviation is unprofitable in the Vickrey auction: the items should be viewed as substitutes. This result depends on the crucial point that bidders are restricted in the way they could bid: they are not allowed to report contingent valuations or equivalently to submit contingent bids. However, if bidders are allowed to report in a more general valuation set with contingent bids, then the Vickrey auction is not guaranteed to be immune from losers' joint deviation as been emphasized in section 2.3 even if the underneath valuations satisfy AM's theorem. Nevertheless, in AM, it is not an issue to restrict the way bids are reported. On the contrary, in proposition 4.1 bidders should be constrained in the way they could report their types insofar as the reported types should fit with the NGDE framework. This restriction corresponds to preclude any report such that the whole set of reported valuations does not satisfy the NGDE framework. But this is not easily done because the fact that a bid is coherent with this framework depends on the bids of the other participants. Those kind of deviations are illustrated in the following example where the Vickrey contingent auction is not immune to a losers' joint deviation with each loser reporting valuations that could fit (independently of each other) with the NGDE framework but such that the whole set of reported valuations does not fit with this framework.

[15]If we consider only joint deviations by losing bidders which are belonging to the same group, then the Vickrey auction is strongly robust against losers' deviation. See the proof of proposition 4.1 for more details. Hence, if bidders are restricted to bid according to the NGDE framework with only one group, then proposition (4.1) is valid with the strong robustness against losers' deviation concept.

Example 4.2 *Consider two items and four unit-demand bidders without externalities. Buyers* 1 *and* 2 *are valuing the item* 10*, whereas bidders* 3 *and* 4 *are valuing the item* 1*. The efficient allocation is* $\{1, 2\}$*. However, bidders* 3 *and* 4 *could jointly and strictly profitably deviate with the following bidding scheme. Buyers* 3 *and* 4 *report that: with a joint purchaser in* $\{1, 2\}$*, they are valuing the item* 0*, else they are valuing the item* 30*. Then bidders* 3 *and* 4 *both obtain the items for a null transfer. Independently of the other reported valuation, bidder* 3 *(respectively* 4*) could fit with the NGDE framework. However, in the NGDE framework, no group structure can fit with the reported valuation of* 3 *and* 4 *which implies that they both belong to bidders* 1 *and* 2*'s group but that they do not belong to the same group which raises a contradiction.*

To summarize, the Vickrey contingent auction requires a special monitoring ability of the seller: the possibility to exclude any report that lies outside the NGDE framework. On the one hand, if the designer can credibly commit to the null allocation, it can be implemented with the augmented grand-mechanism such that the auctioneer cancels the auction if the reported joint preferences do not fit the NGDE framework and such that the same Vickrey contingent auction is then proposed. On the other hand, if the mechanism designer is able ex ante to attribute to each participant its right group, then he can propose the simplified Vickrey mechanism where each bidder is asked to report a mapping $x \to \pi_l(x)$.

5 Robustness of AM-auctions

Our starting example in section 2.6 illustrates the necessity of the condition that the negative externality is concave relative to the number of joint-purchasers in one's group for the coalitional value function w to be bidder-submodular. However, this condition is not sufficient as it will be illustrated by the two next examples. A much stronger condition is needed which considerably reduces the dimensionality of bidder's preferences. Inside a group, bidders valuations must be equal up to a translation: for any $i, j \in N$ such that $G(i) = G(j)$, equalities $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$ must be satisfied for any $x > 0$. This is somehow a restriction of measure null, which could nevertheless have some relevance in some applications.

Now we deliver two examples that illustrate why, for any $x > 0$ and $i < j$ (i.e. bidder i is more efficient than j), both conditions

$$\pi_i(x) - \pi_i(x+1) \leq \pi_j(x) - \pi_j(x+1) \tag{4a}$$

$$\pi_i(x) - \pi_i(x+1) \geq \pi_j(x) - \pi_j(x+1) \tag{4b}$$

are necessary when there is only one group (and a fortiori with several groups). Inequality (4a) (respectively (4b)) means that the externality imposed on a bidder by an additional joint-purchaser in a given group is nonincreasing (nondecreasing) according to the efficiency of that bidder. Those two inequalities do not play a fully symmetrical role in the proof of proposition (5.1). Inequality (4a) implies that the number of purchaser in i's group does not increase if bidder i is removed. We give the intuition below.[16] Suppose that two additional bidders in i's group, say $l1$, $l2$, $l1 < l2$, win an item after bidder i's removal, then the externality imposed by $l2$ on the other winners of his group is greater than the externality imposed by the addition of bidder $l1$ relative to the original efficient allocation. Moreover, the gross contribution of $l2$ to the surplus is smaller than $l1$'s. Finally, the net contribution (incorporating the externalities) of bidder $l2$ (after i's removal) is smaller than $l1$'s in the original allocation problem, which is negative. Then the second additional winner in bidder i's group makes a negative contribution to the surplus, which raises a contradiction. Similarly, inequality (4b) implies than the joint-purchasers of a given bidder i under the optimal allocation are still purchasers under the optimal allocation without bidder i.

Example 5.1 (Failure of condition 4a) *Consider three items and one group. There are five bidders denoted by* $1, 2, 3, 4, 5$ *whose valuations are defined such that:* $\pi_i(x) = 7$ *for* $i = 1, 2$ *and* $x = 1, 2$ *and* $\pi_i(x) = 3$ *in any other case. The coalitional value function* w *is not bidder-submodular because* $w(N) - w(N \setminus \{1\})(= w(N) - w(N \setminus \{2\})) = 4 > 1 = w(N \setminus \{2\}) - w(N \setminus \{1, 2\})$. *Buyer 1 and 2 should be viewed as complement bidders who have to share non-cooperatively the surplus of* 2.

Example 5.2 (Failure of condition 4b) *Consider two items and one group. There are three bidders denoted by* $1, 2, 3$ *whose valuations are defined such that:* $\pi_i(1) = 10$ *for any bidder* i, $\pi_1(2) = \pi_2(2) = 8$ *and* $\pi_3(2) = 0$. *The coalitional value function* w *is not bidder-submodular because* $w(N) - w(N \setminus \{1\}) = 6 > 0 = w(N \setminus \{2\}) - w(N \setminus \{1, 2\})$. *Buyer 1 and 2 should be viewed as complement bidders who have to share non-cooperatively the surplus of* 6.

As illustrated in the above representative examples and as clarified later in the proof of proposition [5.1], the failure of proposition (4a) (respectively 4b) leaves the scope for complementarity between some bidders in a group against an alternative allocation with more (respectively less) bidders in their group.

The following proposition establishes that if both conditions (4a) and (4b) and the previous concavity assumption are satisfied, then w is bidder-submodular in the NGDE framework.

Proposition 5.1 *In the NGDE framework with unit demand or with multi-unit demand, if the functions mapping* $x \to \pi_i(x)$ *are concave and if for*

[16] The complete argument is more complex due to the linkage with the other groups. Refer to the proof of proposition [5.1] and footnote (23) for more details.

any $i, j \in N$ belonging to the same group we have $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$ for any $x > 0$, then w is bidder-submodular.

From lemma [3.1], bidder-submodularity in the multi-unit demand framework is a immediate corollary of the proposition under unit-demand. The proof for the unit-demand framework which is relegated in the appendix contains two part. First, we show that the optimal allocation when one bidder is removed is closely related to the optimal allocation with that bidder included: the purchasers in the latter allocation are still purchasers in the former. Indeed, when a bidder i is removed, three possibilities may arise: the final allocation is unchanged except that the item previously allocated to bidder i goes in the hand of either another bidder in i's group or another bidder in another group, or finally remains in the seller's hands. The second part of the proof is very tedious: the inequalities $w(N) - w(N \setminus \{i\}) \leq w(N \setminus \{j\}) - w(N \setminus \{i, j\})$ are carefully checked according to the different possibilities about the identity (-ies) of the purchaser(s) of the item after bidder i's removal (j and $\{i, j\}$ removals).

Proposition [5.1] requires much stronger conditions on the form of the preferences than proposition [4.1]. In that perspective, it can be viewed as a critic about the relevance of AM-contingent auctions relative to the Vickrey contingent auction. In the perspective of generalizing the substituability condition with allocative externalities, the proposition and the related counterexamples show that a nongeneric congruence relation need to be satisfied such that externalies are aligned in each group. The congruence relation that $\pi_j(x) - \pi_j(x+1)$ is independent of j seems very restrictive, but it could be a reasonable first order approximation in some applications. This point is discussed in the following application studied by Ranger [33] which corresponds to the NGDE framework with multi-unit demand and a single group.

5.1 An application: capacity auctions

Suppose that bidders are competing in an auction for capacity prior to a Cournot market interaction with a concave demand function denoted by $D(p)$. In the whole game, [33] considers the possibility to acquire capacity and not to use it. Here let us consider that a capacity that is purchased must be used.[17] While Ranger's analysis covers a scope without indivisibilies, we find the analog of Ranger's main result, i.e. the vector of valuations of the capacity game satisfies the bidder-submodularity condition, with indivisible good of the same size, say q. The analog result without indivisibilities is obtained by taking the limit $q \to 0$.

[17] In equilibrium, capacities are always used and this restriction point does not modify the analysis. Indeed, preemption and acquisition motivation are disconnected in contingent auctions contrary to standard auctions.

Consider the NGDE framework with only one group: a bidder suffers from negative externalites from all his joint-purchasers insofar as he prefers other items to stay in the seller's hands rather to be sold to his potential opponents. Denote by K the total capacity that is auctioned and $K^T \in [0, K]$ the total capacity purchased by all bidders at the end of the auction. Each bidder i corresponds to the bundle of K identical unit-demand bidders with the cost functions $\{c_l^i(K^T)\}_{l\in[1,K]}$. The profit of the unit-demand entity l of bidder i when it acquires an item equals to: $\pi_l^i(K^T) = D^{-1}(K^T)\times 1 - c_l^i(K^T)$. Then the profit of bidder i when he purchases x_i items equals to: $D^{-1}(K^T)\times x_i - \sum_{l=1}^{x_i} c_l^i(K^T)$. This expression matches without loss of generality [33]'s expression for the profit of a firm since he restricts the analysis to convex cost functions[18]: any cost function $C_i(x_i, K^T)$ which is convex relative to the number x_i of items purchased and zero-valued at the origin can be expressed as the sum $\sum_{l=1}^{x_i} c_l^i(K^T)$. Then to apply Proposition 5.1 on the primitives $\{\pi_l^i\}_{l\in[1,K],i\in N}$ of the model, we have to check the following points in [33]:

- The non-crossing assumption (3.1) is satisfied since profit functions of any unit-demand identities are equals up to a translation.
- The map $\pi_l^i(K^T)$ is concave over K^T, which is satisfied since the demand is concave.
- The condition $\pi_l^i(K^T) - \pi_l^i(K^T+1) = \pi_k^j(K^T) - \pi_k^j(K^T+1)$ for any i, j, l, k is satisfied. Those differences are independent of the couples (l, i) and (k, j) in this application because $\pi_l^i(K^T)$ is additively separable in K^T and the couple (l, i).

First, our analysis shows that most of [33]'s assumptions are binding: in particular the various concavity assumptions. Second the additive separability between the dependance in i and in K^T is also crucial. Nevertheless, a slightly more general form for the cost function could be suitable: $C_i(x_i, K^T) = C_1(K^T)\cdot x_i + C_2^i(x_i)$, where $C_1(\cdot)$ and $C_2^i(\cdot)$ are both convex. It corresponds to the situation where the other producers impose also a negative externality to the cost function due to congestion costs for the supplying of the inputs as an example. The bidder-submodularity condition requires that this congestion cost is independent of the identity of the purchaser: the function $C_1(\cdot)$ does not depend on i (Note also that consequently assumption 3.1 is then automatically satisfied). On the other hand,

[18]In [33], for the cost function of bidder i, Ranger considers the general form $C_i(x_i)$ such that $\frac{\partial C_i(x_i)}{\partial x_i} \geq 0$ and $\frac{\partial^2 C_i(x_i)}{\partial^2 x_i} \geq 0$. So he excludes any dependence on K^T the total capacity sold. Nevertheless, he does not exclude $C_i(0) > 0$, that is a fixed cost introducing a non-convexity. Indeed with such a non-convexity it is possible to construct an example similar to our starting example where w is not bidder-submodular. Thus we fix $C_i(0) = 0$ in the following discussion and then assume that the cost function is convex.

this separability condition is not required for Vickrey to be robust: only the convexity of $C_i(x_i, K^T)$ over the variable K^T is required.[19]

Note that we can also apply proposition [4.1], which means that the Vickrey contingent auction is also robust in this framework. Indeed, there is no monitoring issue in the Vickrey contingent auction in this application because the seller can restrict easily the way bidder could bid because the group structure is clearly common knowledge: bidders should be constrained to report a valuation $(x, y) \to \pi(x, y)$ which is decreasing relative to y the number of capacity units sold to guarantee the robustness to losers' deviation. Thus in this particular application the benefit of AM-contingent auction relative to the Vickrey auction is not clear.

Anyway, we should stay modest relative to the relevance of this analysis for many capacity markets: it assumes that the bidders do not own any capacity prior to the market, i.e. non purchasers do not suffer from identity-dependant externalities. In particular, non-purchasers are indifferent to the total amount of capacity sold at the auction. Otherwise, the analysis does not hold anymore and the classical free-riding issues between incumbents in order to preempt entry arise. Thus this application is relevant only if the scarce resource is in possession of a monopoly who sold it once.

6 Extension with seller preferences

Our preceding results rely on the fact that the auctioneer is reporting non-strategically its true preferences $\Pi_0(\mathcal{A}) = 0$. Nevertheless, the contingent auctions we have defined in a general way take as input the preferences of the auctioneer Π_0. If the auctioneer can increase at some costs the total amount of items available, then it is worthwhile to investigate the scope of validity of our previous results if we allow the auctioneer to report (non strategically) her preferences over the number of items that are sold. The question is how should we restrict the seller's preferences to extend proposition [4.1] and [5.1]. The answer is that the seller's cost function should be convex.

Definition 11 (Convex production function) *The seller's cost function is convex if there exists an increasing and convex production function $c : \mathbb{N} \to \mathbb{R}^+$ such that:*

$$\Pi_0(\mathcal{A}) = -c(\#\mathcal{A}), \ for\ all \ \ \mathcal{A} \in \mathbf{A}$$

[19]Provided that assumption (3.1) is also satisfied, i.e. the most efficient producers without congestion are remaining the most efficient under congestion. As a example, this is satisfied for the general form $C_i(x_i, K^T) = C_1(K^T) \cdot \sum_{l=1}^{x_i} \lambda_l^i + C_2^i(x_i)$, where λ_l^i is the productivity factor of the unit l of producer i and $C_1(K^T)$ corresponds to the price of the input.

The following example illustrates the standard point that the bidder-submodularity condition may fail with a production function exhibiting increasing return to scale.

Example 6.1 *Consider the allocation of two items to two (unit-demand) bidders which are valuing an item* 2*. We suppose that the items are costly to produce for the auctioneer such that to produce one item costs* 2*, whereas the production of two items costs only* 3 *(in case of no production this costs is normalized to zero). Then there is a free-riding issue between the two bidders in order to make the seller produce. The bidder optimal frontier is not a singleton (The bidders have to share a surplus of* 1*) and truthful bidding is not an equilibrium in the AM-auction. To report the valuation* 1 *is a best response if the other bidder is truthful.*

We show that if the seller's cost function is convex then both propositions [4.1] and [5.1] still hold. We have restricted the analysis to a framework where the seller only cares about the number of items she sells and so does not care about the identity of the related purchasers. The proof relies on the fact that the seller can be viewed as additional bidders in an additional group with a neutral seller. Nevertheless, the seller does not have the same incentives to report the truth as any other bidders since she also captures the revenue. Therefore, we have to assume that she is non-strategic.

Proposition 6.1 *In the NGDE frameworks with unit-demand or with multi-unit demand, propositions [4.1] and [5.1] extend if the seller's cost function is convex and if the seller reports her true preferences nonstragically.*

Proof 4 *The assignment problem with the seller's revealed preferences derived from a convex production function is equivalent from the bidders' point of view to the assignment problem where the seller is neutral but where another group of* M *bidders has been added. Denote by* w^* *the coalitional value function of the 'new' assignment problem that we construct. Index by* $N+1,\dots,N+1+M$ *the* M *bidders that we add and who belong to the 'new' group* G_{g+1}*. Consider that they have unit-demand preferences suffering from no identity dependant externalities such that:*

$$\Pi_j = c(N+M+1-j) - c(N+M-j), \quad \forall N+1 \leq j \leq N+M, .$$

Then we obtain that:

$$w(S) = w^*(S \cup \{N+1,\dots,N+1+M\}),\ for\ all\ S \subset N\ and\ 0 \in S$$

The bidder-submodularity of the coalitional value w^* *implies that* w *is also bidder-submodular. Then both propositions 4.1 and 5.1 can be applied to this new framework.*

7 Conclusion: some policy perspectives and limits of the model

A practical issue for policy makers facing an assignment problem is to choose between a centralized procedure and a decentralized market-based procedure as auction mechanisms. This is a very old question in the combinatorial auction literature. In particular, Rassenti, Smith and Bulfin [35] studied the problem of allocating airport slots. Banks, Ledyard and Porter [4]'s experimentations have been motivated by the allocation of the Space Transportation System (sometimes called Space Shuttle) which is a scarce resource for which very different kind of bidders are competing (commercial satellites or scientific experiments for example) for the same homogenous good. Brewer and Plott [7] designed an auction to allocate use of a railroad track. This last allocation issue is even more difficult because allocated goods are determined endogenously by the market. Those papers experiment different combinatorial auction designs in order to argue that those complex assignments may be tackled by a decentralized auction process instead of the prevailing complex systems of hierarchical committees and detailed administrative rules.[20]

This paper is a critic of this literature which endows bidders with very simplified preferences, whereas many technical aspects of the application field have been very finely modelled as in Brewer and Plott [7].

In the Space Transportation System, the value to do a scientific experiment would be considerably reduced if some competing laboratories use the shuttle for the same experiment. Added to the insurance motives, those possible inefficiencies may explain why the competing institutions, i.e. the bidders inside a same group in our framework, may prefer to 'bid' jointly as a single bidder and avoid the inefficiencies due to the impossibility to express those externalities. Indeed, our work suggests that such a level of decentralization may be conceivable. The risk of coordination failure and the resulting duplication of capacities for similar outputs should not be an argument for a joint-bid in a contingent auction or for the persistence of centralized procedures.[21] Between full centralization and full decentraliza-

[20]There is much doubt that those committees are able to allocate efficiently the relating scarce resources. The allocation of airport slots in Europe is a good example. Following the International Air Transport Association's (the airlines' international trade association) guidelines, the European Commission, through the EU Slot allocation directive (95/93), confirms the grandfathering rights in case of disagreement in the committee as it is usually the case with new entrants. The possibility of resale markets (article 8.4) may solve crude inefficiencies in the allocation but the directive leaves each states free to regulate the secondary markets. Anyway, as emphasized in our companion paper, there are few chances that bilateral markets lead to the efficient allocation if externalities intervene at first order. The 'baby-sitting' of slots, i.e. the use of slots for unprofitable markets to prempt entry, is an obvious evidence that the assignment procedure is inefficient.

[21]Such joint-bids could nevertheless be justified by budget-constraints or complemen-

tion, there is midway: the planner determines the number of items that are to be auctioned for each sub-market and then entrusts to the market the assignment among each sub-market. This work suggests that the efficient allocation of the scarce resource among the different usage is achievable with contingent auctions which endogenize efficiently the number of items for each sub-markets.

In the slot allocation issue, the groups could be viewed as the different destinations each representing a specific market. In this case, some diversified airlines may use a slot for different destinations, which is not captured by our model where a bidder is exogenously attached to a group and only one group. The problem at hand is then more complex and is still characterized by externalities. In that case, an airline may be not only concerned about the identity of its joint-purchasers but also on the way its joint-purchasers uses their slots. It suggests that it could be appropriate in practical auction design to consider contingent auctions where bids are conditional not only on the identity of the purchasers but that are also contingent on the way the goods are used: therefore, as an example, an airport slot right should then be defined as a function of its destination. For railroad track use, bids could depend on the use (freight shipment versus different kind of passenger transportation). Such bids that are contingent on the usage of the good may be also relevant if the seller values some external effects that depend on the usage. Then it means that for an assignment problem the right derived from the allocation should depend on the specific use of the scarce resource.

Finally two open questions should be examined before opting for a decentralized assignment procedure. First, what is the practical relevance of the restriction imposed in propositions [4.1] and [5.1]? Second, does AM-auctions perform so bad when truthful reporting is not a dominant strategy? The second question suggests further research in economic experimentation, in particular to test the performance of AM-auctions with allocative externalities and without bidder-submodularity. Although experimental work seems desirable for testing the validity of auction design and determinate the most efficient auction for some families of preferences, such works deserve a lot of caution insofar as it could be difficult to replicate in experimental design the coalitional dimension of real life design. As our analysis, in line with AM, points out, combinatorial designs present some failures relative to joint deviations strategies.

Appendix

tarities.

A Preliminary remark

In all this appendix, in order to alleviate notation, the seller is associated to a group g_s such that if some items are remaining in the sellers' hand, it is considered that there are allocated in group g_s where all purchasers receive a null payoff. The reader can have in mind that there is a reserve of outside bidders which are valuing the item 0 and that do not impose any externality. Thus the number of items that are sold under those notations is a constant. If an item is removed from a group, it should go in the hands of another group.

B Proof Proposition 4.1

Note that any losing bidder receives a null transfer in the Vickrey auction because he suffers from no allocative externality when he receives no item and that consequently his presence do not modify the efficient assignment. Therefore, a deviation is profitable for a losing bidder if and only if he acquires a bundle of items at a smaller price than his valuation in this assignement.

We first derive the proof in the unit-demand framework: it consists in assuming that a profitable deviation of losing bidders exists and then raising a contradiction by proving that at least one deviant bidder makes a strictly negative profit. More precisely, we show that there is a group such that all deviant losers in that group obtain a negative profit. The proof in the multi-demand framework is similar and sketched very briefly.

B.1 Unit-demand

Suppose that a profitable joint-deviation of some losing bidders exists. For each group g, denote by N_g^V (N_g) the set of winning bidders in the Vickrey auction under truthful reporting (under the profitable joint deviation by some losing bidders) and n_g^V (n_g) the cardinality of this set. For each bidder l, denote respectively by π_l^V and p_l^V (π_l and p_l) the type reported by bidder l and the price paid this bidder in the Vickrey auction under truthful reporting (under the profitable joint deviation by some losing bidders) and $\mathcal{A}^V$ ($\mathcal{A}^{dev}$) the Vickrey, i.e. efficient allocation given the reported preferences. Note that throughout the proof we implicitly assume that the deviant losers can not report a type group than their own. Otherwise the report preferences would not fit with the NGDE framework.

Consider first the case where there exists a group g and two bidders l and k such that $l \in N_g \setminus N_g^V$ and $k \in N_g^V \setminus N_g$. It means that through the deviation bidder l has managed to obtain an item whereas a previous winner k under truthful reporting is not a purchaser anymore. Then, from the definition of Vickrey's transfer, $p_i \geq \pi_k(n_g) > \pi_l(n_g)$. The last inequality holds because bidder k is more efficient than bidder l. Finally, bidder k makes a strictly negative profit.

On the hand, if such case do not happen, then two groups g and g' exist such that: $N_g^V \subsetneq N_g$ and $N_{g'} \subsetneq N_{g'}^V$. Then consider $l \in N_g \setminus N_g^V$ and $k \in N_{g'}^V \setminus N_{g'}$. First we derive a condition resulting from $l \notin \mathcal{A}^V$ and $k \in \mathcal{A}^V$. Under the true preferences, the allocation $\mathcal{A}^V(k \curvearrowright l)$ is less efficient that $\mathcal{A}^V$:

$$\sum_{s \in N_g^V} \pi_s^V(n_g^V) + \sum_{s \in N_{g'}^V} \pi_s^V(n_{g'}^V) \geq \sum_{s \in N_g^V} \pi_s^V(n_g^V + 1) + \sum_{s \in N_{g'}^V \setminus \{k\}} \pi_s^V(n_{g'}^V - 1) + \pi_l^V(n_g^V + 1)$$

This expression can be rewritten in the following more suitable form to apply the concavity assumption on the function π_s where s is a winning bidder under truthful reporting.

$$\pi_l^V(n_g^V + 1) < \pi_k^V(n_{g'}^V) + \sum_{s \in N_g^V} \left(\pi_s^V(n_g^V) - \pi_s^V(n_g^V + 1)\right) - \sum_{s \in N_{g'}^V \setminus \{k\}} \left(\pi_s^V(n_{g'}^V - 1) - \pi_s^V(n_{g'}^V)\right) \tag{5}$$

Second we derive a similar condition resulting from $i \in \mathcal{A}^{dev}$ and $j \notin \mathcal{A}^{dev}$.

$$p_l \geq \pi_k(n_{g'} + 1) + \sum_{s \in N_g \setminus \{l\}} (\pi_s(n_g - 1) - \pi_s(n_g)) - \sum_{s \in N_{g'}} (\pi_s(n_{g'}) - \pi_s(n_{g'} + 1)) \tag{6}$$

The price p_l equals to the externality imposed by bidder l on the remaining bidders, whereas the second term is a lower bound for this externality.

Note that $\pi_s^V = \pi_s$ for any winner and thus for $s = j$, $s \in N_g^V$ or $s \in N_{g'}^V$

Then we can compare each term in the right hand of equations 5 and 6:

- The first terms satisfy $\pi_k(n_{g'} + 1) \geq \pi_k(n_{g'}^V)$ because $n_{g'} < n_{g'}^V$ and the mapping $x \to \pi_k(x)$ is decreasing.
- The second terms satisfy

  $$\sum_{s \in N_g \setminus \{l\}} \left(\pi_s^V(n_g - 1) - \pi_s^V(n_g)\right) \geq \sum_{s \in N_g^V} \left(\pi_s^V(n_g^V) - \pi_s^V(n_g^V + 1)\right)$$

  because $N_g \setminus \{l\} \subset N_g^V$, $\pi_s^V(n_g - 1) - \pi_s^V(n_g) \geq \pi_s^V(n_g^V) - \pi_s^V(n_g^V + 1)$ for $s \in N_g^V$ due to our suitable concavity assumption for the types reported by winners and finally $\pi_s^V(n_g - 1) - \pi_s^V(n_g) \geq 0$ for any reported preferences (reported should suffer from no-positive externalities).
- It is proved similarly that the third term satisfies

  $$\sum_{s \in N_{g'}} (\pi_s(n_{g'}) - \pi_s(n_{g'} + 1)) \geq \sum_{s \in N_{g'}^V \setminus \{k\}} \left(\pi_s^V(n_{g'}^V - 1) - \pi_s^V(n_{g'}^V)\right).$$

Finally, we have established that $p_l > \pi_l(n_g^V + 1) \geq \pi_l(n_g)$ which raises a contradiction with bidder l making a profitable deviation.

B.2 Multi-unit

Suppose that proposition [4.1] is false under multi-unit demand. Similarly, we can distinguish two events. First there exists one 'bundled' agents $B_i := \{l_1, \cdots, l_{B_i}\}$ who manages to purchase m items and such that the winners in bidder B_i's group now obtain $m' > m$ items less than they would under truthful reporting. In that event, it is rather immediate that B_i makes a strictly negative profit. Otherwise, there exist one 'bundled' agents B_i and some group $\{g_b\}_{b=1 \dots B_i}$ where $N_{g_b} \subsetneq N^V_{g_b}$ and such that the m items that B_i has purchased with the joint deviation can be decomposed in m' items taken to some winners in his group and $m - m'$ items taken to some winners in a group g_b. Finally, we prove that B_i makes a strictly negative profit in a similar way than in the case with unit-demand. This latter case was simpler because there was only one group g_b. On the other hand, with multi-unit demand, the analogs of condition 5 and 6 involve a double indexed sum.

C Proof Proposition 5.1

From proposition 4, it is sufficient to prove proposition 5.1 in the NGDE framework with unit demand. Thus we restrict the analysis to this case.

Note first that, as shown in Milgrom [29] [Theorem 8.2], bidder-submodularity is equivalent to:

$$w(S \setminus \{l\}) - w(S \setminus \{l, k\}) \geq w(S) - w(S \setminus \{k\}),\ \forall l, k, S \textit{ such that } l, k \in S \subset N$$

Note also that if $\{N, (\Pi_l)_{l \in N}\}$ is an allocation problem which satisfies the conditions of proposition 5.1, then for any $S \subset N$, the allocation problem $\{S, (\Pi_l)_{l \in S}\}$ also satisfies those conditions.

Consequently, to prove proposition 5.1, it is sufficient to prove that the following inequalities are true:

$$w(N \setminus \{l\}) - w(N \setminus \{l, k\}) \geq w(N) - w(N \setminus \{k\}), \quad \forall l, k \in N \tag{7}$$

for any allocation problem under proposition 5.1's assumptions.

For any allocation problem $\{S, (\Pi_l)_{l \in S}\}$, denote by S^* the optimal allocation.

Note that if either $l \notin N^*$ or $k \notin N^*$, then equation [7] is trivially satisfied because if $s \notin S^*, w(S \setminus \{s\}) = w(S)$ and w is an increasing function (we use the assumption that non-purchasers suffer from no allocative externalities). Thus, it is sufficient to prove equations [7] for $l, k \in N^*$.

Then our proof contains two steps. First, we establish a useful lemma which states that, under suitable conditions, if a bidder is a winner in the optimal allocation for a given set of competitors then if the set of competitors is reduced, this bidder remains a winner in the corresponding optimal allocation. Second, we check carefully equation [7], l and k belonging to either the same or different groups, each of this cases being divided in different events according to the groups of the 'new' winners for the allocation problems $N \setminus \{i\}$,$N \setminus \{j\}$ and $N \setminus \{i,j\}$.[22]

Lemma C.1 *In the NGDE framework, if $x \to \pi_i(x)$ is concave and if the equalities $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$ are fulfilled for any $x > 0$, any $i, j \in N$ such that $G(i) = G(j)$, then for any i, $(N \setminus \{i\})^* \supset N^* \setminus \{i\}$.*

Thus it states that if some winners are removed then the original winners are still winners.

Proof 5 *The optimal allocation is characterized by a vector $(x_s)_{s=1,\ldots,g}$ where x_s represents the number of bidders chosen in group g. Immediately, the optimal allocation given x_s is to allocate the item to the more efficient bidders in g which is unambiguous in the NGDE framework with assumption 3.1. For $s = 1, \ldots, g$, denote by H_s (respectively for all $l \in N$, H_s^{-l}) the function mapping x_s into a real number which represents the payoff of the group s (which is independent of the allocation of the other groups in the NGDE framework)(respectively of the group s when bidder l is removed). Denote by $\Gamma_s(x)$ (respectively $\Gamma_s^{-l}(x)$) the set of the x more efficient bidders in group s (respectively in group s when bidder l is removed). Then, $H_s(x) = \sum_{j \in \Gamma_s(x)} \pi_j(x)$ (respectively $H_s^{-l}(x) = \sum_{j \in \Gamma_s^{-l}(x)} \pi_j(x)$). Note that H_s and H_s^{-l} differ only if bidder l belongs to the group indexed by s.*

Lemma C.2 *For all s, H_s is concave on $\mathbb{N}$.*

Proof 6 *First, we can check easily than $2 \cdot H_s(1) \geq H_s(2)$. Second for $x > 0$, we derive: $H_s(x+2) - H_s(x+1) = \pi_{\Gamma_s(x+2) \setminus \Gamma_s(x+1)}(x+2) + \sum_{i \in \Gamma_s(x+1)} (\pi_i(x+2) - \pi_i(x+1))$ and $H_s(x+1) - H_s(x) = \pi_{\Gamma_s(x+1) \setminus \Gamma_s(x)}(x) + \sum_{i \in \Gamma_s(x+1)} (\pi_i(x+1) - \pi_i(x))$. The concavity assumption for the function π_i implies that the sum in the latter expression is superior than the sum in the former. We also have $\pi_{\Gamma_s(x+1) \setminus \Gamma_s(x)}(x) \geq \pi_{\Gamma_s(x+2) \setminus \Gamma_s(x+1)}(x) \geq \pi_{\Gamma_s(x+2) \setminus \Gamma_s(x+1)}(x+2)$. Thus the concavity holds.*

The optimal allocation is the solution of the following optimization program:

$$(x_s)_{s=1,\ldots,g} \in Arg \max_{\sum_{s=1}^{g} y_s = M} \sum_{s=1}^{g} H_s(y_s)$$

[22] This last part of the proof, enumerating the different events, is a bit fastidious and it might be fulfilled with some general formulas covering different events, but we do not believe that it will help to clarify the proof.

Now suppose that a bidder l exists such that $(N \setminus \{l\})^* \supset N^* \setminus \{l\}$ is false. Four cases may happen: first, $n_{g(l)}^{(N\setminus\{l\})^*} = n_{g(l)}^{N^*}$, second $n_{g(l)}^{(N\setminus\{l\})^*} = n_{g(l)}^{N^*} - 1$, third $n_{g(l)}^{(N\setminus\{l\})^*} > n_{g(l)}^{N^*}$ and fourth $n_{g(l)}^{(N\setminus\{l\})^*} < n_{g(l)}^{N^*} - 1$.

Independently of the assumption on the differences $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$, the two first cases can be excluded. It is immediate that if the same number of items is sold in group g_l, then the optimal allocation of the remaining items among the bidders in the other group is independent of bidder l's presence. Now suppose that $n_{g(l)}^{N^*} - 1$ items are sold in bidder l's group after his removal, then the assumption that $(N \setminus \{l\})^* \supset N^* \setminus \{l\}$ is false implies that there exist two groups $s, s' \neq g_l$ such that $n_s^{N^*} > n_s^{(N\setminus\{l\})^*}$ and $n_{s'}^{N^*} < n_{s'}^{(N\setminus\{l\})^*}$. We have:

$$\begin{cases} H_s(n_s^{(N\setminus\{i\})^*}) - H_s(n_s^{(N\setminus\{i\})^*} + 1) \leq H_s(n_s^{N^*} - 1) - H_s(n_s^{N^*}) < \\ < H_{s'}(n_{s'}^{N^*}) - H_{s'}(n_{s'}^{N^*} + 1) \leq H_{s'}(n_{s'}^{(N\setminus\{i\})^*} - 1) - H_{s'}(n_{s'}^{(N\setminus\{i\})^*}) \end{cases}$$

The first inequality (respectively the third) holds from the concavity of H_s ($H_{s'}$) and $n_s^{N^*} > n_s^{(N\setminus\{i\})^*}$ ($n_{s'}^{N^*} < n_{s'}^{(N\setminus\{l\})^*}$). The second inequality holds from the optimality of the allocation such that $x_s = n_s^{N^*}, x_{s'} = n_{s'}^{N^*}$ against $x_s = n_s^{N^*} - 1, x_{s'} = n_{s'}^{N^*} + 1$ everything else being equal.

The strict inequality between the two extreme term implies that

$$H_s^{-l}(n_s^{(N\setminus\{l\})^*}) - H_s^{-l}(n_s^{(N\setminus\{l\})^*} + 1) < H_{s'}^{-l}(n_{s'}^{(N\setminus\{l\})^*} - 1) - H_{s'}^{-l}(n_{s'}^{(N\setminus\{l\})^*}) \tag{8}$$

because H_k and H_k^{-l} are equal for $k = s, s'$ since bidder l does not belong to s and s'. This inequality raises a contradiction with the optimality of $x_s = n_s^{(N\setminus\{l\})^*}, x_{s'} = n_{s'}^{(N\setminus\{l\})^*}$ against $x_s = n_s^{(N\setminus\{l\})^*} + 1, x_{s'} = n_{s'}^{(N\setminus\{l\})^*} - 1$. Then it raises a contradiction with $(N \setminus \{i\})^*$ been optimal.

Now consider the third case: $n_{g(l)}^{(N\setminus\{l\})^*} > n_{g(l)}^{N^*}$. Then there exists a group s such that $n_s^{N^*} > n_s^{(N\setminus\{l\})^*}$. Denote g_l by s'. As in the above calculation, we have:

$$H_s(n_s^{(N\setminus\{l\})^*}) - H_s(n_s^{(N\setminus\{l\})^*} + 1) < H_{s'}(n_{s'}^{(N\setminus\{l\})^*} - 1) - H_{s'}(n_{s'}^{(N\setminus\{l\})^*})$$

Since $H_s = H_s^{-l}$ (l does not belong to s') and due to the assumption (4a) which implies that $H_{s'}(x) - H_{s'}(x+1) \leq H_{s'}^{-l}(x) - H_{s'}^{-l}(x+1)$, then we have that the inequality (8) is satisfied raising the same contradiction as above. The fourth case $n_{g(l)}^{(N\setminus\{l\})^*} < n_{g(l)}^{N^*} - 1$ is treated similarly by using assumption

(4b).[23]

The lemma allows to define[24] k_i and k_j such that: $(N \setminus \{i\})^* = N^* \cup \{k_i\} \setminus \{i\}$ and $(N \setminus \{j\})^* = N^* \cup \{k_j\} \setminus \{j\}$. Then, noting that $N \setminus \{i,j\} = (N \setminus \{i\}) \setminus \{j\}$, lemma C.1 implies that k_i and k_j are belonging to the set $(N \setminus \{i,j\})^*$. Then, if $k_i \neq k_j$, $(N \setminus \{i,j\})^* = N^* \cup \{k_i, k_j\} \setminus \{i,j\}$. On the contrary, if $k_i = k_j = k_*$, then the lemma allows us to define k_{ij} such that $(N \setminus \{i,j\})^* = N^* \cup \{k_*, k_{ij}\} \setminus \{i,j\}$.

First event: $G(i) = G(j) = G^*$ Consider first the case $k_i \neq k_j$. Then either $k_i \in G^*$ and $k_j \notin G^*$ or $k_j \in G^*$ and $k_i \notin G^*$. By symmetry of equation 7, it is sufficient to prove it in one this case, say $k_i \in G^*$ and $k_j \notin G^*$. Now, $w(N) - w(N \setminus \{i\}) = \pi_i(n_G^{N^*}) - \pi_{k_i}(n_G^{N^*})$ whereas $w(N \setminus \{j\}) - w(N \setminus \{i,j\}) = \pi_i(n_G^{N^*} - 1) - \pi_{k_i}(n_G^{N^*} - 1)$. We conclude with the convexity of π_j over j noting that $n_G^{N^*} > 1$.

Now consider the case $k_i = k_j = k_*$. This case is divided in four cases either $k_* \in G^*$ or $k_* \notin G^*$ and $k_{ij} \in G^*$ or $k_{ij} \notin G^*$.

1. $k_*, k_{ij} \in G^*$
   $w(N) - w(N \setminus \{i\}) = \pi_i(n_G^{N^*}) - \pi_{k_*}(n_G^{N^*})$
   $w(N \setminus \{j\}) - w(N \setminus \{i,j\}) = \pi_i(n_G^{N^*} - 1) - \pi_{k_{ij}}(n_G^{N^*} - 1)$.
   The conclusion is straightforward because $i < k_* < k_{ij}$ and that $G(i) = G(j)$ $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$.

2. $k_*, k_{ij} \notin G^*$

$$\begin{cases} w(N) - w(N \setminus \{i\}) = \pi_i(n_{G^*}^{N^*}) - \pi_{k_*}(n_{G(k_*)}^{N^*} + 1) \\ + \sum_{h \in \Gamma_{G^*}(n_{G^*}^{N^*}) \setminus \{i\}} \left\{\pi_h(n_{G^*}^{N^*}) - \pi_h(n_{G^*}^{N^*} - 1)\right\} \\ + \sum_{h \in \Gamma_{G(k_*)}(n_{G(k_*)}^{N^*})} \left\{\pi_h(n_{G(k_*)}^{N^*}) - \pi_h(n_{G(k_*)}^{N^*} + 1)\right\} \end{cases}$$

The expression of $w(N \setminus \{j\}) - w(N \setminus \{i,j\})$ depends on either $G(k_*) =$

[23] Remark the difference between assumption (4a) and (4b). For example, if assumption (4b) is satisfied, then the number of purchasers should not decrease more than 1 in bidder l's group after his removal.

[24] In what follows, k_i, k_j, k_{ij} are not empty due to the preliminary remark of the appendix.

$G(k_{ij})$ or not. In the first case $G(k_*) = G(k_{ij})$,

$$\begin{cases} w(N \setminus \{j\}) - w(N \setminus \{i,j\}) = \pi_i(n_{G^*}^{N^*} - 1) - \pi_{k_{ij}}(n_{G(k_*)}^{N^*} + 2) + \\ \sum_{h \in \Gamma_{G^*}(n_{G^*}^{N^*}) \setminus \{i,j\}} \{\pi_h(n_{G^*}^{N^*} - 1) - \pi_h(n_{G^*}^{N^*} - 2)\} + \\ \sum_{h \in \Gamma_{G(k_{ij})}(n_{G(k_{ij})}^{N^*}) \cup \{k_*\}} \left\{\pi_h(n_{G(k_{ij})}^{N^*} + 1) - \pi_h(n_{G(k_{ij})}^{N^*} + 2)\right\} \end{cases}$$

Then, making the difference and using the fact that π_h is concave, we have the following lower bound for $(w(N \setminus \{j\}) - w(N \setminus \{i,j\})) - (w(N) - w(N \setminus \{i\}))$:

$$\begin{cases} \pi_i(n_{G^*}^{N^*} - 1) - \pi_i(n_{G^*}^{N^*}) \geq 0 \\ +\pi_{k_*}(n_{G(k_*)}^{N^*} + 1) - \pi_{k_*}(n_{G(k_*)}^{N^*} + 2) \geq 0 \\ +\pi_{k_*}(n_{G(k_*)}^{N^*} + 1) - \pi_{k_{ij}}(n_{G(k_{ij})}^{N^*} + 2) \geq 0 \\ +\pi_j(n_{G^*}^{N^*} - 1) - \pi_j(n_{G^*}^{N^*}) \geq 0 \end{cases}$$

Then, inegality 7 holds.

In the second case $G(k_*) \neq G(k_{ij})$,

$$\begin{cases} w(N \setminus \{j\}) - w(N \setminus \{i,j\}) = \pi_i(n_{G^*}^{N^*} - 1) - \pi_{k_{ij}}(n_{G(k_*)}^{N^*} + 1) + \\ \sum_{h \in \Gamma_{G^*}(n_{G^*}^{N^*}) \setminus \{i,j\}} \{\pi_h(n_{G^*}^{N^*} - 1) - \pi_h(n_{G^*}^{N^*} - 2)\} + \\ \sum_{h \in \Gamma_{G(k_{ij})}(n_{G(k_{ij})}^{N^*})} \left\{\pi_h(n_{G(k_{ij})}^{N^*}) - \pi_h(n_{G(k_{ij})}^{N^*} + 1)\right\} \end{cases}$$

Denote by $w(N \setminus \{i\}, k_* \curvearrowright k_{ij})$ the surplus associated with allocation $(N \setminus \{i\})^* \cup \{k_{ij}\} \setminus \{k_*\}$. Then, we derive:

$$w(N) - w(N \setminus \{i\}) = (w(N) - w(N \setminus \{i\}, k_* \curvearrowright k_{ij})) + (w(N \setminus \{i\}, k_* \curvearrowright k_{ij}) - w(N \setminus \{i\}))$$

The second term is negative, whereas we can derive the first term as an expression easily comparable with $w(N \setminus \{j\}) - w(N \setminus \{i,j\})$:

$$\begin{cases} w(N) - w(N \setminus \{i\}, k_* \curvearrowright k_{ij}) = \pi_i(n_{G^*}^{N^*}) - \pi_{k_{ij}}(n_{G(k_*)}^{N^*} + 1) + \\ \sum_{h \in \Gamma_{G^*}(n_{G^*}^{N^*}) \setminus \{i\}} \{\pi_h(n_{G^*}^{N^*}) - \pi_h(n_{G^*}^{N^*} - 1)\} + \\ \sum_{h \in \Gamma_{G(k_{ij})}(n_{G(k_{ij})}^{N^*})} \left\{\pi_h(n_{G(k_{ij})}^{N^*}) - \pi_h(n_{G(k_{ij})}^{N^*} + 1)\right\} \end{cases}$$

Then we can conclude.

The next two remaining cases are solved similarly.

3. $k_* \in G^*$ and $k_{ij} \notin G^*$
4. $k_* \notin G^*$ and $k_{ij} \in G^*$

Second event: $G(i) \neq G(j)$ The proof for this case is completely analogous. Note that there are three possibilities for k_i: it could be either the best remaining bidder of $G(i)$ (denoted by k_1), the best remaining bidder of $G(j)$ (denoted by k_2) or the best remaining bidder of an another group say G' (denoted by k_3). For k_j, it is also only these three possibilities that matters. Then we write the couple (k_i, k_j) as $(k_h, k_{h'}), h, h' \in \{1, 2, 3\}$. Among those nine cases, we can first rule out (k_2, k_1), (k_3, k_1) and (k_2, k_3). Then, it is also quite straightforward that for (k_1, k_2), (k_1, k_3) and (k_3, k_2), $w(N \setminus \{j\}) - w(N \setminus \{i, j\}) = w(N) - w(N \setminus \{i\})$. The three remaining cases are treated similarly as before so we do not detail the proof.

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Chapitre 4: Individual Rationality under Sequential Decentralized Participation Processes

Individual Rationality under Sequential Decentralized Participation Processes*

Laurent Lamy†

Abstract

We consider the implementation of an economic outcome under complete information when the principal cannot commit to a simultaneous participation game. From a general class of sequential decentralized participation processes and without common knowledge on the details of the process, we introduce the concept of implementation under robust sequential individual rationality. We solve the optimal design program: the principal may fail to extract fully agents' surplus relative to the harsher threats but economic efficiency is not damaged.

Keywords: Mechanism Design, Individual Rationality, Imperfect Commitment, Surplus Extraction, Collusion on Participation

JEL classification: C72, D62

1 Introduction

The mechanism design paradigm considers that agents are taking their participation decisions simultaneously. However, the principal in many transactions lacks the ability to commit to close the participation at an exact deadline.[1] In corporate acquisitions and procurement auctions, it is common that the seller violates the announced rules to accept a subsequent better deal. McAdams and Schwarz [17] and Vartiainen [24] consider auction models where the seller is unable to commit not to solicit another round of offers after having publicly disclosed the previous offers. Similarly, in the corruption literature, e.g. Compte et al. [5], the auctioneer may also provide an opportunity for bid readjustments in exchange for a bribe.

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†Laboratoire d'Economie Industrielle, CREST-INSEE, 28 rue des Saints-Pères 75007 Paris. e-mail: laurent.lamy@ensae.fr

[1] In some auction design, as in the online version of the ascending auction used at Amazon and analysed by Ockenfels and Roth [21], the rule of the game explicitly involves an extension of the participation deadline after a submission.

We consider the implementation of an economic outcome under complete information relative to agents' preferences when the principal can not commit to any multi-stage mechanism and has no control on the participation process itself. Hence, the analysis is reduced to feasible participation games such that, sequentially, agents have the opportunity either to accept to participate in the mechanism or to delay their participation decisions, the final outcome depending only on the final set of participants. The general class of participation processes we consider relies on two ingredients that are common knowledge among agents: first, after each acceptation by a given agent, all the remaining agents will have the opportunity to participate. Second, agents that have already accepted the mechanism have the opportunity to secretly provide the evidence that they have accepted the mechanism to any nonparticipant before receiving his last opportunity to participate. Thus contrary to the aforementioned positive literature that assumes that participation decisions or offers are publicly observable, we consider the general case where participation decisions may not be observed but where participants have the opportunity to provide this evidence.

A mechanism is implementable if full participation is the only equilibrium outcome of any participation game. In the same vein as Moldovanu and Winter [20], we require implementation to be independent of the specific structure of the participation game, i.e. the order of the opportunities to participate. Moreover, in the same vein as Chung and Ely [4], we also require implementation to be robust to any kind of beliefs for the agents relative to the specific form of the process. This structure is not necessary for our main insight, i.e. the impossibility of full surplus extraction under decentralized sequential participation processes. But it allows a tractable characterization of the optimal mechanism.[2] The traditional individual rationality constraints are strengthened by requiring *robust sequential individual rationality.* Our implementation concept requires more than the traditional condition that participation is a best-response for agent i given that all the other agents participate. With perfect information, i.e. when participation decisions are publicly observed by all agents, Proposition 5.1 states that robust sequential individual rationality requires that there is no set of agents $S \subset N$ such that all agents in S prefer the outcome where only agents in $N \setminus S$ participate to the outcome where all agents accept the mechanism. Those additional constraints in the mechanism design program are non-linear and the set of implementable mechanisms is thus in general not convex. Nevertheless, the optimal design program can be simplified as done in Proposition 5.3, our main result: it allows us to separate the choice of the final allocation to

[2] Relaxing the common knowledge assumptions on the trading game may seem at odd with the so-called Wilson Doctrine [25] whose agenda is to relax the common knowledge assumption on agents' beliefs about another's preferences or information and not the ones about the trading process itself. We think that we remain coherent with the Wilson doctrine insofar as enforcement on the details of the participation process seems difficult.

the structure of the optimal threats. As under a simultaneous participation game, the coasian logic still applies and we obtain that the optimal mechanism is efficient. Nevertheless, full extraction relative to the harsher threats as in Jehiel et al [13] does not work anymore generally in presence of externalities. In an incomplete information setup, Heifetz and Neeman [11] show that generic priors on the universal type space do not allow for full surplus extraction. Their insight is that, generically, private information implies informational rents. Here, with the common concern for robust mechanism design, we relax the common knowledge assumptions on the details of the participation process and show that the principal may not be able to fully extract agents' surplus relative to their harsher threats in a complete information setup.

The paper is organized as follows. In section 2 we introduce the general allocation problem. Using a simple example, section 3 illustrates our critic of the traditional mechanism design approach and supplies intuition for our characterization of the optimal mechanism. In section 4 we describe a general class of noncooperative sequential participation games. In section 5 we define our main concept- *robust sequential individual rationality* -and prove the main results. In section 6 we provide two general examples where our alternative mechanism design approach may be relevant and change some insights. Concluding remarks are gathered in section 7.

2 The Model

Let $N = \{1, 2, \ldots, n\}$ be a set of agents and $A = \{a_1, a_2, \ldots, a_K\}$ be a finite set of possible outcomes. Denote by $\Sigma(N)$ the set of the permutations over the set N. For a given permutation $\sigma : N \to N$, denote by T_i^σ the subset $\{\sigma(1), \sigma(2), \ldots, \sigma(i-1)\}$, i.e. the $i-1$ first smallest agents according to the implicit order defined by σ. We assume that the agents and the principal, characterized by the subscript 0, have quasilinear preferences over outcomes and (divisible) money. Preferences are assumed to be common knowledge. The utility of a player i over outcome $a \in A$ and the money transfer t_i is:

$$\mathcal{U}_i(a, t_i) = V_i^a - t_i.$$

We first describe the class of procedures among which the principal chooses an optimal mechanism. In step 1, the principal designs a mechanism. In a complete information setting, a mechanism, denoted by $(\mathbf{a}, \mathbf{t})$, specifies a final outcome $\mathbf{a}(S)$ and a vector of monetary transfers $\mathbf{t}(S)$ for each possible set of participants $S \subset N$. We emphasize that such a reduced form mechanism should not be viewed as resulting from some Revelation Principle, a logic that can not be invoked in our framework. On the contrary, we consider implicitly that the principal has a very limited commitment power: he can not commit to any multi-stage game, i.e. he can not commit not to change the rule of the game after observing some report, but rather only in one shot mechanism. We make the further restriction that the outcome depends only on the set of participants, a maxmin foundation for such an approach being given in section 5.3.[3] In step 2, the agents are playing a sequential decentralized participation process described in next section. In the previous mechanism design literature, the decisions whether to participate or not in the proposed mechanism are assumed to be taken simultaneously. Here we consider that the principal cannot commit to such a simultaneous participation game: an agent will always have at least one opportunity to participate in the mechanism after each decision to accept the mechanism by an agent. In step 3, the mechanism is implemented according to the participation set $S \subset N$. A mechanism is said to be *feasible* if:

- For each set of participants S, the final outcome belongs to $\mathcal{A}(S)$, the subset of A of accessible or feasible outcome with the consent of agents in S.
- If agent i decides not to participate the principal cannot extract a positive payement from that agent: $\mathbf{t}_i(S) \leq 0$, if $i \in N \setminus S$.
- Transfers are budget-balanced: $\sum_{i=0}^{n} \mathbf{t}_i(S) = 0$, for any $S \subset N$.

The second and third restrictions are standard. The first restriction means that some outcome in A may not be feasible if some agents refuse to participate. For example, in the case of the sale of an indivisible good, Jehiel et al. [13] considers that one cannot 'dump' the object on a non-participating agent. We do not impose any specific structure on the feasibility sets $\{\mathcal{A}(S)\}_{S \subset N}$ except that:

Assumption 1 *$\mathcal{A}(S) \subset \mathcal{A}(T)$, whenever $S \subset T$.*

Assumption 1 states that if the consent of the agents in S is enough to implement a given final outcome a, then the extra consent of some agents outside S cannot make this outcome unfeasible. Then, there is no loss of generality to consider that $\mathcal{A}(N) = A$. To simplify the exposition, we assume that, for a given utility level, an agent strictly prefers to participate in the mechanism. With this trick, the set of implementable mechanisms - which is defined in section 5 -is a closed set and has thus an optimal element. Furthermore, in the sequential participation process, we also assume that agents prefers to accept the mechanism as soon as possible, for a given

[3] Implicitly, we also exclude any mechanism that depends on the precise order of the participation decisions of the agents. One argument is that the timing of the participation decisions is not verifiable. Note nevertheless that Jehiel et al [13]'s full extraction mechanism with the optimal threats for each agent can not always be reached with such richer mechanisms confirming thus our main insight that individual rationality constraints should be strengthened (it can be shown in our simple example).

outcome. This additional trick is also innocuous, but allows us to consider a general universal belief space à la Mertens-Zamir [19] still avoiding related pathological phenomena.

For an agent i and a set of participants $S \subset N \setminus \{i\}$, denote by $a_i^*(S)$ the harsher feasible threat that the principal can inflict on i given that the agents in S have accepted the mechanism: $a_i^*(S) \in Arg \min_{a \in \mathcal{A}(S)} V_i^a$. Denote by $V_i^*(S) = V_i^{a_i^*(S)}$ the corresponding utility level. In mechanism design under simultaneous participation, only the threats $a_i^*(N \setminus \{i\})$ do matter. In the optimal design, if one agent refuses the mechanism, the remaining ones commit to this harsher threat also called 'minmax punishment' as in Jehiel et al. [13] or Dequiedt [8]. On the other hand, in mechanism design under robust sequential individual rationality, the whole set of the feasible threats $a_i^*(S)$ will play an active role in the computation of the optimal mechanism.

Finally, our framework is characterized by the 4-uple: $(N, A, \{V_i^a\}_{i \in N, a \in A}, \{\mathcal{A}(S)\}_{S \subset N})$. Let us define two special subsets among those frameworks: *externality-free* and *negative-externality-free* frameworks.

Definition 1
- *A framework is said to be* externality-free *if for any agent i, the map $a \to V_i^a$ is constant over the set $\mathcal{A}(N \setminus \{i\})$.*
- *A framework is said to be* negative-externality-free *if the optimal threat $V_i^*(S)$ for any agent i is independent of the set of participant $S \subset N \setminus \{i\}$: $V_i^*(S) = V_i^*(\emptyset)$ for any i.*

A framework is said to be externality-free if the agents do not care about the final outcome in the event where they do not participate in the mechanism. For the sale of some goods and under the assumption that a non-participant does not receive any good, it corresponds to the standard case where agents care only on the set of goods they obtain and in particular are indifferent to the final allocation when they are non-purchaser. Negative-externality-free is less restrictive: it only requires that the principal can credibly threat any agent with the minmax punishment independently to the other participants, i.e. by retaining all goods in the above example.

3 A Simple Example

The following example consider the sale of a single object involving identity-dependent externalities. It formalizes the starting examples of Jehiel and Moldovanu [12] and Das Varma [7] where two potential buyers suffering from important reciprocal negative externalities prefer not to participate in the bidding process for a single item and let a third buyer win at a low price. We emphasize that those previous modellings can not embrace our kind of strategic nonparticipation.

Let $n = 3$ and $A = \{0, 1, 2, 3\}$ where allocation i corresponds to the allocation of the item to player i. We consider that the seller is able to allocate the item only to participating agents: $\mathcal{A}(S) = \{i | i \in S \cup \{0\}\}$. Let V_i^i be equal respectively to V, v and 0 for $i \in \{1, 2\}$, $i = 3$ and $i = 0$. Let V_i^j, $i \neq j$ be equal to $-\alpha$ if $i, j \in \{1, 2\}$ and 0 otherwise. Assume that $V > v > V - \alpha > 0$. Thus the efficient allocation consists in allocating the item to agent 3. Nevertheless, agents 1 and 2 are valuing the item more than agent 3. They are also chosen symmetric only to simplify the exposition. The same kind of results holds in the neighborhood of the parameter values or even with huge asymmetries between agents 1 and 2 provided that the reciprocal negative-externalities between them are big enough.

Standard Auctions Consider first a simultaneous participation game as in [12]: the buyers have first the opportunity to decide whether or not they want to participate in the auction. Those decisions are made simultaneously and are publicly revealed before the auction takes place. We consider the first price auction, but the results are similar for any other standard auction as the English button auction considered in [7]. In any equilibrium, the item is sold either to agent 1 or to agent 2. In the unique symmetric equilibrium, agents 1 and 2 both participate with probability 1 and are submitting the bid $V + \alpha$. They are both suffering from a loss of α compared to their profit in the case where they could jointly coordinate themselves not to participate. In our example, non-participation from agent 1 is vain and cannot prevent the purchase by agent 2 in the auction because $V > v$.[4]
Now consider a sequential participation game with agents 1 and 2 such that potential buyers are always eligible to participate after one has decided to participate and such that participation decisions are publicly observable among agents (the proper formalization is done in section 4). Now it is a subgame perfect equilibrium for agents 1 and 2 not to participate if and only if his 'feared' opponent did so. Under sequential participation, we obtain the paradox that seems to correspond to the stories reported in [12, 7] and that cannot emerge in previous models with simultaneous participation: an agent may prefer not to submit a bid though his intrinsic value for the good, i.e. excluding the motivations to outbid resulting from the fear of negative externalities, is greater than the final bid.

The Optimal Mechanism Under a simultaneous participation game, Jehiel et al. [13] presents an optimal mechanism where participation is a strictly dominant strategy. The optimal mechanism is efficient and the seller can extract surplus from agents who do not obtain the object by using the optimal threats $a_i^*(N \setminus \{i\})$ for each agent i. Here the efficient allocation is to allocate the object to agent 3 and the optimal mechanism raises the revenue $v + 2 \cdot \alpha$:

[4]Strategic non-participation as in [12] emerges only if $v > V$ and thus not here.

each non-purchaser has to pay α in order to avoid that the seller gives the object to his most feared opponent. But what can be implemented if agents 1 and 2 can coordinate their participation decisions thanks to a sequential participation process? Then the seller can not allocate the object to agent 3 and extract a strictly positive surplus from both agents 1 and 2. In particular, she cannot threat simultaneously agents 1 and 2 with their respective tougher threat. Otherwise, they could jointly not participate and obtain a null payoff since the seller is assumed to be unable to 'dump' the object. To maximize her revenue, the seller should use a *divide and conquer* strategy: it consists in giving the incentive to participate for one agent, say 1, independently of the participation decision of agent 2. Then given that agent 1 participates, she could really threat agent 2 to allocate the object to agent 1 in case of non-participation. Indeed we will show that it is the optimal mechanism and it raises the revenue $v + \alpha$. It illustrates several features that are generalized in section 5: first, the optimal selling procedure is still efficient under the *robust sequential individual rationality* constraints; second, those constraints reduce the revenue. Finally, we find surprisingly that although agents 1 and 2 are symmetric, they should not be treated in a symmetric way in an optimal mechanism. That is the reason why standard auctions that are intrinsically symmetric were leading to joint non-participation.

4 Sequential Participation Processes

We describe a simple sequential participation procedure based on a given mechanism $(\mathbf{a}, \mathbf{t})$. Suppose that the agents in S have already accepted the mechanism, then the remaining agents are playing a participation game where each agent has once the possibility to accept the mechanism or to delay his decision. If all those agents do not accept the mechanism, then the participation game stops and the outcome $(\mathbf{a}(S), \mathbf{t}(S))$ is implemented. Otherwise, if at least one agent i accepts the mechanism, then his acceptance is followed by a participation game given the consent of $S \cup \{i\}$. Observe that, in this informal described situation, the order according to which agents have the possibility to accept or not the mechanism has not been specified. Indeed, each order generates a different extensive form game. Moreover, we should also specify the structure of the information sets. We wish to compare final outcomes in these different games, and therefore we proceed to a formal description of the participation games.

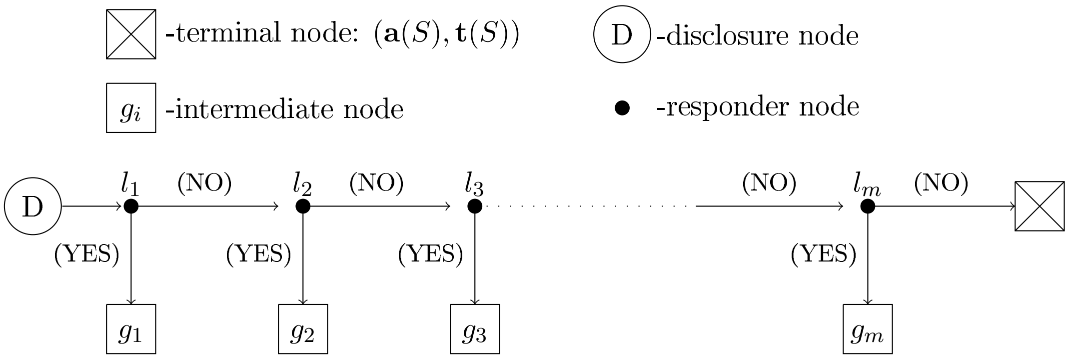

Figure 1

For a given mechanism $(\mathbf{a}, \mathbf{t})$, we define recursively the set of participation games as a function of the cardinality of the set of the agents that have already accepted the mechanism. We denote by $\mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$ the set of participation games if the agents in S have accepted the mechanism and where $\beta = \{\beta_i\}_{i \in N \setminus S}$ represents the vector of the initial priors about the acceptance of the other agents. If $S = N$, this set corresponds to the (unique) degenerate game where agents make no choice and the final outcome $(\mathbf{a}(N), \mathbf{t}(N))$ is implemented. If $S \subsetneq N$, we consider a participation game $g = ((\mathbf{a}, \mathbf{t}), S, \{l_i\}_{i=1,\ldots,m}, \{g_i\}_{i=1,\ldots,m}, \{\beta_i\}_{i \in N \setminus S})$ where $(\mathbf{a}, \mathbf{t})$ is a feasible mechanism, S is the set of the agents that have previously accepted the mechanism, $\{l_i\}_{1,\ldots,m}$, where $m = \# N \setminus S$, is an ordered list of the agents in $N \setminus S$ and g_i is a participation game in $\mathcal{G}(\mathbf{a}, \mathbf{t}, S \cup \{l_i\}, \{\beta_i\}_{i \in N \setminus S \cup \{l_i\}})$ which is properly defined by the induction hypothesis. See Figure 1.

There are four kinds of positions in $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$:

1. Disclosure nodes of the form $S_{current}$ where $S_{current}$ is the current set of the agents that have previously accepted the mechanism $(\mathbf{a}, \mathbf{t})$.

2. Responder nodes of the form (l_i, S), where $S \subset N$ is the set of the agents that have previously accepted the mechanism and $l_i \in N \setminus S$ is the identity of the agent with the initiative.

3. Intermediate nodes of the form g_i, where g_i is a participation game in $\mathcal{G}(\mathbf{a}, \mathbf{t}, S \cup \{l_i\}, \beta)$.

4. Terminal nodes of the form $(\mathbf{a}, \mathbf{t}, S)$ where S is the set of the agents that have previously accepted the mechanism $(\mathbf{a}, \mathbf{t})$.

At an intermediate node g_i, agents have no choice and the game moves to the disclosure node of the game g_i or moves to the terminal node $(\mathbf{a}, \mathbf{t}, N)$ if all agents give their consent. At a terminal node $(\mathbf{a}, \mathbf{t}, S)$, the game ends and the outcome $(\mathbf{a}(S), \mathbf{t}(S))$ is implemented.

At any responder position (l_i, S) there is the choice:

1. (l_{i+1}, S) if $i < m$ which means that agent l_i delays participation and l_{i+1} becomes the new responder. It corresponds to the two first arrays (NO) at the left of Fig. 1.

2. $(\mathbf{a}, \mathbf{t}, S)$ if $i = m$ which means that agent l_i refuses participation and the game ends at this terminal node. It corresponds to the array (NO) at the extreme right of Fig. 1.

3. g_i which means that agent l_i accepts the mechanism and the game moves to the intermediate node g_i. It corresponds to the arrays (YES) in Fig. 1.

At a disclosure node $S_{current}$, each agent in $S_{current}$ can secretly give the evidence that he has accepted the mechanism to any subset of his opponents. This evidence can also be revealed by the nature in a purely exogenous way.

The disclosure structure in the participation game has been incorporated to go beyond participation games with perfect information, where participation decisions are publicly observed by the agents. The precise structure of the information states will not matter: the crucial ingredient is that each participant has the opportunity at least once to prove to any of the remaining potential participants that he has accepted the mechanism.

The closure of the participation game after a finite number of delays may seem incoherent with our paradigm that agents do not decide whether they accept or reject the mechanism but rather that they either accept the mechanism or delay their acceptance decision. Indeed, apart from technical details, our following analysis is unchanged with the related infinite participation games, i.e. if $\{l_i\}_{i=1,\dots,m}$ with $m = \infty$ is an infinite ordered list of agents, where an infinite delay for the agents in $N \setminus S$, given that agents in S gave their consent, results in the implementation of the outcome $(\mathbf{a}(S), \mathbf{t}(S))$. Participation games with a finite number of nodes and a unique opportunity to accept the mechanism at a given stage have been chosen to ease the backward induction argument and the presentation.

Let $\mathcal{G} = \bigcup_{S \subset N} \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$ be the set of all participation games and $\mathcal{G}_{PI} \subsetneq \mathcal{G}$ the subset of the participation games with perfect information, i.e. the set of current participants is publicly revealed at the disclosure node. Since the number of participation games is finite, the parameter-space of the game they are playing about which they are uncertain is compact. Then we can build a universal type space Ω à la Mertens-Zamir [19] to represent agents' beliefs in the participation game (respectively Ω_{PI} with perfect information). Hence, our analysis does not hinge on any specific bidders' beliefs about irrelevant details of the participation process.

Then we assume that, whenever possible, beliefs are updated according to Bayes' rule. When an agent (or the Nature) provides the evidence to some agents that he has accepted the mechanism then their first-order beliefs are then stuck to the probability one that he has accepted the mechanism.

5 Optimal Design under robust sequential individual rationality

We now define a rationality constraint that removes the dependence of the participation decisions on the exact structure of the participation game and the corresponding beliefs of the agents.

Definition 2

- *A mechanism* $(\mathbf{a}, \mathbf{t})$ *is* robustly sequentially individually rational *(respectivelly robustly sequentially individually rational with perfect information) if* $(\mathbf{a}(N), \mathbf{t}(N))$ *is the final outcome in any subgame perfect equilibrium of any participation game (with perfect information)* $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$.

- *A mechanism is* implementable *(with perfect information) if it is feasible and robustly sequentially individually rational (with perfect information).*

- *A mechanism* $(\mathbf{a}, \mathbf{t})$ *is* implementable under simultaneous participation *if it is feasible and if full participation is an equilibrium of the simulatenous participation game, i.e. if participation is a best response conditionally on the participation of all the other agents.*

The sets of implementable, implementable with perfect information and implementable under simultaneous participation are respectively denoted by $\daleth$, $\daleth_{PI}$ and $\daleth_{sim}$. Note that our implementation concept under simultaneous participation is milder than the dominant strategy implementation concept according to which Jehiel et al. [13] shows that the principal obtains full extraction of the optimal threats under complete information.

Note the difference of the 'order independent' nature of our implementation concept with the 'order independent equilibrium' concept of Moldovanu and Winter [20]. In a nutshell, [20] considers *strategy profiles* that are *an* equilibrium independently of the specific structure of their coalition formation game. Here we consider *final outcomes*, more precisely the outcome $(\mathbf{a}(N), \mathbf{t}(N))$ derived from full participation, that are the equilibrium outcome of *any* equilibrium independently of the specific structure of the participation game and agents' initial beliefs.

5.1 Participation processes with perfect information

Proposition 5.1 *A mechanism* $(\mathbf{a}, \mathbf{t})$ *is implementable with perfect information if and only if it is feasible and for any* $S \subset N$

$$\max_{i \in N \setminus S} \{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)}\} \geq 0 \tag{1}$$

Proof 1 *We first prove the 'Only if' part. Suppose that* $(\mathbf{a}, \mathbf{t})$ *is implementable and that there exists a subset* S *such that* $V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)} < 0$ *for any agent* $i \in N \setminus S$*. Then consider a participation game* $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, N, \beta)$ *where* β *is such that it is common knowledge among the agents in* S *believe that the agents in* $N \setminus S$ *have accepted the mechanism. Hence, agents in* S *are playing as in a participation game in* $\mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$ *where it is common knowledge that the agents in* $N \setminus S$ *have accepted the mechanism. At any node where he is the last responder, the best response of an agent in* S *is to refuse the mechanism (if he accepts, the only equilibrium outcome is full participation since we have assumed that* $(\mathbf{a}, \mathbf{t})$ *is implementable). By backward induction, the best response of an agent in* S *at any node is to delay. Consequently, any subgame perfect equilibrium of the game* g *leads to the non-participation of the agents in* $N \setminus S$ *which raises a contradiction.*

The sufficiency part is proved by induction on the cardinality of the set of the agents that have already accepted the mechanism. The initial step where this set has the cardinality n *is immediate. Now consider that all agents in* $S \subsetneq N$ *have accepted the mechanism and suppose that* $\max_{i \in N \setminus S} \{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)}\} \geq 0$*. By the induction hypothesis, we obtain that every agents accept the mechanism in any subgame perfect equilibrium of any subgame* $\{g_i\}_{i=1,\ldots,m}$ *of the participation game* $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$*. It remains to show that, for any game* $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$*, it cannot belong to any equilibrium path that all agents refuse the mechanism at the responder nodes* $\{l_i\}_{i=1,\ldots,m}$*. In such a case, the agent* i *such that* $V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)} \geq 0$ *has a profitable deviation: he accepts (with probability one) the mechanism when he is the responder, i.e. for a responder node such that* $l_k = i$*, which exists from the structure of the participation game. Note that in the case where he believes that the equilibrium outcome is still full participation if he delays, then he prefers strictly to accept immediately the mechanism as it has been assumed.*

The inequality (1) with $S = N \setminus \{i\}$ corresponds to the standard individual rationality constraint of agent i in the standard mechanism design approach under a simultaneous participation game. Thus the lack of commitment in the participation game results in a limitation of the set of implementable mechanisms. On the other hand, in an externality-free framework, the standard individual rationality constraints $V_i^{\mathbf{a}(N)} + \mathbf{t}_i(N) \geq V_i^{\mathbf{a}(N \setminus \{i\})}$ imply that $V_i^{\mathbf{a}(N)} + \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)} \geq 0$ for any S and any $i \in N \setminus S$, the inequalities (1) are thus satisfied. Those points are summed up in the following corollary.

Corollary 5.2 *Any implementable mechanism with perfect information is implementable under simultaneous participation. In an externaltity-free framework, the converse holds: a mechanism that is implementable under simultaneous participation is implementable with perfect information.*

In the previous literature on mechanism design (with possibly incomplete information), the set of constraints that makes a mechanism implementable, i.e. feasibility, incentive compatibility and individual rationality constraints, results from inequalities that are linear according to the mechanisms $(\mathbf{a}, \mathbf{t})$.[5] Thus the set of the mechanisms that are implementable is a convex set. Moreover, the payoff of the principal depends linearly on the mechanism. From an optimal design perspective, there is thus no loss of generality to consider mechanisms that are symmetric if agents are symmetric. Suppose that a given asymmetric mechanism m is optimal. Then consider the permutations m_σ of this mechanism where $\sigma \in \Sigma(N)$. By symmetry, those mechanisms *implement* the same revenue for the principal. Finally, the mechanism $\frac{1}{n!} \sum_{\sigma \in \Sigma(N)} m_\sigma$ implements the same revenue in a symmetric way. On the contrary, the robust sequential individual rationality constraint results from inequalities involving the maximum of some linear maps and is thus not linear. Let us reconsider our simple example to illustrate the possible non-convexity of the set of implementable mechanisms.

Example 5.1 ***A simple example (suite)*** *Let $\mathbf{a}(S) = 1$, $\mathbf{t}_1(S) = V$ and $\mathbf{t}_i(S) = 0$ if $i \neq 1$ in the event where $1 \in S$ and $2 \notin S$. Let $\mathbf{a}(S) = 0$, $\mathbf{t}_i(S) = 0$ for any $i \in N$ in the event where $1 \notin S$. Let $\mathbf{a}(S) = 3$, $\mathbf{t}_1(S) = 0$, $\mathbf{t}_2(S) = \alpha$ and let $\mathbf{t}_3(S) = v$, if $S = \{1, 2, 3\}$ and $\mathbf{a}(S) = 0$, $\mathbf{t}_1(S) = 0$, $\mathbf{t}_i(S) = \alpha$ for any $i \in N$ in the event where $S = \{1, 2\}$. It is easily checked that this mechanism is feasible. Agents 1 and 3 obtain the same utility level (zero) independently of the final set of participants. Thus the inequalities (1) are satisfied if either 1 or 3 belongs to $N \setminus S$. Thus, it remains to check that the inequality (1) is satisfied if $S = \{1, 3\}$. Finally, the mechanism $(\mathbf{a}, \mathbf{t})$ is implementable. The mechanism $(\mathbf{a}', \mathbf{t}')$ where the roles of 1 and 2 have been switched is implementable by symmetry. Now consider the mechanism where, at a terminal node, each mechanisms $(\mathbf{a}, \mathbf{t})$ and $(\mathbf{a}', \mathbf{t}')$ are implemented with probability one half. This mechanism is of course feasible. Nevertheless, it is not robustly sequentially individually rational. The constraint (1) with $S = \{3\}$ is violated. If agents 1 and 2 do not jointly participate, they obtain a null payoff. On the contrary, under full participation, their expected payoff is equal to $-\frac{\alpha}{2}$. Indeed the (efficient) mechanism $(\mathbf{a}, \mathbf{t})$ is the optimal design as it will appear as an application of proposition 5.3.*

[5] The implicit space structure according to which linearity applies is the following. For two mechanisms, $(\mathbf{a}, \mathbf{t})$ and $(\mathbf{a}', \mathbf{t}')$ and a real number $\lambda \in [0, 1]$, the mechanism $\lambda \cdot (\mathbf{a}, \mathbf{t}) + (1 - \lambda) \cdot (\mathbf{a}', \mathbf{t}')$ is the mechanism that implements the mechanism $(\mathbf{a}, \mathbf{t})$ (respectively $(\mathbf{a}', \mathbf{t}')$) with probability λ (resp. $(1 - \lambda)$).

There is no loss of generality to invite all agents to the mechanism since the set of feasible allocations does not shrink when some participants are added (Assumption 1). It suffices to extent the mechanism to the additional agents such that they do not modify the final outcomes and that they receive no transfer. The optimal design program is thus:

$$\max_{(\mathbf{a},\mathbf{t})} V_0^{\mathbf{a}(N)} + \sum_{i=1}^{n} \mathbf{t}_i(N)$$

subject to

$$\forall S \subset N, \max_{i \in N \setminus S} \{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)}\} \geq 0,$$

where $(\mathbf{a}, \mathbf{t})$ is a feasible mechanism.

Nevertheless, in this form, the program is hardly tractable and it is unclear whether the optimal design is efficient. We simplify the program by showing that there is no loss of generality to restrict the maximisation to a subclass of implementable mechanisms which are fully characterized by a couple $(\alpha, \sigma) \in A \times \Sigma(N)$. Let us introduce a last useful notation: for a given set $S \subset N$ and a permutation $\sigma \in \Sigma(N)$, denote by $j(S, \sigma)$ the smallest agent according to the order σ that is not belonging to S. Formally, $j(S, \sigma) = \max\{j \in N | T_j^\sigma \subset S\}$. This agent plays a key role in the subclass that we define below and such that if the set of participants is S, the principal will inflict the minmax punishment to the agent $j(S, \sigma)$.

Definition 3 *For $(\alpha, \sigma) \in A \times \sigma(N)$, we define the (α, σ)- optimal threat mechanism as the mechanism $(\mathbf{a}, \mathbf{t})$ defined in the following way:*

- $\mathbf{a}(N) = \alpha$
- $\mathbf{a}(S) = a^*_{j(S,\sigma)}(S)$, *if* $S \subsetneq N$
- $\mathbf{t}_i(N) = V_i^\alpha - V_i^*(T^\sigma_{\sigma^{-1}(i)})$
- $\mathbf{t}_i(S) = 0$, *if* $S \subsetneq N$

Those mechanisms can be interpreted in the following way: take one agent, $\sigma(1)$, and give him the incentive to participate independently to the participation decision of the other agents by using the optimal threat among $\mathcal{A}(\emptyset)$; then take another agent, $\sigma(2)$, and give him the incentive to participate taken as given that $\sigma(1)$ surely participates and independently to the participation decisions of the other agents in $N \setminus \{\sigma(1)\}$ by using the optimal threat among $\mathcal{A}(\{\sigma(1)\})$; and so on. In particular, for the last agent, $\sigma(N)$, in this new order σ, the principal uses the optimal threat in $\mathcal{A}(N \setminus \{\sigma(N)\})$ as in the standard literature with simultaneous participation.

We first show that this restricted class of mechanisms is a subset of the implementable mechanisms.

Lemma 5.1 *Any (α,σ)- optimal threat mechanism is implementable.*

Proof 2 *It is immediately feasible by definition of $a^*_{j(S,\sigma)}(S)$ which is the minmax punishment for agent $j(S,\sigma)$ given the participation set S. Consider $S \subset N$ and the agent $j(S,\sigma)$ who does not belong to S. We have:*

$$V_{j(S,\sigma)}^{\mathbf{a}(N)} - \mathbf{t}_{j(S,\sigma)}(N) - V_{j(S,\sigma)}^{\mathbf{a}(S)} = V^*_{j(S,\sigma)}(T^\sigma_{\sigma^{-1}(j(S,\sigma))}) - V^*_{j(S,\sigma)}(S) \geq 0$$

The equality comes from the definition of $\mathbf{t}_{j(S,\sigma)}(N)$ and because $\mathbf{a}(S) = a^*_{j(S,\sigma)}(S)$. The inequality is satisfied because $T^\sigma_{\sigma^{-1}(j(S,\sigma))} = \{\sigma(1),\ldots,\sigma(j(S,\sigma)-1)\} \subset S$ (the inclusion comes from the definition of $j(S,\sigma)$). Thus we have proved that the inequality (1) holds for any $S \subset N$.

Then we show that there is no loss of generality to look for an (α,σ)-optimal threat mechanism to solve the optimal design program.

Proposition 5.3 *For any implementable mechanism $(\mathbf{a},\mathbf{t})$, there exists an implementable mechanism that belongs to the class of (α,σ)- optimal threat mechanisms and that raises at least the same utility level for the principal. The optimal design program becomes:*

$$\max_{(\alpha,\sigma)\in A\times\sigma(N)} \left\{ \sum_{i=0}^{n} V_i^\alpha - \sum_{i=1}^{n} V_i^*(\{\sigma(1),\ldots,\sigma(\sigma^{-1}(i)-1)\}) \right\} \tag{2}$$

Proof 3 *For a given mechanism $(\mathbf{a},\mathbf{t})$, we define a corresponding (α,σ)- optimal threat mechanism in the following way: $\alpha = \mathbf{a}(N)$, σ is defined by induction such that $\sigma(1) = Arg\max_{i\in N}\{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(\emptyset)}\}$ (initial step) and $\sigma(i) = Arg\max_{i\in N\setminus\{\sigma(1),\ldots,\sigma(i-1)\}}\{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(\{\sigma(1),\ldots,\sigma(i-1)\})}\}$ (inductive step). The map σ is by definition a permutation. From lemma 5.1, the (α,σ)- optimal threat mechanism is implementable. It remains to show that it raises a greater utility for the principal than the original mechanism $(\mathbf{a},\mathbf{t})$. More precisely, the principal implements the same economic outcome and extracts more surplus from each agent. Let $\mathbf{t}_i^{(\alpha,\sigma)}(N)$ be the transfer for agent i in the (α,σ)- optimal threat mechanism at equilibrium. We have:*

$$\mathbf{t}_i^{(\alpha,\sigma)}(N) = V_i^{\mathbf{a}(N)} - V_i^*(T^\sigma_{\sigma^{-1}(i)}) \geq V_i^{\mathbf{a}(N)} - V_i^{\mathbf{a}(T^\sigma_{\sigma^{-1}(i)})} \geq \mathbf{t}_i(N)$$

The first equality results from the definition of $\mathbf{t}_i^{(\alpha,\sigma)}(N)$ and that $\alpha = \mathbf{a}(N)$. The first inequality comes from the definition of the map $V_i^*(.)$ and since $a(T^\sigma_{\sigma^{-1}(i)}) \in \mathcal{A}(T^\sigma_{\sigma^{-1}(i)})$. The last inequality results from our subtle construction of σ and the inequality (1) for the set $T^\sigma_{\sigma^{-1}(i)}$. This latter inequality states *that* $\max_{j \in N \setminus \{\sigma(1),\ldots,\sigma(\sigma^{-1}(i)-1)\}} \{V_j^{\mathbf{a}(N)} - t_j(N) - V_j^{\mathbf{a}(\{\sigma(1),\ldots,\sigma(\sigma^{-1}(i)-1)\})}\} \geq 0$ *if* $(\mathbf{a}, \mathbf{t})$ *is implementable. The construction of* $\sigma(i)$ *guarantees that the expression in the 'max' is positive for* $j = \sigma(i)$*, i.e.* $V_{\sigma(i)}^{\mathbf{a}(N)} - V_{\sigma(i)}^{\mathbf{a}(T^{\sigma}_{\sigma^{-1}(i)})} \geq \mathbf{t}_{\sigma(i)}(N)$*. To sum up, we have proved that* $\alpha = \mathbf{a}(N)$ *and* $\mathbf{t}_i^{(\alpha,\sigma)}(N) \geq \mathbf{t}_i(N)$ *for all agents. The utility level of the principal is thus higher in the* (α, σ)*-optimal threat mechanism we have constructed than in* $(\mathbf{a}, \mathbf{t})$*.*

The optimal program (2) allows us to separate the choice of the final outcome α to the choice of the optimal threat structure, which is indeed reduced to the choice of a permutation that specifies the order according to which agents will be threat taken as given the participation decision of the agents that are lower in this order. The optimal choice of α thus coincides with the maximisation of the allocative efficiency.

Corollary 5.4 *Optimal robustly sequentially individually rational feasible mechanisms are efficient.*

The expression (2) of the utility level of the principal should be compared with the standard expression under simultaneous participation:

$$\max_{\alpha \in A} \left\{ \sum_{i=0}^{n} V_i^{\alpha} \right\} - \sum_{i=1}^{n} V_i^*(N \setminus \{i\}) \tag{3}$$

In general, the possibility to commit to a simultaneous participation game leads to a greater payoff for the principal since $V_i^*(S)$ is decreasing in S. Under robust sequential individual rationality and in an (α, σ)-optimal threat mechanism, the set of implementable threats is reduced to $V_{\sigma(i)}^*(\{\sigma(1), \ldots, \sigma(i-1)\})$ for the agent $\sigma(i)$. Nevertheless, in a negative-externality-free framework, the optimal threat $V_i^*(N \setminus \{i\})$ against agent i requires economic an outcome a that is always feasible independently to the set of participant, i.e. $a \in \mathcal{A}(\emptyset)$, and is thus always equal to $V_i^*(\{\sigma(1), \ldots, \sigma(\sigma^{-1}(i) - 1\})$. We obtain the following corollary:

Corollary 5.5 *In a negative-externality-free framework, the optimal revenue under simultaneous participation can be implemented under robustly sequentially individually rationality.*

5.2 Participation processes with imperfect information

The optimal design derived in Proposition 5.3 extends to a general information states structure with partial observability of participations decisions but where participants can strategically disclose evidence that they have accepted the mechanism. We exclude any strategic disclosure by the seller of evidence on participation decisions to remain coherent with our assumption that the seller can not control the participation process. It would not alter the following result.

Proposition 5.6 *The optimal outcome* $(\mathbf{a}(N), \mathbf{t}(N))$ *implementable with perfect information is implementable for any participation process with possibly imperfect information.*

Proof 4 *The construction is exactly the same as with perfect information with only an additional care on the transfers* $\mathbf{t}_i(S)$ *when* S *is a strict subset of* N. *Remind that those transfers did not play any role with perfect information. A mechanism that is implementable with perfect information may not be implementable in general because some agents may prefer to participate without disclosing this information. There may exist some subgame perfect equilibria where agents uses mixed strategies leading in some cases to incomplete participation. Nevertheless, if the transfers with incomplete participation are sufficiently high, e.g.* $\mathbf{t}_i(S) > \mathbf{t}_i(N) - V_i^{\mathbf{a}(N)} + V_i^{\mathbf{a}(S)}$ *for* $S \subsetneq N$, *then it guarantees that any participant would disclose that he has accepted the mechanism, if there were a terminal node with incomplete participation in the equilibrium path.*

5.3 A maxmin foundation

In the same way as a maxmin principal which is uncertain about agents' beliefs lays the foundation of dominant strategy implementation (see Chung and Ely [4]), our implementation concept requires that full acceptance is the only equilibrium for any kind of beliefs for the agents. Moreover, we made the somehow *ad hoc* assumption that the mechanism's outcomes depend only on the set of participants and so that agents can not report their beliefs. With a maxmin decision maker, that maximizes the worst-case performance, there is no loss of generality to search for a detail-free optimal design where a participant does not report any message.

Proposition 5.7 *The use of implementable mechanisms has a maxmin foundation; i.e.,*

$$\sup_{\substack{(a,t):\Omega^* \to \mathcal{A} \times \mathbb{R}^N \\ (a,t) \text{feasible}}} \inf_{\mu \in \mathcal{M}(\Omega^*)} E_\omega[V_0^{\mathbf{a}(\omega)} + \sum_{i=1}^n \mathbf{t}_i(\omega)] = \sup_{(a,t) \in \gimel} V_0^{\mathbf{a}(N)}) + \sum_{i=1}^n \mathbf{t}_i(N),$$

where $\mathcal{M}(\Omega^*)$ *is the set of all probability measures on* Ω^*. *The same equality holds for by replacing* Ω *and* $\gimel$ *by respectively* Ω^*_{PI} *and* $\gimel_{PI}$.

Proof 5 *Suppose that we can implement a higher revenue with a more complex mechanism with a arbitrary set of messages. Hence, by proposition 5.1, there is a set* S *such that the constraint (1) is violated. If it is common* *knowledge among the agents in $N \setminus S$ that there are playing a given participation process where the agents in S have already accepted the mechanism, then any subgame perfect outcome involves nonparticipation of the agents in $N \setminus S$. Thus we have raised a contradiction.*

6 Examples

As illustrated by our starting example, an important class of applications where our new rationality constraints are binding is auctions with negative externalities as in [12, 13, 14, 7, 9]. The scope of application may seem quite limited since the optimal design is unchanged in a negative-externality-free framework. Our two following examples show how robust sequential individual rationality may be fruitful first to model general collusion mechanisms and second contracting in dynamic environments when long-term contracts are not available.

6.1 Example 1: The design of collusion mechanisms

In the recent mechanism design literature on collusion as in Che and Kim [3], one agent or a third party proposes a mechanism that can be vetoed by each agent. When an agent breaks the collusion process, the game is played in a non-cooperative way under passive-beliefs. Thus contrary to the mainstream mechanism design literature, the principal is significantly limited in the way she can punish non-participants. In an auction framework, Caillaud and Jehiel [1] relax slightly this veto power assumption by also considering the case where a defection leads to a collusive report from the agents that are remaining in the collusion process. Dequiedt [8] considers that the remaining agent can commit to the harsher punishment if the other agent refuses the collusion mechanism. The reluctance to adopt the standard mechanism design approach to model collusion may come from the seemingly excessive commitment power that it requires and which is slightly softened under our approach.

Let us discuss those differences in a simple example under complete information: a symmetric triopoly under Cournot competition. Each firm has a constant null marginal cost and a maximum capacity $q_{max} = 0.5$. Inverse demand is given by $P = 1 - Q$, where Q denotes the total quantity supplied. Without collusion, the quantity supplied by each firm in equilibrium is equal to $1/4$ and the corresponding total profit of the triopoly is $\Pi_{nc} = 3/16$. The collusive outcome corresponds to the total production $Q = 1/2$ and the joint profit $\Pi_{col} = 1/4$. Suppose that a collusion mechanism, which specifies the quantities produced by each participant and balanced monetary transfers among participants for each possible set of participant, is proposed by one firm, say 1. Under complete information, all the different models, leads to the collusive outcome in the optimal mechanism. Nevertheless, the distribution of the profits from collusion that can be implemented are very different according to the model for collusion. Under veto power, an assumption that is often made, each firm is guaranteed to obtain her non-cooperative profit 1/16. The proposer is able to capture all the rents from collusion $\Pi_{col} - \Pi_{nc} = 1/16$. At the other extreme as in Dequiedt [8], a non-participant can be punished by the minmax punishment which leads here to a null payoff: the two remaining participants commit to produce $q = 0.5$ which leads to a null price. Nevertheless, this mechanism may seem poorly convincing since firm 1 manages to extract all the surplus from trade (1/4) from both firms by threatening each to flood the market with the help of the other one. With our model, the maximal surplus that firm 1 can extract is intermediate: she can extract the full surplus only to one firm and has to leave the surplus 1/36 to the other one, the profit corresponding to the Cournot outcome after the commitment to produce $q = 0.5$ by firm 1. Thus she should use a divide and conquer strategy.

6.2 Example 2: Dynamic processes of social and economic interactions

Gomes and Jehiel [10] consider a model of dynamic interactions in complete information where, at each period, an agent is selected to make an offer to a subset of the other agents to move the state of the economy. They do not only assume that long-term contracts are not available but also restrict the analysis to simple-offer contracts where each approached agent can veto the proposed move. Indeed, as they emphasize, this restriction is with no loss of generality if a third party can coordinate the approached agents by means of a 'strong' collusion contract with transfers. With general contracts -i.e. without any form of collusion- the economy moves immediately to the efficient state. On the contrary, with simple-offer contracts, efficiency is no longer guaranteed. This negative result compared with the Coasian intuition depends critically on the model for collusion. If collusion is modeled without any monetary tranfers only by means of robust sequential individual rationality, interpreted here as a mild collusion device, then the transposition of corollary 5.4 in their framework restores efficiency: all Markov Perfect Equilibria of the economy with general spot contract that are robustly sequentially individually rational are efficient, entailing an immediate move to the efficient state, where it remains forever. Note however that, under our milder collusion device, the expected payoff of the selected proposer is lower than with general contracts. At the other extreme, under a mildly stronger form of collusion where the third party can also contract with non-approached agents and where collusion is not observable by the proposer, the economy also moves immediately to the efficient state.

7 Concluding Remarks

We relax the commitment ability of the principal in some minimal way and give some theoretical foundations for such a refinement of the standard mechanism design approach. The scope of application may seem relatively restricted since the optimal design is unchanged in a negative-externality-free framework. Nevertheless, jointed with other commitment failures as the unability to commit not to propose a new mechanism if the first one fails to work, e.g. the unability to commit never to attempt to resell the good if she fails to sell it as in McAfee and Vincent [18] and Skreta [23], robust sequential individual rationality may have some bite even in pure private value trading framework without externality. For example, in a procurement auction, the designer may be unable to set a high reserve price since this would trigger a joint boycott of the main market participants that will force the designer to propose a new mechanism.

Our sequential participation game can be also interpreted as a minimal collusive device for the agents. The main contributions on collusion-proof implementation [15, 16, 3] preclude any collusion on the participation decisions themselves and restrict the collusive activity to the reports. In this literature, the collusion technologies allow agents to fully contract (with monetary transfers) their reports to the principal. Surprisingly, Che and Kim [3] show that optimal noncollusive mechanism can be made collusion-proof in a broad class of circumstances including economic environment with (allocative) externalities. Here our collusive device is much weaker: neither monetary transfers nor binding agreements on the reports are available. Nevertheless, it consists in a form of collusion that includes the participation decisions. We show that in general, except when the framework is negative-externality free, the principal may raise a lower revenue at the optimal design under this device. It contrasts with the insights of Pavlov [22] and Che and Kim [2], where the collusion mechanism proposed by a third party takes place before the participation decisions, and where the second best is still implementable with collusion.[6] Those papers consider the auction of a single item in the independent private value framework and thus exclude any kind of externality. We thus shed some light on the impact of collusion on participation -any stronger collusive device as the ones in [8, 22, 2] above would only strengthen our results- independently of any informational asymmetry.

Finally, we have restricted attention to a complete information setup. It is left for further research how to extend the notion of implementation under sequential participation processes in incomplete information in order to analyse the interactions with the incentive compatibility constraints and

[6] In this line, Dequiedt [8] is an exception: in a binary type environment, he shows that asymmetric information do not prevent bidders to collude efficiently, i.e. to act as a single agent when the third party can manipule the participation decisions.

ask whether this constraint is beneficial or not to the welfare. As for the concept of ratifiability introduced by Cramton and Palfrey [6], incomplete information requires a careful treatment of how agents revise their beliefs relative to the participation decisions of their opponents.

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Chapitre 5: Nonparametric Identification and Estimation of the Private Value Auction Model under Anonymity

Nonparametric Identification and Estimation of the Private Value Auction Model under Anonymity*

Laurent Lamy†

Abstract

We consider standard auction models when bidders' identities are not observed by the econometrician. First, we adapt the definition of identifiability to a framework with anonymous bids and we explore the extent to which anonymity reduces the possibility to identify private value auction models. Second, in the asymmetric independent private value model which is nonparametrically identified, we adapt Guerre, Perrigne and Vuong [10]'s two-stage estimation procedure. Our multi-stage kernel-based estimator achieves the optimal uniform convergence rate when identities are observed.

Keywords: Auctions, asymmetric, nonparametric identification, nonparametric estimation, anonymous bids, uniform convergence rate
JEL classification: D44, C14

1 Introduction

Motivated by the fact that the identities of the bidders are lacking to the econometrician in some auction data either because this information is confidential or have been lost, or because submissions are structurally anonymous as in internet auctions, we consider a setup where bidders' identities are not observed by the econometrician.[1] At first glance, anonymity reduces considerably the scope of the economic analysis and invites the econometrician to assume that bidders are ex ante symmetric.[2] On the other hand, the presence of asymmetries has been the key determinant of many empirical studies of auction data. In Porter and Zona [23, 24] and Pesendorfer

*I would like to thank Philippe Février for very helpful discussions.

†Laboratoire d'Economie Industrielle, CREST-INSEE, 28 rue des Saints-Pères 75007 Paris. e-mail: laurent.lamy@ensae.fr

[1]The same motivation is the starting point of Yokoo et al. [30]'s analysis of combinatorial auctions when bidders have the possibility to submit false-name bids.

[2]See Song [27] and Sailer [26] for eBay auction models with symmetric bidders. Thus those models exclude any shill bidding activity from the seller, a pervasive phenomenon that is analyzed in Lamy [14].

[22], the bidding behavior of alleged cartel participants is compared to the ones of non-cartel bidders through reduced form approaches. In Hendricks and Porter [11], neighbor firms are shown to be better informed in auctions for drainage leases. The aim of this paper is to lay the foundations of the econometric of auctions under anonymous data and to show how we can deal with asymmetric models. We adopt the so-called structural approach (see Paarsch and Hong [21]) and focus on the private value single-unit auction model.

First, we adapt the definition of identifiability to a framework with anonymous bids by requiring the unique characterization of bidders' primitives up to a permutation of bidders' identities. Then, in the spirit of Laffont and Vuong [13] we explore the extent to which anonymity reduces the possibility to identify private value models in standard auctions with risk neutral buyers.[3] We show in Proposition 3.1 that anonymity prevents the identification of the asymmetric affiliated private value model, contrary to Campo, Perrigne and Vuong [7]'s analysis when bidders' identities are observed by the econometrician. When the identities of the bidders are not observed, the method that is currently implemented is to assume symmetry as an identifying restriction and to develop Guerre, Perrigne and Vuong [10]'s nonparametric methodology (henceforth GPV), which reaches the best rate of uniform convergence for the symmetric independent private value model. However, the validity of this method relies on the assumption that bidders are symmetric, an assumption that can not be rejected on any testable restriction without further restrictions if bids are anonymous. Furthermore, for auction models that explicitly involve asymmetry, e.g. models with collusion, with shill bidding or if the underlying market is intrinsically asymmetric, this identification route is not appropriate. We propose another identification route. We show in Proposition 3.1 that the asymmetric independent private value (IPV) model is identified. One crucial step in the resolution of this inverse problem is to recover the underlying cumulative distribution functions (CDFs) $(F_{\mathbf{B}_\mathbf{i}^*})_{i=1,\cdots,N}$ of each buyers' bids from the CDFs $(F_{\mathbf{B}_p})_{p=1,\cdots,N}$ of the order statistics of the anonymous bids. By exploiting independence, the vector of the N bidders CDFs $(F_{\mathbf{B}_\mathbf{i}^*})_{i=1,\cdots,N}$ corresponds to the roots of a polynomial of degree N whose coefficients are linear combinations of the CDFs $(F_{\mathbf{B}_p})_{p=1,\cdots,N}$.

Second, we propose a multi-stage kernel-based estimation procedure to recover the underlying distributions of bidders' private values. We mainly adapt GPV's two-stage estimation procedure. We establish the uniform consistency of our estimator and show that it attains the best uniform convergence rate for estimating the latent density of private values from observed bids. Indeed, we obtain the same rate as with nonanonymous bids. Our

[3] Risk aversion adds new caveats in the first price auction. See Campo, Guerre, Perrigne and Vuong [6].

estimation procedure also fits to the setup where the econometrician may benefit from some additional information as the identity of the winner, e.g. in Li and Perrigne [17], or the identities of the second-highest and highest bidders, e.g. in Baldwin et al [3]. In those latter cases, we know from Athey and Haile [2] that the asymmetric IPV model is identified. Nevertheless, in this framework, the existing nonparametric methodology from GPV may not perform very well in small data set because it uses only the highest bidding statistics. In particular, in the second stage of GPV's estimation procedure, the pseudo-values are computed only for those bids that are not anonymous. On the contrary, our estimation procedure uses the complete vector of bids at both stages. In particular, we obtain for each bid a pseudo private value according to each possible identities of the bidder. Then, to estimate the distribution of private values, we should estimate for each bid the probability that it comes from a given bidder.

In a nutshell, we face two identification routes with anonymous bids: either to assume symmetry and to apply GPV's method allowing for correlated signals as in Li, Perrigne and Vuong [18] or to assume independence and to apply ours. Furthermore, with partially anonymous data, our estimation procedure may be more suitable for small data set since it exploits all the bids.

The paper is organized as follows. In Section 2, we introduce the model and the definition of identification under anonymity. In Section 3, we consider nonparametric identification of private value models under anonymity. In section 4, for the asymmetric independent private value model which is identified, we propose a multi-stage kernel-based estimator that corresponds to the natural extension of GPV's procedure. We establish its asymptotic properties allowing for heterogeneity across auctions and variations in the set of participants. In section 5, we conclude by indicating some future lines of research. Two Appendices contain the proofs of our results.

2 The Model

Consider an auction of a single indivisible good with $n \geq 2$ risk-neutral bidders. We consider the first price and second price sealed-bid auctions with no reserve price and when all bids are collected by the econometrician. We mention in section 5 how to extend our methodology with bidding reserve prices and with incomplete sets of bids. Nevertheless, if the econometrician can observe the amounts submitted by all bidders, we assume that bids are anonymous, i.e. she can not observe their corresponding identities. Hence, she observes the ordered vector of bids $B = (B_1, \cdots, B_p, \cdots, B_n)$, where B_p denotes the pth order statistic of the vector of bids B. But she does not observe $B^* = (B_1^*, \cdots, B_i^*, \cdots, B_n^*)$, where B_i^* denotes the amount submitted by bidder i. Subsequently, we use the indices i, j for bidders' identities and

p, r for bidding order statistics.

We consider the private value paradigm: each participant $i = 1, \cdots, n$ is assumed to have a private value x_i for the auctioned object. Hence, bidder i would receive utility $x_i - p$ from winning the object at price p. In the first price and second price auctions, the price p is equal to B_n and B_{n-1}, respectively. Let $F_{\mathbf{X_i}}(.)$ and $F_{\mathbf{X}}(.)$ denote the cumulative distribution functions of X_i and $X = (X_1, \cdots, X_n)$, respectively, which are assumed to be absolutely continuous with probability density functions (PDF) $f_{\mathbf{X_i}}(.)$ and $f_{\mathbf{X}}(.)$ and compact support $[\underline{x}, \overline{x}]$ and $[\underline{x}, \overline{x}]^n$, respectively.[4,5] Each bidder is privately informed about x_i, whereas the common distribution $F_{\mathbf{X}}(.)$ is assumed to be common knowledge among bidders. When we refer to models with *symmetric* bidders we assume that the joint distribution of $\mathbf{X}$ is exchangeable with respect to buyers' indices. On the other hand, when we treat models allowing *asymmetric* bidders we drop the exchangeability assumption. For a generic random variable $\mathbf{S}$ and a class of events $\mathbf{E}$, we denote respectively by $F_{\mathbf{S}|\mathbf{E}}(.|e)$ and $f_{\mathbf{S}|\mathbf{E}}(.|e)$ the CDF and PDF of $\mathbf{S}$ conditionally on an event e in $\mathbf{E}$.

Our analysis falls into two classes of models:
Independent Private Values (IPV): $F_{\mathbf{X}}(x) = \prod_{i=1}^{n} F_{\mathbf{X_i}}(x_i)$.
Strictly Affiliated Private Value (APV): $\frac{\partial^2 \log f_{\mathbf{X}}}{\partial x_i \partial x_j} > 0$ for $i \neq j$

Assumption A 1 *The joint density $f_{\mathbf{X}}$ is bounded, atomless and strictly positive on $[\underline{x}, \overline{x}]^n$.*

We restrict attention to Bayesian Nash Equilibrium in weakly undominated pure strategies, denoted by $(\beta_1(.), \cdots, \beta_n(.))$, where $\beta_i(.)$ is the bidding function of bidder i. In the equilibrium of the second price auction, buyers are thus bidding their private value. Hence, the link between bids and private types is straightforward:

$$x_i = b_i \equiv \xi_i^{nd}(b_i, F_{\mathbf{B}}) \tag{1}$$

In the first price auction, under assumption (1), Athey [1] guarantees the existence of an increasing pure strategy equilibrium if private values are affiliated and thus in the IPV and APV models. The link between bids and types for each bidder i is made by a standard rewriting of the first order differential equation derived from bidder i's optimization program:

[4] Throughout, uppercase letters are used for distributions, while lowercase letters are used for densities. We also follow the standard notation by using an uppercase letter for a statistic and the corresponding lowercase letter for its realization.

[5] We restrict ourselves to the common-support case that guarantees that almost all bids are 'serious' bids, i.e. win with a strictly positive probability. Otherwise identification is obtained only for 'serious' types. See Lebrun [16] for the analysis of the first-price auction with different supports.

$$x_i = b_i + \frac{F_{\mathbf{B}^*_{-i}|\mathbf{B}^*_i}(b_i|b_i)}{f_{\mathbf{B}^*_{-i}|\mathbf{B}^*_i}(b_i|b_i)} \equiv \xi_i^{rst}(b_i, F_{\mathbf{B}}), \tag{2}$$

where, for bidder i, $\mathbf{B}^*_{-i}$ denotes the maximum of the bids from bidder i's opponents.

Following Laffont and Vuong [13], we extend the identification issues of private value models to the case where bids are remaining anonymous. On the one hand, if bidders' identities are observed, then identifiability corresponds to the condition that, if two possible underlying distributions $F_{\mathbf{X}}(.)$ and $F'_{\mathbf{X}}(.)$ of private signals lead to the same distribution of bids $F_{\mathbf{B}^*}(.)$, then it follows that $F_{\mathbf{X}}(.)$ and $F'_{\mathbf{X}}(.)$ are equal. On the other hand, the following definition introduces the notion of identifiability that makes sense under anonymity.

Definition 1 (Identifiability under anonymity) *Under anonymous bidding, an auction model is said to be identifiable if for two possible underlying distributions $F_{\mathbf{X}}(.)$ and $F'_{\mathbf{X}}(.)$ of private values leading to the same distribution of bids $F_{\mathbf{B}}(.)$, then it follows that $F_{\mathbf{X}}(.)$ and $F'_{\mathbf{X}}(.)$ are equal up to a permutation of the potential buyers, i.e. there exists a permutation $\pi : [1, n] \to [1, n]$ such that $F_{\mathbf{X}}(x_1, \cdots, x_n) = F'_{\mathbf{X}}(x_{\pi(1)}, \cdots, x_{\pi(n)})$ for almost any vector of types X.*

Our definition of identifiability corresponds to the possibility of recovering an anonymous joint distribution of buyers' private values. Note that this information is not sufficient with asymmetric PV models for the computation of the optimal mechanism à la Myerson [20] that requires the knowledge of bidders' identities. Nevertheless, it is sufficient for the computation of the optimal anonymous mechanism or the optimal reserve price in a standard auction.

3 Nonparametric Identification

Anonymity restricts the degree of information of the data and thus it can only reduce the identification possibilities. In particular we show that asymmetric affiliated private value models are not indentified on the contrary to Campo, Perrigne and Vuong [7]'s identification result in a framework where bidders' identities are observed. Nevertheless, we show in Proposition [3.1] that, for a complete set of bids, either symmetry or independence restores identification. The surprising result is that anonymity does not prevent the identification of asymmetric IPV models. Our proof is constructive as it gives $F_{\mathbf{X}}(.)$ as a function of $F_{\mathbf{B}}(.)$. The empirical counterparts of this construction will then be used in the section devoted to nonparametric estimation. The proof of this result is thus given in the body of the text. The resolution of this inverse problem contains two steps. First we derive bidder's distribution from the distribution of B, the vector of the bidding order statistics. It is the innovative step which relies on the root-finding of a well chosen polynomial. The second step is the identification of bidders' signals and is well-known: it is straightforward in the second price auction, whereas the first price auction has been treated by GPV. Remark that identification under anonymity can not be proved directly from local arguments as in Roehring [25] (see also Benkard and Berry [4]) as it was the case with nonanonymous bids as in GPV. The reason is that anonymity breaks differentiability at some points: the function that maps the vector of bidders' private values X into the bidding order statistics B is not differentiable at the point x such that $\beta_i(x_i) = \beta_j(x_j)$ for $i \neq j$.

Proposition 3.1 *Under the full observation of any submitted bids and under anonymous bids, in the first price and second price auctions and for $n \geq 2$:*

- *The asymmetric APV model is not identified. For any distribution $F_{\mathbf{X}}(.)$ from the asymmetric APV model, there exists a continuum of local perturbations of $F_{\mathbf{X}}(.)$ that stay in the asymmetric APV model and that are observationally equivalent to $F_{\mathbf{X}}(.)$, i.e. that leads to the same distribution of bids.*
- *The symmetric APV model is identified.*
- *The asymmetric IPV model is identified.*

The second point is satisfied since the identification result in Li, Perrigne and Vuong [18] does not rely on the observability of bidders' identities. For the first point, we can construct, as it is done in the appendix, a continuum of local perturbations of the primitives that are observationally equivalent. For any IPV model, we can easily check that the local perturbations constructed in the proof of the first point of Proposition 3.1 break the independence, which illustrates, in this framework, the more general point that any unordered (i.e. observable up to a permutation) vector of independent components is observationally equivalent to a model where the components are correlated. In other word, the econometrician has to assume independence in order to identify asymmetry, an assumption which can not be fully tested.[6] Nevertheless, independence involves some testable restrictions un-

[6]The nonparametric approaches in the literature that test whether the different components of a vector $X = (x_1, \cdots, x_m) \in \mathbb{R}^m$ are independent, e.g. the Blum, Kiefer and Rosenblatt [5] test, consider that the statistician observes ordered vectors, i.e. she can distinguish $X = (x_1, \cdots, x_m)$ from $X' = (x_{\pi(1)}, \cdots, x_{\pi(m)})$ where π is a permutation of bidders' indices. With respect to our setup, those tests are requiring nonanonymous bids.

der anonymity and some partial tests could be build. Such developments are left for further research.[7]

The rest of this section is devoted to the proof of the third point which will guide our estimation procedure. We observe the distributions $F_{\mathbf{B_p}}$ for any $p = 1, \cdots, n$. Independence implying exchangeability, we can identify the CDFs $F_{\mathbf{B}}^{(r:m)}(u)$, $r \leq m$, that corresponds to the rth order statistic among m bidders that would result by exogenous variation of the number of bidders, by recursive use of the formula (see Athey and Haile [2] p.2128)

$$\frac{m-r}{m} F_{\mathbf{B}}^{(r:m)}(u) + \frac{r}{m} F_{\mathbf{B}}^{(r+1:m)}(u) = F_{\mathbf{B}}^{(r:m-1)}(u), \quad \forall u, r, m, r \leq m-1, m \leq n. \tag{3}$$

Remark that the corresponding induction is initialized by noting that $F_{\mathbf{B}}^{(p:n)} = F_{\mathbf{B_p}}$. In particular, it implies the identification of the CDFs $F_{\mathbf{B}}^{(r:r)}$ for any $r \in [1, n]$. Indeed, the expression of $F_{\mathbf{B}}^{(r:r)}$ corresponds to a linear combination of the CDFs $F_{\mathbf{B_p}}$, for $p = 1, \cdots, n$. Moreover, independence allows us to express $F_{\mathbf{B}}^{(r:r)}(b)$ as a function of the distributions $F_{\mathbf{B_i^*}}(b), i = 1, \cdots, n$ in the following way.

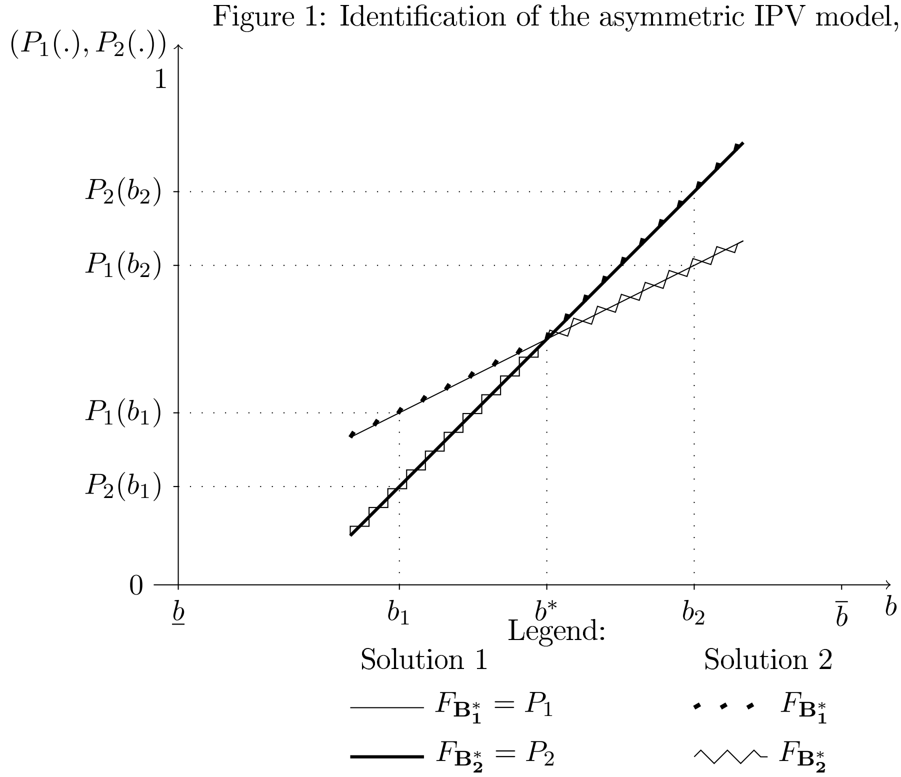
$$\begin{aligned}
F_{\mathbf{B}}^{(1:1)}(b) &= \frac{1}{n} \cdot \sum_{i=1}^{n} F_{\mathbf{B_i^*}}(b) \\
F_{\mathbf{B}}^{(2:2)}(b) &= \frac{1}{n(n-1)} \cdot \sum_{i_1, i_2, i_1 \neq i_2} F_{\mathbf{B_{i_1}^*}}(b) \cdot F_{\mathbf{B_{i_2}^*}}(b) \\
&\;\;\vdots \\
F_{\mathbf{B}}^{(r:r)}(b) &= \frac{1}{n(n-1)\cdots(n-r+1)} \cdot \sum_{i_1, \cdot, i_r, i_k \neq i_{k'}} \prod_{i_k \in \{i_1, \cdot, i_n\}} F_{\mathbf{B_{i_k}^*}}(b) \\
&\;\;\vdots \\
F_{\mathbf{B}}^{(n:n)}(b) &= \frac{1}{n!} \cdot \sum_{i_1, \cdot, i_n, i_k \neq i_{k'}} \prod_{i_k \in \{i_1, \cdot, i_n\}} F_{\mathbf{B_{i_k}^*}}(b)
\end{aligned} \tag{4}$$

The CDFs $(F_{\mathbf{B_i^*}}(b))_{i=1,\cdots,n}$ in the above system of equations correspond exactly to the n roots of the polynomial of degree n: $u \rightarrow \sum_{i=0}^{n} a_i(b) \cdot (-1)^{n-i} \cdot u^i$, where $a_n(b) = 1$ and $a_i(b) = \frac{n(n-1)\cdots(i+1)}{(n-i)!} \cdot F_{\mathbf{B}}^{(n-i:n-i)}(b)$, for $i < n$. When b is fixed, such a solution is unique. By continuity of the coefficients of the polynomial as a function of b and since the roots of a polynomial depends continuously on its coefficients (see [29]), there exists a continuous function $b \rightarrow (P_1(b), \cdots, P_n(b))$ mapping the vector of solutions. What

[7] If the alternative to independence is not strict correlation but rather affiliation, then it is an open question whether independence can be tested.

remains to show is the more restrictive condition that the CDFs $F_{\mathbf{B_i^*}}(b)$, $i = 1, \cdots, n$, are unique up to a permutation. If the n roots of the above polynomial were always distinct for any b in the interior of the bidding support $(\underline{b}, \overline{b})$, then by continuity of $F_{\mathbf{B}^*}(.)$ the only candidate solution would be $(F_{\mathbf{B_1^*}}(.), \cdots, F_{\mathbf{B_n^*}}(.)) = (P_1(.), \cdots, P_n(.))$ (up to a permutation). On the contrary, if the maps $P_i(b)$ cross then the way we construct the continuous selection of the roots $(P_1(.), \cdots, P_n(.))$ is no more unique as it is illustrated in Figure 1 where two candidate solutions are depicted for $n = 2$ and when the roots cross at least once.

Figure 1: Identification of the asymmetric IPV model, $n = 2$



Indeed, the sole knowledge of the CDFs $F_{\mathbf{B}}^{(p:m)}$ for any p, m such that $p \leq m \leq n$ can not discriminate between these two possible solutions. Nevertheless, the knowledge of the joint distribution $F_{\mathbf{B}}$ of all order statistics selects a unique solution among those. For example, consider the case $n = 2$ and a point b^* where $P_1(.)$ and $P_2(.)$ strictly cross as in Figure 1. We consider a point b_2 at the right of the junction (respectively b_1 at the left of the junction) such that the derivative of the upper root as a function of b, $P_2'(b_2)$ (resp. $P_1'(b_1)$), is strictly bigger (resp. strictly smaller) than the derivative of the lower root, $P_1'(b_2)$ (resp. $P_2'(b_1)$). Such a point exists in right (resp. left) neighborhood of b^* since the junction is strict. Then the two candidate solutions lead to different predictions in term of the joint density of the order statistics: $f_{\mathbf{B}}(b_1, b_2) = f_{\mathbf{B}_1^*}(b_1) \cdot f_{\mathbf{B}_2^*}(b_2) + f_{\mathbf{B}_1^*}(b_2) \cdot f_{\mathbf{B}_2^*}(b_1)$. The difference of the density $f_{\mathbf{B}}(b_1, b_2)$ between the two depicted solutions is equal to $(P_2'(b_2) - P_1'(b_2)) \cdot (P_2'(b_1) - P_1'(b_1)) \neq 0$. The argument remains valid for any number of bidders and also for more general junctions where the roots may coincide on an interval.

4 Nonparametric Estimation

In practice the auctioned objects can be heterogeneous and the number and the identities of the participants can vary across auctions. Consider a set of potential bidders, denoted by $\mathbb{I}$, among which a subset I participates in an auction for a single and indivisible object. We consider that the number of participants, denoted by n_{I}, and their identities are common knowledge among bidders and are also observed by the econometrician.[8]

In this section, we adapt GPV's two step estimation procedure to recover the densities of bidders' private values in the first price auction.[9] Two caveats arise. First we can not directly estimate with kernel techniques the ratio $F_{\mathbf{B}_{-\mathbf{i}}^*|\mathbf{b_i}}(.|.)/f_{\mathbf{B}_{-\mathbf{i}}^*|\mathbf{b_i}}(.|.)$ since identities are not observed. An indirect procedure leading to the same uniform convergence rate in any inner closed subset of the bidding support is obtained. Second, if $F_{\mathbf{B}_{-\mathbf{i}}^*|\mathbf{b_i}}(.|.)/f_{\mathbf{B}_{-\mathbf{i}}^*|\mathbf{b_i}}(.|.)$ is suitably estimated, we can apply (2) to define pseudo private values in the first price auction. For each bid, a vector of pseudo private values, i.e. for each possible identities of the bidder. With anonymity, an additional step is needed: for a given vector of bid $b = (b_1, \cdots, b_p, \cdots, b_n)$, we have to estimate the probability that buyer i's bid b_i^* is equal to b_p for any $k \in [1, n]$. Then instead of a unique pseudo private value for a given bidder, we obtain a weighted vector of n pseudo private values that is used to estimated nonparametrically buyers' private values densities. When buyers' CDFs $F_{\mathbf{X_i}}(.|Z)$ have R bounded continuous derivatives and if d denotes the dimension of the (continuous) covariates Z, we obtain the same optimal uniform rate as in GPV: $(L/\log L)^{R/(2R+d+3)}$.

We also lead in parallel the analysis for the second price auction which is not straightforward as it was with nonanonymous bids. If bidders' identities were observed, private values would be directly observed by applying (1) and the optimal uniform rate of convergence for estimating private values

[8]The observation of the identities of the participants by the econometrician may appear in contradiction with our paradigm of anonymous bids. If we could not observe participants identities, as on eBay, we can adapt our method if we are prepared to make specific assumptions about the identities of the fluctuating bidders (real bidder versus shill bidder). Anyway, in an asymmetric framework, the exogenous participation assumption that is often made for identification as in Athey and Haile [2] may not be suitable since the expected payoffs in the auction differ across bidders.

[9]See Flambard and Perrigne [9] for the the implementation of this procedure in the asymmetric private value model with nonanonymous bids.

densities is $(L/\log L)^{R/(2R+d+1)}$ (see Stone [28]). Under anonymous bids, our procedure for the second price auction reaches this optimal rate.

Denote by Σ_{I} the set of the $n_{\mathrm{I}}!$ bijections between participants' identities and the order statistics of the bids. Such an assignment of the bids to the participants is denoted by $\pi : \mathrm{I} \to [1, n_{\mathrm{I}}]$ where $\pi(i) = p$ means that the pth order statistic of the bids corresponds to bidder i, i.e. $b_i^* = b_p$. To cover both the case where bidders' identities remain fully anonymous with the common case where only the identity of the winner is disclosed, we consider the most general case where the econometrician may benefit from some information linking some submitted bids with the identities of some participants. This information is modelled as a subset $\sigma_{\mathrm{I}} \subset \Sigma_{\mathrm{I}}$, e.g. $\sigma_{\mathrm{I}} = \Sigma_{\mathrm{I}}$ corresponds to the case where bids are fully anonymous. The opposite case where σ_{I} is always a singleton corresponds to nonanonymous bids and where GPV's procedure should be preferred. Our estimation procedure is flexible and imposes no restriction on the way σ_{I} varies across auctions.

4.1 Regularity Assumptions and Key Properties

Let Z_l denote the vector of relevant continuous characteristics for the lth auctioned object and I_l the set of participants in the lth auction. The vector (Z_l, I_l) is assumed to be common knowledge among bidders and is observed by the econometrician. Relative to our previous notation, in this section, we will work with conditional distributions and densities of private values and bids given (Z_l, I_l). E.g., $F_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(.|Z_l, I_l)$ denotes the CDF of bidder i's private value X_{il} for the lth auction. Thus (1) and (2) for the first and second price auction become respectively:

$$X_{il} = B_{il}^* + \frac{F_{\mathbf{B}_{-i}^*|\mathbf{B}_i^*,\mathbf{Z},\mathrm{I}}(B_{il}^*|B_{il}^*, Z_l, \mathrm{I}_l)}{f_{\mathbf{B}_{-i}^*|\mathbf{B}_i^*\mathbf{Z},\mathrm{I}}(B_{il}^*|B_{il}^*, Z_l, \mathrm{I}_l)}, \tag{5}$$

and

$$X_{il} = B_{il}^*. \tag{6}$$

The next assumptions concern the underlying generating process as well as the smoothness of the latent joint distribution of $(X_{il}, Z_l, \mathrm{I}_l)$ for any $i \in \mathrm{I}_l$.

Assumption A 2 (i) *The $(d+1)$-dimensional vectors $(Z_l, \mathrm{I}_l), l = 1, 2, \cdots$, are independently and identically distributed as $F_{\mathbf{Z},\mathbf{I}}(.,.)$ with density $f_{\mathbf{Z},\mathbf{I}}(.,.)$.*

(ii) *For each l the variables $X_{il}, i \in \mathrm{I}_l$ are independently distributed conditionally upon (Z_l, I_l) as $F_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(.|.,.)$ with density $f_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(.|.,.)$, for $i \in \mathrm{I}_l$.*

As in Campo et al. [6], we consider here that the support of buyers' private values does not depend on the (Z, I) to simplify the presentation, while the general case can be fully treated as in GPV. It implies that the lower bound of the support of buyers' bids does not depend on the variables I and Z. Throughout we denote by $S(*)$ and $S^o(*)$ the support of $*$ and its interior, respectively. Let $\mathcal{I} \subset \mathbb{I}$ be the set of possible values for I_l. Note that $\mathcal{I}$ is finite.

Assumption A 3 *For each bidder $i \in I \subset \mathcal{I}$,*

(i) $S(F_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}) = \{(x, z, I) : z \in [\underline{z}, \overline{z}], x \in [\underline{x}, \overline{x}], I \subset \mathcal{I}\}$; *with* $\underline{z} < \overline{z}$;

(ii) *for* $(x, z, I) \in S(F_{\mathbf{X_i},\mathbf{Z},\mathbf{I}})$, $f_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(x|z, I) \geq c_f > 0$, *and for* $(z, I) \in S(F_{\mathbf{Z},\mathbf{I}})$, $f_{\mathbf{Z},\mathbf{I}}(z, I) \geq c_f > 0$;

(iii) *for each* $I \subset \mathcal{I}$, $F_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(.|., I)$ *and* $f_{\mathbf{Z},\mathbf{I}}(., I)$ *admit up to* $R+1$ *continuous bounded partial derivatives on* $S(F_{\mathbf{X_i},\mathbf{Z},\mathbf{I}})$ *and* $S(F_{\mathbf{Z},\mathbf{I}})$, *with* $R \geq 1$.

The next assumption is not necessary for identification as established in Proposition 3.1 without heterogeneity across objects. Nevertheless, heterogeneity requires an additional structure to identify the model. Similar junctions as the one in Figure 1 when b varies may arise when the variable capturing heterogeneity Z varies. But the different solutions are observationally equivalent without some mild additional assumptions. Here to preserve identification, we make the (strong) assumption that bidding distributions can be ordered according to first order stochastic dominance. With two classes of bidders, Maskin and Riley [19] show that first order stochastic dominance for private values is sufficient for first order stochastic dominance for equilibrium bids.[10] Moreover, to simplify our estimation procedure, we also assume that the dominance is strict in the interior of the bidding support.

Assumption A 4 (Strict Stochastic Dominance) *The bid densities $F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}(.|z, I)$ are strictly ordered according to first order stochastic dominance:*

$$F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}(b|z, I) > F_{\mathbf{B_{i+1}^*}|\mathbf{Z},\mathbf{I}}(b|z, I), \;\; if \;\; b \in S^0(f_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}})$$

for any $i \in I$ and any z, I.

A crucial step in deriving uniform rates of convergence in some inverse problem is to study the smoothness of the observables that is implied by the smoothness of the latent distributions of the primitives of the model. Here,

[10] An alternative identification strategy with two classes of bidders is to make assumptions on the comparative statics of the bidding distribution according to Z. Another strategy would rely on the point that, generically, at a junction point, only one candidate solution is differentiable at this point.

relative to GPV, we do not observe the vector of bids B^* but only the vector of bidding order statistic B. Thus we are interested in the smoothness of the densities $f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(.|.,I)$ for any $p \in I$. This is the purpose of the next proposition which is the analog of proposition 1 in GPV which derives similar results for the bid densities $f_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}(.|.,.)$.

Proposition 4.1 *Given A3, the conditional distribution $F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(.|.,I)$, $p \in I$ and $I \subset \mathbb{I}$, satisfies for both the first and second price auctions (if not specified):*

(i) *its support $S(F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}})$ is such that $S(F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}) = \{(b,z,I) : z \in [\underline{z},\overline{z}], b \in [\underline{b}(z,I,p), \overline{b}(z,I,p)], I \subset \mathcal{I}\}$ with $\overline{b}(z,I,p) > \underline{b}(z,I,p)$ for any I,p. Moreover, $(\underline{b}(.,I,p), \overline{b}(.,I,p))$ admit up to $R+1$ continuous bounded derivatives on $[\underline{z},\overline{z}]$ for each $I \subset \mathcal{I}$ and $p = 1,\cdots,n_I$. We have $\underline{b}(z,I,p) = \underline{x}$. In the second price auction, $\overline{b}(z,I,p) = \overline{x}$. In the first price auction $\overline{b}(z,I,n_I) = \overline{b}(z,I,n_I-1)$.*

(ii) *for $(b,z,I) \in \mathcal{C}(B_n)$, $f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(b,z,I) \geq c_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}} > 0$, where $\mathcal{C}(B_n)$ is a closed subset of $S^0(F_{\mathbf{B_n}|\mathbf{Z},\mathbf{I}})$;*

(iii) *for each (I,p), $p = 1,\cdots,n_I$, $F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(.|.,I)$ admits up to $R+1$ continuous bounded partial derivatives on $S(F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}) \setminus (\{\overline{b}(z,I,p)\}_{p=1,\cdots,n_I-1})$;*

(iv) *in the first price auction, for each (I,p), $p = 1,\cdots,n_I$, if $\mathcal{C}(B_p)$ is a closed subset of $S^o(F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}) \setminus (\{\overline{b}(z,I,p)\}_{p=1,\cdots,n_I})$, then $f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(.|.,I)$ admits up to $R+1$ continuous bounded partial derivatives on $\mathcal{C}(B_p)$;*

(v) *in the second price auction, for each (I,p), $p = 1,\cdots,n_I$, $f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(.|.,I)$ admits up to R continuous bounded partial derivatives on $S(F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}) \setminus (\{\overline{b}(z,I,p)\}_{p=1,\cdots,n_I-1})$.*

Remark that by comparing (iv) and (v), the bid densities in the first price auction are smoother than for the second price auction. Thus $f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(.|.,I)$ can be estimated uniformly at a faster rate, namely $(L/\log L)^{R+1/(2R+d+3)}$, in the first price than in the second price auction, namely $(L/\log L)^{R/(2R+d+1)}$. In particular, the optimal bandwidths -that we specify later in assumption A6- are asymptotically smaller for the second price auction than for the first price auction. Nevertheless the optimal uniform convergence rate will be smaller in the first price auction than in the second price auction. This is due to the more indirect nature of the link between observables and latent distributions in the first price auction, see equation (5) versus (6).

Proposition 4.1 differs from the one appearing in GPV as irregularities of the CDF of the observed variables may appear in the interior of their support, more precisely at the upper bound of the bidding support of the (at most $n_I - 2$) bidders such that $\overline{b}(z,I,p) < \overline{b}(z,I,n)$. In the following, to alleviate notations, we make the simplifying assumption A5 that the bidding supports of all bidders coincide, i.e. $\bar{b}(z, I, p)$ does not depend on p. Our uniform consistency results extend provided that the neighborhood of the bidders' signals than make them bid $\bar{b}(z, I, p)$ are removed. In the same way as the support of bidders' private values is consistently estimated in GPV and that the neighborhoods of the lower and upper bounds of the support are removed with an suitable trimming, we can trim those inner neighborhoods.

Assumption A 5 (Common bidding support) *All bidders have the same bidding support: $\bar{b}(z, I, p)$ does not depend on p.*

4.2 Optimal Uniform Convergence Rate

In this section, we adopt a minmax approach to obtain bounds for the rate at which the latent density of private values can be estimated uniformly from observed bids. The next proposition gives an upper bound for the optimal uniform convergence rate for estimating $f_{\mathbf{X}|\mathbf{Z},\mathbf{I}}(.|.,.)$ from observed (anonymous) bids. GPV derives the same bound for the symmetric IPV model and nonanonymous bids. Here we extend their result to the asymmetric IPV model. In the following, for a given density function f, denote by $||f||_r$ (resp. $||f||_{r,\mathcal{C}}$) the maximum of f and all its derivatives up to the rth order on $S(F)$ (resp. on $\mathcal{C}$).

Proposition 4.2 *Assume A2-A5 and $||f_{\mathbf{X},\mathbf{Z},\mathbf{I}}(x,z,I)||_R < M$. Let $\mathcal{C}(X)$ be an inner compact subset of $S(f^o_{\mathbf{X}|\mathbf{Z},\mathbf{I}})$ with nonempty interior. There exists a constant $\kappa > 0$ such that*

$$\lim_{\epsilon \to 0} \varliminf_{L \to +\infty} \inf_{\widehat{f}_L} \sup_{f \in U_\epsilon(f^o_{\mathbf{X}|\mathbf{Z},\mathbf{I}})} Prob_f \left[(\frac{L}{\log L})^{\frac{R}{(2R+d+3)}} \sup_{(x,z,I) \in \mathcal{C}(X)} ||\widehat{f}_{\mathbf{X}|\mathbf{Z},\mathbf{I}}(x|z,I) - f_{\mathbf{X}|\mathbf{Z},\mathbf{I}}(x|z,I)||_0 > \kappa \right] > 0$$

in the first price auction, and

$$\lim_{\epsilon \to 0} \varliminf_{L \to +\infty} \inf_{\widehat{f}_L} \sup_{f \in U_\epsilon(f^o_{\mathbf{X}|\mathbf{Z},\mathbf{I}})} Prob_f \left[(\frac{L}{\log L})^{\frac{R}{(2R+d+1)}} \sup_{(x,z,I) \in \mathcal{C}(X)} ||\widehat{f}_{\mathbf{X}|\mathbf{Z},\mathbf{I}}(x|z,I) - f_{\mathbf{X}|\mathbf{Z},\mathbf{I}}(x|z,I)||_0 > \kappa \right] > 0$$

in the second price auction, where the infimums are taken over all possible estimators $\widehat{f}_L$ of $f_{\mathbf{X}|\mathbf{Z},\mathbf{I}}(.|.,.)$ based upon $(B_{pl}, Z_l, \mathrm{I}_l)$ for any $p = 1, \cdots, n_{I_l}$ and $l = 1, \cdots, L$ and where $U_\epsilon(f^o_{\mathbf{X}|\mathbf{Z},\mathbf{I}})$ is a neighborhood of $f^o_{\mathbf{X}|\mathbf{Z},\mathbf{I}}$ defined as

$$U_\epsilon(f^o_{\mathbf{X}|\mathbf{Z},\mathbf{I}}) \equiv \left\{ f; \sup_{(x,z,i) \in S(F^o_{\mathbf{X}|\mathbf{Z},\mathbf{I}})} ||f(x,z,I) - f^o_{\mathbf{X}|\mathbf{Z},\mathbf{I}}(x,z,I)||_0 < \epsilon, ||f(.,.,.)||_R < M \right\},$$

where $M > 0$.

The set of possible estimators based upon anonymous bids is tautologically smaller than those based upon $(B^*_{il}, Z_l, \mathrm{I}_l)$ for any $i \in I_l$ and $l =$

$1, \cdots, L$. Thus it is sufficient to prove the above proposition with this richer set of estimators. In this latter case, for the second price auction where observed bids correspond exactly to private values, the above result has been proved by Khas'minskii [12]. In the first price auction, the above proposition has been proved when the model is restricted to be symmetric among bidders by GPV who adapts Khas'minskii [12]'s arguments. It is intuitive that a faster local rate of uniform convergence is not available in the general case with asymmetric bidders. Nevertheless, due to the local nature of the above result, the argument is not tautologic. Indeed, since a general asymmetric model with n bidders involves n overlapped differential equations for bidders' distributions, the asymmetric structure may 'smooth' the link between observables and the latent private values. We show in the appendix how GPV's proof has to be adapted.

4.3 Definition of the Estimator

The purpose of this section is to adapt GPV's two step procedure to asymmetric auctions with anonymous bids. Using independence, (5) and (6) can be rewritten as

$$X_{il} = B^*_{il} + \psi_i(B^*_{il}, Z_l, \mathrm{I}_l), \tag{7}$$

where $\psi_i(.,.,.)$ is defined as

$$\psi_i(b,z,I) = \begin{cases} \left[\sum_{j \in I_l, j \neq i} \frac{f_{\mathbf{B}^*_j|\mathbf{Z},\mathrm{I}}(B^*_{il}|Z_l,\mathrm{I}_l)}{F_{\mathbf{B}^*_j|\mathbf{Z},\mathrm{I}}(B^*_{il}|Z_l,\mathrm{I}_l)} \right]^{-1}, & \text{in the first price auction} \\ 0, & \text{in the second price auction} \end{cases} \tag{8}$$

The first step in GPV's approach consists in estimating the maps $\psi_i(.,.,.)$. The main caveat is that we do not observe the variables B^*_{il} but only the order statistics B_{pl}. Thus we need to convert our estimations of the CDFs and PDFs of B_{pl}, that can be done with the standard kernel estimation techniques, into estimations for the CDFs and PDFs of B^*_{il}.

Using the observations $\{(B_{pl}, Z_l, \mathrm{I}_l); p \in \mathrm{I}_l, l = 1, \cdots, L\}$, our first step consists in estimating the CDFs and the PDFs of the pth ordered statistics of the bids for $pin[1, n_I]$.

$$\widehat{F}_{\mathbf{B_p},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}) = \min\left\{ \frac{1}{Lh^d_{F_{\mathbf{B_p}|\mathbf{Z}}}} \sum_{l=1}^{L} \mathbf{1}(B_{pl} \leq b) K_{F_{\mathbf{B_p}|\mathbf{Z}}}\left(\frac{z - Z_l}{h_{F_{\mathbf{B_p}|\mathbf{Z}}}}\right) \mathbf{1}(\mathrm{I}_l = \mathrm{I}), 1 \right\} \tag{9}$$

$$\widehat{f}_{\mathbf{B_p},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}) = \frac{1}{Lh^d_{f_{\mathbf{B_p}|\mathbf{Z}}}} \sum_{l=1}^{L} \mathbf{1}(B_{pl} \leq b) K_{f_{\mathbf{B_p}|\mathbf{Z}}}\left(\frac{b - B_{pl}}{h_{f_{\mathbf{B_p}|\mathbf{Z}}}}, \frac{z - Z_l}{h_{f_{\mathbf{B_p}|\mathbf{Z}}}}\right) \mathbf{1}(\mathrm{I}_l = \mathrm{I}) \tag{10}$$

Here $h_{F_{\mathbf{B_p|Z}}}, h_{f_{\mathbf{B_p|Z}}}$ are some bandwidths, and $K_{F_{\mathbf{B_p|Z}}}(.)$ and $K_{F_{\mathbf{B_p|Z}}}(.,.)$ are kernels with bounded supports. By recursive use of the empirical counterpart of the formula (3), we estimate $\widehat{F}^{(r:r)}_{\mathbf{B,Z,I}}(b,z,I)$ and $\widehat{f}^{(r:r)}_{\mathbf{B,Z,I}}(b,z,I)$ for $r = 1, \cdots, n$, which respectively corresponds (up to a known multiplicative coefficient) to the coefficients and their derivatives of the polynomial whose roots is the vector of bidders' bidding distribution $\{F_{\mathbf{B_i^*,Z,I}}\}_{1\leq i\leq n}$.

For $r \leq m \leq n$, we define $\widehat{F}^{(r:m)}_{\mathbf{B,Z,I}}(b,z,I)$ and $\widehat{f}^{(r:m)}_{\mathbf{B,Z,I}}(b,z,I)$ by recursive use of the formulas:

$$\frac{m-r}{m}\widehat{F}^{(r:m)}_{\mathbf{B,Z,I}}(b,z,I)+\frac{r}{m}\widehat{F}^{(r+1:m)}_{\mathbf{B,Z,I}}(b,z,I)=\widehat{F}^{(r:m-1)}_{\mathbf{B,Z,I}}(b,z,I), \forall b,z,r\leq m-1 \quad (11)$$

$$\frac{m-r}{m}\widehat{f}^{(r:m)}_{\mathbf{B,Z,I}}(b,z,I)+\frac{r}{m}\widehat{f}^{(r+1:m)}_{\mathbf{B,Z,I}}(b,z,I)=\widehat{f}^{(r:m-1)}_{\mathbf{B,Z,I}}(b,z,I), \forall b,z,r\leq m-1 \quad (12)$$

As a weighted sum of the estimators $\widehat{F}_{\mathbf{B_p,Z,I}}$ which are confined in the interval $[0,1]$, the estimators $\widehat{F}^{(r:m)}_{\mathbf{B,Z,I}}(b,z,I)$ are confined in the interval $[0,1]$.

Let $\Upsilon : [0,1]^n \to \mathbb{Z}^n$ be the function such that $(\omega_1, \cdots, \omega_n) = \Upsilon(a_0, \cdots, a_{n-1})$ (where $\omega_1 \geq \cdots \geq \omega_n$) is the ordered vector of the roots (possibly complex number) counted with their order of multiplicity of the polynomial $Q(u) = u^n + \sum_{i=0}^{n-1} a_i \cdot (-1)^{n-i} u^i$, i.e. $Q(u) = \prod_{i=1}^{n}(u-\omega_i)$. Uherka and Sergott [29] show that Υ is continuous and hence uniformly continuous on the compact $[0,1]^n$.

In a second step, in view of (4), it would be natural to estimate the CDFs $\widehat{F}_{\mathbf{B_i^*,Z,I}}(.,.), i \in I$ by

$$(\widehat{F}_{\mathbf{B^*_{j_1},Z,I}}(b,z,I), \cdots, \widehat{F}_{\mathbf{B^*_{j_{n_I}},Z,I}}(b,z,I)) = \mathcal{R}[\Upsilon(\widehat{F}^{(n:n)}_{\mathbf{B,Z,I}}(b,z,I), \cdots, \widehat{F}^{(1:1)}_{\mathbf{B,Z,I}}(b,z,I))], \quad (13)$$

where $\mathcal{R}[z]$ denotes the real part of the complex vector z and $I = (j_1, \cdots, j_{n_I})$, where $j_1 < \cdots < j_{n_I}$.

The derivation of the polynomial relation with respect to b leads to:

$$\begin{aligned}\frac{\partial Q(u)}{\partial b} &= \sum_{i=0}^{n_I-1} f^{(n-i:n-i)}_{\mathbf{B,Z,I}}(b,z,I)\cdot(-1)^{n-i}\cdot u^i \\ &= -\sum_{i\in I}\prod_{j\in I,\ j\neq i}\left(u - F_{\mathbf{B_j^*,Z,I}}(b,z,I)\right)\cdot f_{\mathbf{B_i^*,Z,I}}(b,z,I), \forall u,b,z,I\end{aligned}$$

Thus under assumption (4) that there are no multiple roots, we have a natural estimator for bidders' densities.

$$\widehat{f}_{\mathbf{B_i^*,Z,I}}(b,z,I) = \frac{\sum_{k=0}^{n_I-1}\widehat{f}^{(n-k:n-k)}_{\mathbf{B,Z,I}}(b,z,I)\cdot(-1)^{n_I-k+1}\cdot\left[\widehat{F}_{\mathbf{B_i^*,Z,I}}(b,z,I)\right]^k}{\prod_{j\in I,\ j\neq i}\left(\widehat{F}_{\mathbf{B_i^*,Z,I}}(b,z,I)-\widehat{F}_{\mathbf{B_j^*,Z,I}}(b,z,I)\right)} \quad (14)$$

Note that we have assumed strict asymmetry to avoid singularity points in the estimation of $f_{\mathbf{B}_{\mathbf{i}}^*,\mathbf{Z},\mathbf{I}}$ in any closed subset of $S^o(F_{\mathbf{B}_{\mathbf{i}}^*,\mathbf{Z},\mathbf{I}})$. Now we have all the elements to estimate the function $\psi_i(.,.,.)$ in the first price auction.[11]

In view of (7) and similarly to GPV, it would be natural to construct pseudo private values for each order statistic $p = 1, \cdots, n_I$ and for each potential bidder $i \in I_l$:

$$\widetilde{X}_{ipl} = B^*_{pl} + \widetilde{\psi}_i(B^*_{pl}, Z_l, \mathrm{I}_l), \tag{15}$$

Unfortunately, as it has been emphasized by GPV, the estimator of $\psi_i(.,.,.)$ in the first price auction is asymptotically biased at the boundaries of the support and a trimming is needed. The same trimming is also needed in the second price auction.

In this aim we first estimate the boundary of the support of the joint distribution of (B, Z, I), which is unknown. Since the support of (Z, I) can be assumed to be known, we focus on the estimation of the support $[\underline{b}(z, I), \overline{b}(z, I)]$ of the conditional distribution of B given (Z, I). By our simplifying assumption $\underline{b}(z, I)$ does not depend on (z, I) and is estimated by the minimum of all submitted bids. On the other hand, $\underline{b}(z, I)$ should be estimated as in GPV. Let $h_\delta > 0$. We consider the following partition of $\mathbb{R}^d$ with a generic hypercube of side h_δ:

$$\vartheta_{k_1,\cdots,k_d} = [k_1 h_\delta, (k_1+1)h_\delta) \times \cdots \times [k_d h_\delta, (k_d+1)h_\delta),$$

where $k_1, \cdots, k_d$ runs over $\mathbb{Z}^d$. This induces a partition of $[\underline{z}, \overline{z}]$. Given a set of participant I and a value z, the estimate of the upper boundary $\overline{b}(z, I)$ is the maximum of those bids for which $I_l = I$ and the corresponding value of X_l falls in the hypercube $\vartheta_{k_1,\cdots,k_d}(z)$ containing z. Formally, our estimators for the lower and upper boundaries are respectively given by:

$$\widehat{\overline{b}}(z, I) = \sup\{B_{n_I l}, l = 1, \cdots, L; X_l \in \vartheta_{k_1,\cdots,k_d}(z), I_l = I\} \tag{16}$$

$$\widehat{\underline{b}} = \inf\{B_{1l}, l = 1, \cdots, L\} \tag{17}$$

Our estimator for $S(F_{\mathbf{B_p},\mathbf{Z},\mathrm{I}})$ is $\widehat{S}(F_{\mathbf{B_p},\mathbf{Z},\mathrm{I}}) = \{(b, z, I) : b \in [\widehat{\underline{b}}, \widehat{\overline{b}}(z, I)], z \in [\underline{z}, \overline{z}], I \in \mathcal{I}\}$.

We now turn to the trimming. It is well known that kernel estimators are asymptotically biased at the boundaries of the support. Following GPV, we have to trim out observations that are close to the boundaries of the support. Because $\underline{b} \leq \widehat{\underline{b}} \leq \widehat{\overline{b}}(z, I) \leq \overline{b}$, $\widehat{f}_{\mathbf{B}_p,Z,I}(B_{pl}, Z_l, I)(.,.,.)$ and thus $\widehat{f}_{\mathbf{B}_{\mathbf{j}}^*,\mathbf{Z},\mathbf{I}}(.,.,.)$ are asymptotically unbiased on $[\widehat{\underline{b}} + \frac{\rho_{f_{\mathbf{B_p}|\mathbf{Z}}} \cdot h_{f_{\mathbf{B_p}|\mathbf{Z}}}}{2}, \widehat{\overline{b}}(z, I) - \frac{\rho_{f_{\mathbf{B_p}|\mathbf{Z}}} \cdot h_{f_{\mathbf{B_p}|\mathbf{Z}}}}{2}]$. This leads to defining the sample of pseudo private values $\{\widehat{X}_{ipl}, i \in I_l; p =$

[11] Our procedure easily adapts if the multiplicity of the root $F_{\mathbf{B}_{\mathbf{j}}^*,\mathbf{Z},\mathbf{I}}$ is $k > 1$ by considering the polynomial $u \to \frac{\partial^k Q(u)}{\partial b(\partial u)^{k-1}}$ evaluated at $u = F_{\mathbf{B}_{\mathbf{j}}^*,\mathbf{Z},\mathbf{I}}(b, z, I)$.

$1, \cdots, n; l = 1, \cdots, L\}$ where $\widehat{X}_{ipl}$, the estimate of the private value of bidder i would it be the bidder of the pth order statistic of the vector of bids B_l, is defined by

$$\widehat{X}_{ipl} = \begin{cases} B_{pl} + & \left[\sum_{j \neq i} \frac{\widehat{f}_{\mathbf{B}_j^*}(B_{pl}, Z_l)}{\widehat{F}_{\mathbf{B}_j^*}(B_{pl}, Z_l)}, \right]^{-1} \\ & \text{if } \ \widehat{\underline{b}} + h_{f_{\mathbf{B_p|Z}}} \leq B_{pl} \leq \widehat{\overline{b}}(Z_l, I_l) \\ +\infty, & \text{otherwise,} \end{cases} \tag{18}$$

in the first price auction and

$$\widehat{X}_{ipl} = \begin{cases} B_{pl} & \text{if } \ \widehat{\underline{b}} + h_{f_{\mathbf{B_p|Z}}} \leq B_{pl} \leq \widehat{\overline{b}}(Z_l, I_l) \\ +\infty, & \text{otherwise,} \end{cases} \tag{19}$$

in the second price auction.

On the contrary to GPV, we should not use directly this pseudo sample of private values in a standard kernel estimation to estimate $f_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x, z, I)$. Each pseudo values do not have the same weighted since for a given order statistic B_p the probability that it results from a given bidder i depends on the identity of this bidder. Thus we have to estimate the corresponding probability weights. Under anonymity, there are at most $n_I!$ vectors of private values that can rationalize a given vector of bids $(B_{1l}, \cdots, B_{n_I l})$. Denote by $\widetilde{\pi} \in \Sigma_I$ the permutation that matches a given vector of bidding order statistics $(B_{1l}, \cdots, B_{n_I l})$ with the unobserved vector of bids $(B_{1l}^*, \cdots, B_{nl}^*)$.

The following expression gives the theoretical probability, denoted by $Prob(\widetilde{\pi} = \pi | (b_1, \cdots, b_{n_I}, z, I))$, that the assignment of bidders to the observed order statistics corresponds to the permutation π:

$$Prob(\widetilde{\pi} = \pi | (b_1, \cdots, b_{n_I}, z, I)) = \frac{\prod_{i \in I} f_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I}}(b_{\pi(i)} | z, I)}{\sum_{\pi' \in \sigma_I} \prod_{i \in I} f_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I}}(b_{\pi'(i)} | z, I)} \cdot \mathbf{1}\{\pi \in \sigma_I\}. \tag{20}$$

Note that we use the information set σ_I to refine our beliefs on $\widetilde{\pi}$. Then the probability, denoted by P_{ip}, that the pth order statistic results from bidder i equals to the sum of the above probabilities for all the permutations that assign i to the pth order statistic, i.e.

$$P_{ip} = \sum_{\pi \in \sigma_I \ s.t. \ \pi(i) = p} Prob(\widetilde{\pi} = \pi | (b_1, \cdots, b_n, z, I)). \tag{21}$$

Their empirical counterparts, $\widehat{P}_l(\pi)$ and $\widehat{P}_{ipl}$ are given straightforwardly by means of our previous estimators and are thus asymptotically unbiased if order statistics belong to the interval $S^0(F_{\mathbf{B_p},\mathbf{Z},\mathbf{I}})$.

$$\widehat{P}_l(\pi) = \pi|(B_l, Z_l, I_l)) = \frac{\prod_{i\in I_l} \widehat{f}_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I}}(B_{\pi(i)l}|Z_l, I_l)}{\sum_{\pi'\in\sigma_{I_l}} \prod_{i\in I_l} \widehat{f}_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I}}(B_{\pi'(i)l}|Z_l, I_l)} \cdot \mathbf{1}\{\pi \in \sigma_{I_l}\} \tag{22}$$

$$\widehat{P}_{ipl} = \sum_{\pi\in\sigma_{I_l}\ s.t.\ \pi(i)=p} \widehat{Prob}(\widetilde{\pi} = \pi|(B_l, Z_l, I_l)) \tag{23}$$

In the final step we use the pseudo sample $\{(\widehat{X}_{ipl}, \widehat{P}_{ipl}, Z_l), i = 1, \cdots, n, p = 1, \cdots, n, l = 1, \cdots, L\}$ to estimate nonparametrically the densities $f_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(x|z, I)$ by $\widehat{f}_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(x|z, I) = \widehat{f}_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(x, z, I)/\widehat{f}_{\mathbf{Z},\mathbf{I}}(z, I)$, where

$$\widehat{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x, z, I) = \frac{1}{Lh^{d+1}_{f_{\mathbf{X_i},\mathbf{Z}}}} \sum_{l=1}^{L} \sum_{p\in \mathrm{I}_l} \widehat{P}_{ipl} \cdot K_{f_{\mathbf{X_i},\mathbf{Z}}}(\frac{x - \widehat{X}_{ipl}}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}, \frac{z - Z_l}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}) \cdot \mathbf{1}(I_l = I), \tag{24}$$

$$\widehat{f}_{\mathbf{Z},\mathbf{I}}(z, I) = \frac{1}{Lh^{d}_{\mathbf{Z}}} \sum_{l=1}^{L} \sum_{p=1}^{n} K_{f_{\mathbf{Z}}}(\frac{z - Z_l}{h_{f_{\mathbf{Z}}}}) \cdot \mathbf{1}(I_l = I), \tag{25}$$

$h_{f_{\mathbf{X_i},\mathbf{Z}}}$ and $h_{f_{\mathbf{Z}}}$ are bandwidths, and $K_{f_{\mathbf{X_i},\mathbf{Z}}}$ and $K_{f_{\mathbf{Z}}}$ are kernels.

We now turn to the choice of kernels and bandwidths defining our multistep estimators.

Assumption A 6 • ***KERNELS***

(i) *The kernels $K_{F_{\mathbf{B_p}|\mathbf{Z}}}(.)$, $K_{f_{\mathbf{B_p}|\mathbf{Z}}}(., .)$, $K_{f_{\mathbf{X_i},\mathbf{Z}}}(., .)$ and $K_{f_{\mathbf{Z}}}(.)$ are symmetric with bounded hypercube supports of length equal to 2 and continuous bounded first derivatives with respect to their continuous argument.*

(ii) *$\int K_{F_{\mathbf{B_p}|\mathbf{Z}}}(z)dz = 1$, $\int K_{f_{\mathbf{B_p}|\mathbf{Z}}}(b, z)dbdz = 1$, for any $p = 1, \cdots, n$, $\int K_{f_{\mathbf{X_i},\mathbf{Z}}}(x, z)dxdz = 1$ for any $i = 1, \cdots, n$ and $\int K_{f_{\mathbf{Z}}}(z)dz = 1$.*

(iii) *$K_{F_{\mathbf{B_p}|\mathbf{Z}}}(.)$, $K_{f_{\mathbf{B_p}|\mathbf{Z}}}(., .)$, $K_{f_{\mathbf{X_i},\mathbf{Z}}}(., .)$ and $K_{f_{\mathbf{Z}}}(.)$ are of order $R+1$, $R + 1$,R and $R + 1$ respectively, i.e. moments of order strictly smaller than the given order vanish.*

• ***BANDWIDTHS***

(i) *In the first price auction, the bandwidths $h_{F_{\mathbf{B_p}|\mathbf{Z}}}, h_{f_{\mathbf{B_p}|\mathbf{Z}}}$, for $p = 1, \cdots, n$, $h_{f_{\mathbf{X_i},\mathbf{Z}}}$ for $i = 1, \cdots, n$ and $h_{f_{\mathbf{Z}}}$ are of the form:*

$$h_{F_{\mathbf{B_p}|\mathbf{Z}}} = \lambda_{F_{\mathbf{B_p}|\mathbf{Z}}}(\frac{\log L}{L})^{\frac{1}{(2R+d+2)}}, \quad h_{f_{\mathbf{B_p}|\mathbf{Z}}} = \lambda_{f_{\mathbf{B_p}|\mathbf{Z}}}(\frac{\log L}{L})^{\frac{1}{(2R+d+3)}},$$

$$h_{\mathbf{f_{X_i,Z}}} = \lambda_{\mathbf{f_{X_i,Z}}} (\frac{\log L}{L})^{\frac{1}{(2R+d+3)}}, \quad h_{f_{\mathbf{Z}}} = \lambda_{f_{\mathbf{Z}}} (\frac{\log L}{L})^{\frac{1}{(2R+d+2)}},$$

where the λ's are strictly positive constants.

(ii) *In the second price auction, the bandwidths $h_{F_{\mathbf{B_p|Z}}}, h_{f_{\mathbf{B_p|Z}}}$, for $p = 1, \cdots, n$, $h_{f_{\mathbf{X_i,Z}}}$ for $i = 1, \cdots, n$ and $h_{f_{\mathbf{Z}}}$ are of the form:*

$$h_{F_{\mathbf{B_p|Z}}} = \lambda_{F_{\mathbf{B_p|Z}}} (\frac{\log L}{L})^{\frac{1}{(2R+d)}}, \quad h_{f_{\mathbf{B_p|Z}}} = \lambda_{f_{\mathbf{B_p|Z}}} (\frac{\log L}{L})^{\frac{1}{(2R+d+1)}},$$

$$h_{\mathbf{f_{X_i,Z}}} = \lambda_{\mathbf{f_{X_i,Z}}} (\frac{\log L}{L})^{\frac{1}{(2R+d+1)}}, \quad h_{f_{\mathbf{Z}}} = \lambda_{f_{\mathbf{Z}}} (\frac{\log L}{L})^{\frac{1}{(2R+d+2)}},$$

(iii) *The "boundary" bandwidth is of the form $h_\delta = \lambda_\delta (\frac{\log L}{L})^{\frac{1}{d+1}}$ if $d > 0$*
where the λ's are strictly positive constants.

4.4 Uniform Consistency

Our main result establishes the uniform consistency of our multistage kernel-based estimators for the first and second price auctions and with their rates of convergence. Before, next proposition, the analog of propositions 2 and 3 in GPV, establishes the uniform consistency with their rates of convergence of our nonparametric estimators of the upper and lower boundaries $\bar{b}(z, I)$ and $\underline{b}$ and also the rates at which the pseudo private values $\widehat{X}_{ipl}$ and the pseudo probabilities $\widehat{P}_{ipl}$ converge uniformly to their true values.

Proposition 4.3 *Under A1-A6, for any closed subset $\mathcal{C}$ of $S^o(F_{\mathbf{X},\mathbf{Z},\mathrm{I}})$, we have almost surely:*

$$\sup_{(z,I)\in[\underline{z},\bar{z}]\times\mathcal{I}} |\widehat{\bar{b}}(z, I) - \bar{b}(z, I)| = O(\frac{\log L}{L})^{\frac{1}{d+1}},$$

$$|\widehat{\underline{b}} - \underline{b}| = O(\frac{\log L}{L})^{\frac{1}{d+1}}$$

for both the first and second price auctions. The pseudo values and pseudo probabilities are satisfying almost surely:

(i)

$$\sup_{i,p,l} \mathbf{1}_{\mathcal{C}}(X_{ipl}, Z_l, \mathrm{I}_l) |\widehat{X}_{ipl} - X_{ipl}| = O\left((\frac{\log L}{L})^{\frac{R+1}{(2R+d+3)}} \right)$$

(ii)

$$\sup_{i,p,l} \mathbf{1}_{\mathcal{C}}(X_{ipl}, Z_l, \mathrm{I}_l)|\widehat{P}_{ipl} - P_{ipl}| = O\left((\frac{\log L}{L})^{\frac{R+1}{(2R+d+3)}} \right)$$

in the first price auction and

(i)

$$\sup_{i,p,l} \mathbf{1}_{\mathcal{C}}(X_{ipl}, Z_l, \mathrm{I}_l)|\widehat{X}_{ipl} - X_{ipl}| = O\left((\frac{\log L}{L})^{\frac{R}{(2R+d+1)}} \right)$$

(ii)

$$\sup_{i,p,l} \mathbf{1}_{\mathcal{C}}(X_{ipl}, Z_l, \mathrm{I}_l)|\widehat{P}_{ipl} - P_{ipl}| = O\left((\frac{\log L}{L})^{\frac{R}{(2R+d+1)}} \right)$$

in the second price auction.

In the same way as the vector of pseudo private values are not sufficient to estimate the CDFs of each bidders private values (on the contrary to GPV), the estimation of conditional mean, variance or quantiles of a given bidder's private values requires the joint use of the pseudo private values with the associated vector of pseudo probabilities.

We now state our main result. The study of uniform convergence is restricted to inner closed subset of the support to avoid boundary effects.

Proposition 4.4 *Suppose that A1-A6 hold, then* $(\widehat{f}_{\mathbf{X_1}|\mathbf{Z},\mathbf{I}}(.|.,.), \cdots, \widehat{f}_{\mathbf{X_n}|\mathbf{Z},\mathbf{I}}(.|.,.))$ *is uniformly consistent as* $L \to \infty$ *with rate* $(L/\log L)^{R/(2R+d+3)}$ *on any inner compact subset of the support of* $(f_{\mathbf{X_1}|\mathbf{Z},\mathbf{I}}(.|.,.), \cdots, f_{\mathbf{X_n}|\mathbf{Z},\mathbf{I}}(.|.,.))$ *in the first price auction and respectively the rate* $(L/\log L)^{R/(2R+d+1)}$ *in the second price auction.*

In addition to establishing the uniform consistency of our multistep estimator, Proposition 4.4 implies that the upper bounds that have been derived for the first and second price auctions in Proposition 4.2 are in fact the optimal uniform convergence rates for estimators of the conditional density $F_{\mathbf{X}|\mathbf{Z},\mathbf{I}}(.|.,.)$ from observed anonymous bids and that our procedure is asymptotically optimal. On the contrary, if the interest of the econometrician lies only in the estimation of the distributions $F_{\mathbf{B}^*|\mathbf{Z},\mathbf{I}}(.|.,.)$, then, in the first price auction, our bandwidths are suboptimal and the same bandwidths as those for the second price auction should be used.

We present the proof of Proposition 4.4 as it helps to identify the additional points relative to GPV's two step procedure and why it does not change the asymptotical rates of convergence.

Proof 1 *We have* $\widehat{f}_{\mathbf{X_i}|\mathbf{Z},\mathbf{I}}(x|z,I) = \widehat{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I)/\widehat{f}_{\mathbf{Z},\mathbf{I}}(z,I)$*. Given the optimal bandwidth choice for* $h_{f_\mathbf{Z}}$ *in assumption A(6), we know that* $\widehat{f}_{\mathbf{Z},\mathbf{I}}(z,I)$

converges uniformly to $f_{\mathbf{Z},\mathbf{I}}(z,I)$ at the rate $(L/\log L)^{(R+1)/(2R+d+1)}$ on any inner compact of its support. Because this rate is faster than that of the theorem (for both the first and second price auction) and $f_{\mathbf{Z},\mathbf{I}}(z,I)$ is bounded away from 0 by assumption A3-(ii), it suffices to show that $\widehat{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I)$ converges at the rate $(\frac{\log L}{L})^{R/(2R+d+3)}$ and $(\frac{\log L}{L})^{R/(2R+d+1)}$ in the first and second price auctions respectively.

We decompose the difference $\widehat{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I) - f_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x|z,I)$ into three terms.

$$\begin{aligned}
&\widehat{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I) - f_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I) \\
&\qquad= \frac{1}{Lh^{d+1}_{f_{\mathbf{X_i},\mathbf{Z}}}} \sum_{l=1}^{L} \sum_{p\in \mathrm{I}_l} (\widehat{P}_{ipl} - P_{ipl}) \cdot K_{f_{\mathbf{X_i},\mathbf{Z}}}(\frac{x-X_{ipl}}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}, \frac{z-Z_l}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}) \cdot \mathbf{1}(I_l = I) \\
&\qquad+\frac{1}{Lh^{d+1}_{f_{\mathbf{X_i},\mathbf{Z}}}} \sum_{l=1}^{L} \sum_{p\in \mathrm{I}_l} \widehat{P}_{ipl} \cdot \mathbf{1}(I_l = I) \\
&\qquad\times \left(K_{f_{\mathbf{X_i},\mathbf{Z}}}(\frac{x-\widehat{X}_{ipl}}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}, \frac{z-Z_l}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}) - K_{f_{\mathbf{X_i},\mathbf{Z}}}(\frac{x-X_{ipl}}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}, \frac{z-Z_l}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}) \right) \\
&\qquad+\widetilde{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I) - f_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I),
\end{aligned} \tag{26}$$

where $\widetilde{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}$ is the (infeasible) nonparametric estimator of the density of (X_i, Z, I) using the unobserved values X_{ipl} and the unobserved probabilities P_{ipl}:

$$\widetilde{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I) = \frac{1}{Lh^{d+1}_{f_{\mathbf{X_i},\mathbf{Z}}}} \sum_{l=1}^{L} \sum_{p\in \mathrm{I}_l} P_{ipl} \cdot K_{f_{\mathbf{X_i},\mathbf{Z}}}(\frac{x-X_{ipl}}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}, \frac{z-Z_l}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}) \cdot \mathbf{1}(I_l = I).$$

In the second price auction, the bandwidth $h_{\mathbf{f_{X_i},Z}}$ is optimal and thus leads to a uniform convergence of $\widetilde{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I)$ to $f_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I)$ at the rate $(L/\log L)^{R/(2R+d+1)}$ in any inner compact of its support. In the first price auction, the suboptimal bandwidth leads to the rate $(L/\log L)^{R/(2R+d+3)}$ as in GPV. Thus we are left with the first two terms, the first one resulting explicitly from the anonymous nature of the bids is new, whereas the second term appears already in GPV.

First consider the second price auction. Since $\widehat{X}_{ipl} = X_{ipl}$, the second term vanishes and we are left with the first term which is bounded by:

$$\sup_{p,l} \mathbf{1}_{\mathcal{C}}(X_{ipl}, Z_l, \mathrm{I}_l)|\widehat{P}_{ipl} - P_{ipl}| \cdot \left[\frac{1}{Lh^{d+1}_{f_{\mathbf{X_i},\mathbf{Z}}}} \sum_{l=1}^{L} \sum_{p\in \mathrm{I}_l} K_{f_{\mathbf{X_i},\mathbf{Z}}}(\frac{x-X_{ipl}}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}, \frac{z-Z_l}{h_{f_{\mathbf{X_i},\mathbf{Z}}}}) \cdot \mathbf{1}(I_l = I) \right]$$

The above term appearing in the bracket may be viewed as a kernel estimator, and hence converges uniformly on $\mathcal{C}$ to

$$\sum_{p \in I} f_{\mathbf{X_{ip}},\mathbf{Z},\mathbf{I}}(x,z,I) \cdot \int f_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I)dxdz$$

Thus this term stays bounded almost surely. Finally the difference $\widehat{f}_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I) - f_{\mathbf{X_i},\mathbf{Z},\mathbf{I}}(x,z,I) = O(\log L/L)^{R/2R+d+1}$.

In the first price auction, similarly to GPV, a first-order Taylor expansion establishes that the second term has the order $O(\log L/L)^{R/2R+d+3}$*, whereas the same argument as above establishes that the first term has the order* $O(\log L/L)^{R+1/2R+d+3}$*. Thus with anonymity, it is still the second term that results from the gap between estimated and real private values that is the 'binding' term relative to the uniform convergence rate.*

5 Conclusion

Our identification methodology has been limited to the independent private value framework with risk neutral bidders, no reserve price and a complete set of bids. For the second price auction, we can be reluctant to propose identification and estimation methods that are relying on the observation of the complete set of bids, in particular on the observation of the highest bid which may remain unobserved. Moreover, this excludes any direct application for the ascending (English) auction.

All our analysis of the first-price auction can be adapted to risk averse bidders under a conditional quantile restriction and a parametrization of bidders' utility function following Campo et al. [6]. As in GPV, our analysis can also be adapted to a binding reserve price provided that we are prepared to assume that the number of potential bidders is constant. Naturally, identification is obtained only for the truncated distribution of types that are above the reserve price. More involved is the extension of our methodology with incomplete sets of bids, whose developments are left for further research. Let us briefly precise the issues. Each ordered statistic leads to an equation leading thus to an n equations system, whereas we face n unknowns. Thus the least unobserved bidding statistic leads to unidentification. There are two routes to restore identification. First, to impose more symmetry by assuming that some bidders are symmetric: it corresponds to a reduction of the number of unknowns. Second, to exploit some exogenous variations in the number of bidders: it corresponds to an expansion of the number of equations.

Note that the symmetric APV model is not identified if we do not observe the highest bid (Theorem 4 in Athey and Haile [2]) and that identification could not be tackled even if we do observe some exogenous variation in the number of bidders. Thus the way we exploit independence could be developed in further research to obtain identification with an incomplete set of anonymous bids and which goes beyond the symmetric IPV framework that is currently used in such a case.

Our approach can also be used for alternative asymmetric auction models with independent private signals as the one developed by Landberger et al. [15] where the ranking of bidders' private valuations is common knowledge among bidders, but not to the econometrician. A promising avenue for research, which was the initial motivation of this work, is the structural analysis of models with shill bidding as developed by Lamy [14]. In a private value framework, models with shill bidding are strategically equivalent to models with a secret reserve price. It differs only from the econometrician point of view: in the latter, she distinguishes a submitted bid from the reserve price which facilitates the estimation as in [8, 17], whereas, in the former, the strategic bidding activity of the seller is indistinguishable from any other bid. Nevertheless, our methodology can be adapted.

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A Proofs of Mathematical Properties

A.1 Proof of Proposition [3.1]

We write the proof for the first price auction, the most difficult case where the correspondance between bids and private signals is not immediate. In the second price auction, bids are equal to private values and the following proof can be easily adapted.

Remind that under observability of bidders' identities, Li, Perrigne and Vuong [18] show that the symmetric APV model is identified whereas Campo, Perrigne and Vuong [7] extend this result to the asymmetric APV model. Let us see why [18]'s proof remains valid under anonymity whereas [7]'s proof does not.

The main step to obtain identification is the equilibrium equation (7) that allows to express bidder i's private value x_i as the function of his bid b_i, the CDF $G_{\mathbf{B}_{-i}|\mathbf{b}_i}(.|.)$ and the PDF $g_{\mathbf{B}_{-i}|\mathbf{b}_i}(.|.)$ of his opponents bids conditional on his bid. Under observed identities, it is possible to obtain the full distribution of the vector of private valuations X since $G_{\mathbf{B}_{-i}|\mathbf{b}_i}(y|x)$ is observed. Under anonymity, only $\frac{1}{n} \cdot \sum_{i=1}^{n} G_{\mathbf{B}_{-i}|\mathbf{b}_i}(y|x)$ is observed, which prevents the use of the above equation except in the symmetric case where $G_{\mathbf{B}_{-i}|\mathbf{b}_i}(y|x) = \frac{1}{n} \cdot \sum_{i=1}^{n} G_{\mathbf{B}_{-i}|\mathbf{b}_i}(y|x)$. Therefore the symmetric APV model is identified.

For the asymmetric APV model, for any distribution of bids $F_{\mathbf{B}}$ and a given distribution of signals $F_{\mathbf{X}}$ that rationalizes $F_{\mathbf{B}}$, let us construct a distribution of signals $F'_{\mathbf{X}}$ that differs from $F_{\mathbf{X}}$ (up to any permutation) and that leads to $F_{\mathbf{B}}$. Consider two bids $\underline{b}$ and $\overline{b}$, $\overline{b} > \underline{b}$, used by all bidders, take $\epsilon < \frac{\overline{b}-\underline{b}}{2}$ such that bidders are bidding in the intervals $[\underline{b}-\epsilon, \underline{b}+\epsilon]$ and $[\overline{b}-\epsilon, \overline{b}+\epsilon]$. For any bidder i, define $\underline{x}_i^{-\epsilon}$, $\underline{x}_i$ and $\underline{x}_i^{+\epsilon}$ by the equations:

$$\begin{aligned} \underline{b}-\epsilon &= \beta_i(\underline{x}_i^{-\epsilon}) \\ \underline{b} &= \beta_i(\underline{x}_i) \\ \underline{b}+\epsilon &= \beta_i(\underline{x}_i^{+\epsilon}). \end{aligned} \tag{27}$$

We define $\overline{x}_i^{-\epsilon}$, $\overline{x}_i$ and $\overline{x}_i^{+\epsilon}$ in the same way. For a couple of bidders (i,j), define

$$\begin{aligned} c(x_1, \cdots, x_n; \epsilon, i, j) \equiv & \prod_{k \neq i,j} \left(\mathbf{1}\{x_k \in [\underline{x}, \underline{x}_k]\}\right) \cdot \mathbf{1}\{x_i \in [\overline{x}_i^{-\epsilon}, \overline{x}_i]\} \\ & \cdot \Big(\mathbf{1}\{x_j \in [\underline{x}_j, \underline{x}_j^{+\epsilon}]\} - \mathbf{1}\{x_j \in [\underline{x}_j^{-\epsilon}, \underline{x}_j]\}\Big) \\ & - \prod_{k \neq i,j} \left(\mathbf{1}\{x_k \in [\underline{x}, \underline{x}_k]\}\right) \cdot \mathbf{1}\{x_i \in [\overline{x}_i, \overline{x}_i^{+\epsilon}]\} \\ & \cdot \Big(\mathbf{1}\{x_j \in [\underline{x}_j, \underline{x}_j^{+\epsilon}]\} - \mathbf{1}\{x_j \in [\underline{x}_j^{-\epsilon}, \underline{x}_j]\}\Big) \end{aligned} \tag{28}$$

For sufficiently small $\gamma > 0$, $f'_{\mathbf{X}}(.) \equiv f_{\mathbf{X}}(.) + \gamma \cdot (c(.;\epsilon,i,j) - c(.;\epsilon,j,i))$ is a PDF with the functions c shifting probability weight from some regions to others.

In a first step, we prove that the first order conditions that characterizes equilibrium bidding functions do not change when the map $c(.;\epsilon,i,j)$ is added to the original PDF $f_{\mathbf{X}}(.)$. It results from the fact that both $G_{\mathbf{B}_{-s}|\mathbf{b}_s}(x|x)$ and $g_{\mathbf{B}_{-s}|\mathbf{b}_s}(x|x)$ do not change. On the one hand, for $s \neq j$, it is easy to see that the term $\mathbf{1}\{x_k \leq \underline{x}_k, \forall k \neq i,j\} \cdot \mathbf{1}\{x_i \in [\overline{x}_i^{-\epsilon}, \overline{x}_i]\} \cdot \Big(\mathbf{1}\{x_j \in [\underline{x}_j, \underline{x}_j^{+\epsilon}]\} - \mathbf{1}\{x_j \in [\underline{x}_j^{-\epsilon}, \underline{x}_j]\}\Big)$ does not modify $G_{\mathbf{B}_{-s}|\mathbf{b}_s}(x|x)$ and $g_{\mathbf{B}_{-s}|\mathbf{b}_s}(x|x)$. On the other hand, the second term in expression (28) has been explicitly added to guarantee that $G_{\mathbf{B}_{-s}|\mathbf{b}_s}(x|x)$ and $g_{\mathbf{B}_{-s}|\mathbf{b}_s}(x|x)$ do not change for $s = j$. Indeed, if γ is small enough $\gamma < \gamma_1$, then the original equilibrium bid functions still satisfy the global equilibrium conditions.

In a second step, we have to check that the full distribution of the complete set of bids remains the same. To understand our construction, first remark that the perturbation $\gamma \cdot c(.;\epsilon,i,j)$ alone changes the final distribution of bids. It shifts probability weight from regions where two bids are respectively in the intervals $[\underline{b}-\epsilon, \underline{b}]$ and $[\overline{b}-\epsilon, \overline{b}]$ (respectively in the intervals $[\underline{b}, \underline{b}+\epsilon]$ and $[\overline{b}, \overline{b}+\epsilon]$) to regions where two bids are respectively in the intervals $[\underline{b}-\epsilon, \underline{b}]$ and $[\overline{b}, \overline{b}+\epsilon]$ (respectively in the intervals $[\underline{b}, \underline{b}+\epsilon]$ and $[\overline{b}-\epsilon, \overline{b}]$). Subtracting the (symmetric) permutation $\gamma \cdot c(.;\epsilon,j,i)$ allows to restore those shifts in the bids joint distribution making it identical to the original one.

Finally, we have to check that $f'_{\mathbf{X}}(.)$ and $f_{\mathbf{X}}(.)$ do not coincide up to a permutation. By coincidence, for a given γ, there may exist a permutation π such that $f'_{\mathbf{X}}(x_1, \cdots, x_n) = f_{\mathbf{X}}(x_{\pi(1)}, \cdots, x_{\pi(n)})$. Our construction is valid for any γ which is sufficient small, thus an infinite number of γ are potential candidates. On the other hand, there exists only a finite number of permutation. Now suppose that for any $\gamma < \gamma_1$ there exists a permutation π_γ such that $f'_{\gamma,\mathbf{X}}(x_1, \cdots, x_n) = f_{\mathbf{X}}(x_{\pi_\gamma(1)}, \cdots, x_{\pi_\gamma(n)})$. Then there exists γ^a and γ^b such that $\pi_{\gamma^a} = \pi_{\gamma^b}$, which implies that the function $(c(.;\epsilon, i, j) - c(.;\epsilon, j, i))$ should be null, which raises a contradiction.

For instance, we have only proved that, for any asymmetric PV model, there exists a local perturbation which corresponds to an asymmetric PV model and that leads to the same distribution of bids. Indeed the above perturbation may break affiliation due to the non-smoothness of the indicator function. Let $\phi(.)$ be a smoothed version of the indicator function on the interval $[0, 1]$: $\phi(x) > 0$ if and only if $x \in [0, 1]$ and $\int \phi = 1$. Then in the above perturbations, replace the expressions of the kind $\mathbf{1}\{a \in [\underline{a}, \overline{a}]\}$ by $\phi(\frac{a-\underline{a}}{\overline{a}-\underline{a}})$. The resulting modified perturbations are still shifting probability weight from some regions to others for γ sufficiently small. Moreover, by setting γ sufficiently small, the expressions

$$\partial^2 log(1 + \frac{\gamma \cdot (c(x_1, \cdots, x_n; \epsilon, i, j) - c(x_1, \cdots, x_n; \epsilon, j, i))}{f_{\mathbf{X}}(x_1, \cdots, x_n)})/\partial x_i \partial x_j,$$

for any (i, j), $i \neq j$, can be made arbitrarily small, which guarantees that strict affiliation is preserved if γ is small enough.

A.2 Proof of Proposition [4.1]

In their proposition 1, GPV obtain the same properties for the CDFs $F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}$ instead of $F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}$. From (3) and (4), we obtain that any CDF $F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(b, z, I)$ can be expressed as a linear combination of terms which are product of $F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}(b, z, I)$, i.e. as a continuous function of the CDFs $F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}$. The CDF $F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}$ have the desired smoothness properties on the set $S^0(F_{\mathbf{B_n}|\mathbf{Z},\mathbf{I}}) \setminus \{\overline{b}(z, I, i)\}$: on the set $S^0(F_{\mathbf{B_i}|\mathbf{Z},\mathbf{I}})$, it comes from GPV, whereas $F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}$ is equal to 1 above $\overline{b}(z, I, i)$ and is thus C^∞. Thus all the regularity properties (iii-v) that are valid for $F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}$ are still valid for $F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}$ if the points $\{\overline{b}(z, I, i)\}$ have been appropriately removed. The image of a closed interval by a continuous function is a closed interval. Thus (i) holds also for $F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}$. Finally we are left with (ii). Note the difference between the similar point in GPV which holds for the whole support and not only for a closed subset of the $S^o(F_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}})$ as above. By deriving (4) and (3), we obtain an another expression of $f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(b, z, I)$ as a function of $F_{\mathbf{B_i^*}|\mathbf{Z},\mathbf{I}}(b, z, I)$ and

$f_{\mathbf{B}_\mathbf{i}^*|\mathbf{Z},\mathbf{I}}(b,z,I)$:

$$f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(b,z,I) = \frac{1}{(p-1)!(N_I-p-1)!} \cdot \sum_{\pi\in\Sigma_I}\left[\prod_{i=1}^{p-1} F_{\mathbf{B}^*_{\pi(\mathbf{i})}|\mathbf{Z},\mathbf{I}}(b,z,I)\cdot f_{\mathbf{B}^*_{\pi(\mathbf{p})}|\mathbf{Z},\mathbf{I}}(b,z,I)\cdot \prod_{i=p+1}^{n_I}(1-F_{\mathbf{B}^*_{\pi(\mathbf{i})}|\mathbf{Z},\mathbf{I}}(b,z,I))\right]$$

Thus we obtain that $f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(b,z,I)$ is strictly positive on $S^o(F_{\mathbf{B_n}|\mathbf{Z},\mathbf{I}})$. Remark that $f_{\mathbf{B_p}|\mathbf{Z},\mathbf{I}}(b,z,I)$ is null at the lower bound $b=\underline{b}(z,I,p)$ for $p>1$ (respectively at the upper bound $b=\overline{b}(z,I,p)$ for $p<n$).

B Proofs of Statistical Properties

The proofs of the statistical properties are very closely related to GPV. The proof for the derivation of the asymptotic uniform rate of convergence of bidders' private values uses intensively the rates derived previously by GPV. Less obvious is the adaption of GPV's proof for the upper bound on the uniform convergence rate. We follow their proof very carefully and focus only on the new ingredients.

B.1 Optimal Uniform Convergence Rate

We adapt GPV's proof to the asymmetric framework. To ease the exposition, we consider the case where there is a positive probability that $n_I=2$. Without loss of generality, this set is $\{1,2\}$ and is denoted by I_2. The first step is identical to GPV: it is sufficient to prove the proposition by replacing $f_{\mathbf{X}|\mathbf{Z},\mathbf{I}}$ by $f_{\mathbf{X},\mathbf{Z},\mathbf{I}}$. The set $U_\epsilon(f^o_{\mathbf{X},\mathbf{Z},\mathbf{I}})$ can also be replaced by any subset $U\subset U_\epsilon(f^o_{\mathbf{X},\mathbf{Z},\mathbf{I}})$. Then the second step consists in the construction of a discrete subset U of the form $\{f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}(.,.,.), k=1,\cdots,m^{d+1}\}$, where m is increasing with the sample size L, that are suitable perturbations of $f^o_{\mathbf{X},\mathbf{Z},\mathbf{I}}$.

We consider a nonconstant and odd C_∞-function ϕ, with support $[-1,1]^{d+1}$, such that

$$\int_{[-1,0]}\phi(b,z)db=0,\quad \phi(0,0)=0,\quad \phi'(0,0)\neq 0, \tag{29}$$

where ϕ' denotes the derivative of ϕ according to its first component.

Let $\mathcal{C}_{I_2}(B^*)$ be the image of $\mathcal{C}(X)$ by the function that maps bidders' types into observed bids and conditionally on $I=I_2$. It is a nonempty inner compact subset of $S(f^o_{\mathbf{B}^*,\mathbf{Z},\mathbf{I}})$. Let $(b_k,z_k), k=1,\cdots,m^{d+1}$ be distinct points in the interior of $\mathcal{C}_{I_2}(B^*)$ such that the distance between (b_k,z_k) and (b_j,z_j), $j\neq k$, and the distance between (b_k,z_k) and any point outside $\mathcal{C}_{I_2}(B^*)$ are larger than λ_1/m. Thus, one can choose a constant $\lambda_2>1/\lambda_1$ such that the m^{d+1} functions

$$\phi_{mk}(b,z) = \frac{1}{m^{R+1}}\phi(m\lambda_2(b-b_k), m\lambda_2(z-z_k)) \quad (k=1,\cdots,m^{d+1})$$

have disjoint hypercube supports. Let C_3 be a positive constant (chosen below), for each $k = 1, \cdots, m^{d+1}$ define:

$$f_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I},mk}(b,z,I_2) \equiv \begin{cases} f^o_{\mathbf{B_1^*},\mathbf{Z},\mathbf{I}}(b,z,I_2) & \text{if } \ i=1 \\ f^o_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b,z,I_2) - C_3\phi_{mk}(b,z) & \text{if } \ i=2, \end{cases} \tag{30}$$

whereas define $f_{\mathbf{B}^*,\mathbf{Z},\mathbf{I},mk}(b,z,I) \equiv f^o_{\mathbf{B}^*,\mathbf{Z},\mathbf{I}}(b,z,I)$ for $I \neq I_2$. That is $f_{\mathbf{B}^*,\mathbf{Z},\mathbf{I},mk}$ differs from $f^o_{\mathbf{B}^*,\mathbf{Z},\mathbf{I}}$ only for $I = I_2$ and in the neighborhood of (b_k, z_k). The function $f_{\mathbf{B}^*,\mathbf{Z},\mathbf{I},mk}(b,z,I)$ is a density if C_3 is small enough (integrates to 1 from (29) and is bounded away from 0) with the same support as $f^o_{\mathbf{B}^*,\mathbf{Z},\mathbf{I}}(b,z,I)$. Now consider the functions $\xi_{i,mk}(b,z) = b + \frac{F_{\mathbf{B^*_{3-i}},\mathbf{Z},\mathbf{I},mk}(b,z,I_2)}{f_{\mathbf{B^*_{3-i}},\mathbf{Z},\mathbf{I},mk}(b,z,I_2)}$ for $i = 1,2$. If C_3 is small enough, then $\xi_{i,mk}(b,z), i = 1,2$ is increasing in b with a differentiable inverse denoted by $\xi^{-1}_{i,mk}(x,z)$. Then we define for $I = I_2$ and $i = 1,2$

$$f_{\mathbf{X_i},\mathbf{Z},\mathbf{I},mk}(x,z,I_2) = f_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I},mk}(\xi^{-1}_{i,mk}(x,z),z,I_2)/\xi'_{i,mk}(\xi^{-1}_{i,mk}(x,z),z) \tag{31}$$

$$= \frac{f_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I},mk}(\xi^{-1}_{i,mk}(x,z),z,I_2)\cdot(f_{\mathbf{B^*_{3-i}},\mathbf{Z},\mathbf{I},mk}(\xi^{-1}_{3-i,mk}(x,z),z,I_2))^2}{2(f_{\mathbf{B^*_{3-i}},\mathbf{Z},\mathbf{I},mk}(\xi^{-1}_{i,mk}(x,z),z,I_2))^2 - F_{\mathbf{B^*_{3-i}},\mathbf{Z},\mathbf{I},mk}(\xi^{-1}_{i,mk}(x,z),z,I_2)f'_{\mathbf{B^*_{3-i}},\mathbf{Z},\mathbf{I},mk}(\xi^{-1}_{i,mk}(x,z),z,I_2)}$$

For $I \neq I_2$, let $f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}(.,.,.) = f^o_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}(.,.,.)$. From the above expression, $f_{\mathbf{X_i},\mathbf{Z},\mathbf{I},mk}(x,z,I_2) > 0$ if and only if $f_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I},mk}(b,z,I_2) > 0$, where $b = \xi^{-1}_{i,mk}(x,z)$. This completes the construction of the densities $f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}(.,.,.)$, $k = 1,\cdots,m^{d+1}$, which composes the set U. Note that the supports of $f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}(.,.,.)$ and $f_{\mathbf{B}^*,\mathbf{Z},\mathbf{I},mk}(.,.,.)$ coincide respectively with the supports of $f^o_{\mathbf{X},\mathbf{Z},\mathbf{I}}(.,.,.)$ and $f^o_{\mathbf{B}^*,\mathbf{Z},\mathbf{I}}(.,.,.)$.

Then to adapt GPV's proof, we need the analog of their lemma B1 where the notation $f_{mk}(.,.,.)$ should be replaced by $f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}(.,.,.)$, where the first argument x is now the vector of bidders' private values instead of a single unidimensional private value. The analog of Lemma B1 gives two points. First, an appropriate asymptotic lower bound is given for the uniform distance between two elements, i.e. the norm $||.||_{0,\mathcal{C}(X)}$, in the set U as a function of λ_2, m and R. With this bound we can apply Fano's lemma exactly in the same way as in GPV: the step 3 in their proof is unchanged. Second, an asymptotic approximation is given for the distance between $f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}$ and $f^0_{\mathbf{X},\mathbf{Z},\mathbf{I}}$ in the norm $||.||_{r,\mathcal{C}(X)}$, which guarantees that $f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}$ belongs to the set U if m is large enough.

Lemma B.1 (Analog of lemma B1 in GPV) *Given A2-A3, the following properties hold for m large enough:*

(i) *For any $k = 1, \cdots, m^{d+1}$, the supports of $f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}(.,.,.)$ and $f_{\mathbf{B}^*,\mathbf{Z},\mathbf{I},mk}(.,.,.)$ are $S(f^o_{\mathbf{X},\mathbf{Z},\mathbf{I}}(.,.,.))$ and $S(f^o_{\mathbf{B}^*,\mathbf{Z},\mathbf{I}}(.,.,.))$.*

(ii) *There is a positive constant C_4 depending upon ϕ, $f^o_{\mathbf{B}^*,\mathbf{Z},\mathbf{I}}(.,.,.)$ and $\mathcal{C}(X)$ such that for $j \neq k$,*

$$||f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk} - f_{\mathbf{X},\mathbf{Z},\mathbf{I},mj}||_{0,\mathcal{C}(X)} \geq C_4 \cdot \frac{C_3\lambda_2}{m^R}.$$

(iii) *Uniformly in $k = 1, \cdots, m^{d+1}$, we have*

$$||f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk} - f^0_{\mathbf{X},\mathbf{Z},\mathbf{I}}||_r = C_3\lambda_2^{r+1}O(\frac{1}{m^{R-r}}), r = 0 \cdots R-1$$

$$||f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk} - f^0_{\mathbf{X},\mathbf{Z},\mathbf{I}}||_R = C_3\lambda_2^{R+1} \cdot O(1) + o(1).$$

where the big $O(.)$ depends upon ϕ and $f^0_{\mathbf{B}^*,\mathbf{Z},\mathbf{I}}$

Let us detail the proof of (ii) and what has changed relative to GPV's framework. Remind that $(b_k, z_k) \in \mathcal{C}_{I_2}(B^*)$ implies $(x_k, z_k) \in \mathcal{C}(X)$. As in GPV, it then suffices to prove that $|f_{\mathbf{X},\mathbf{Z},\mathbf{I},mk}(x_k, z_k, I_2) - f_{\mathbf{X},\mathbf{Z},\mathbf{I},mj}(x_k, z_k, I_2)| \geq C_4 \cdot \frac{C_3\lambda_2}{m^R}$, where $x_k = \xi^0(b_k, z_k, I_2)$.

From (29), we have: $F_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I},mk}(x_k, z_k, I_2) = F^0_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I}}(x_k, z_k, I_2)$ and $f_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I},mk}(x_k, z_k, I_2) = f^0_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I}}(x_k, z_k, I_2)$ for $i = 1, 2$. The difference is for the expression of $f'_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I},mk}(x_k, z_k, I_2) - f'^0_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I}}(x_k, z_k, I_2)$ which equals to 0 for $i = 1$ and to $-C_3\frac{\lambda_2}{m^R}\phi'(0,0) \neq 0$ for $i = 2$. Thus $f_{\mathbf{X_2},\mathbf{Z},\mathbf{I},mk}(x_k, z_k, I_2) = f_{\mathbf{X_2},\mathbf{Z},\mathbf{I},\mathbf{mj}}(x_k, z_k, I_2)$ which is bounded away from zero and we are left with the term $f_{\mathbf{X_1},\mathbf{Z},\mathbf{I},mk}(x_k, z_k, I_2) - f_{\mathbf{X_1},\mathbf{Z},\mathbf{I},\mathbf{mj}}(x_k, z_k, I_2)$.

Then, from equation (31), we have:

$$f_{\mathbf{X_1},\mathbf{Z},\mathbf{I},mk}(x_k, z_k, I_2) = \frac{f^0_{\mathbf{B_1^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2) \cdot (f^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2))^2}{2(f^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2))^2 - F^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2)(f'^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2) - C_3\lambda_2\phi'(0,0)/m^R)} \tag{32}$$

and

$$f_{\mathbf{X_2},\mathbf{Z},\mathbf{I},mj}(x_k, z_k, I_2) = \frac{f^0_{\mathbf{B_1^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2) \cdot (f^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2))^2}{2(f^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2))^2 - F^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2)(f'^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}(b_k, z_k, I_2))} \tag{33}$$

Now compare (32) and (33). As $\phi'(0,0) \neq 0$ and $F^0_{\mathbf{B_2^*},\mathbf{Z},\mathbf{I}}$ is bounded away from zero since (b_k, z_k) are far enough from the boundaries, the desired result (ii) follows. The proof of (iii) is more involved and follows GPV's proof with the same modification as above by carefully separating the cases $i = 1$ and $i = 2$. More precisely, we have $||f_{\mathbf{X_1},\mathbf{Z},\mathbf{I},mk} - f^0_{\mathbf{X_1},\mathbf{Z},\mathbf{I}}||_r = C_3\lambda_2^{r+1}O(\frac{1}{m^{R-r}})$ and $||f_{\mathbf{X_2},\mathbf{Z},\mathbf{I},mk} - f^0_{\mathbf{X_2},\mathbf{Z},\mathbf{I}}||_r = O(1)$ and the result follows for the product.

B.2 Proof of Proposition [4.3]

There are two new points relative to GPV's analysis. First, their proof is based on the uniform rates of convergence for the CDF, the PDF and also the boundaries estimators of the variable B^* that is observed by the econometrician. Here we do not observe B^* but only the vector of order statistics B. Second, the pseudo probabilities are a new ingredient that do not appear in GPV.

The first issue is then to prove that the same uniform rates of convergence are still valid for B^* though it is not observed. Nevertheless, the uniform rates of convergence they obtained for B^* are still valid under anonymity for the variable B that is observed and with our similar choices for the kernels and the bandwidth parameters. On the contrary to GPV, note that the observed variable B is multidimensional: it does not modify their analysis which immediately adapts.

First the bidding support of the bidders are coinciding with the support of the order statistics. Thus all the results for the estimator of the support of B are immediately converted into results for B^*. From GPV (lemma B2), we obtain the following uniform rate of convergence for the kernel estimators of $\widehat{F}_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})$ and $\widehat{f}_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})$ on any inner closed compact subset of the bidding support $\mathcal{C}(B)$.

$$\sup_{(b,z,I)\subset\mathcal{C}(B)} ||\widehat{F}_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}) - F_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})||_0 = O(\frac{\log L}{L})^{\frac{R+1}{2R+d+2}}$$

$$\sup_{(b,z,I)\subset\mathcal{C}(B)} ||\widehat{f}_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}) - f_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})||_0 = O(\frac{\log L}{L})^{\frac{R+1}{2R+d+3}}$$

In GPV, the corresponding uniform rates of convergence are obtained for the bidding distributions and densities $\widehat{F}_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})$ and $\widehat{f}_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})$ since bidders' identities are observed. However, note that the function mapping the vector of the order statistics CDF $(F_{\mathbf{B_p},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}))_{p=1,\cdots,n_I}$ into $(F_{\mathbf{B_i^*},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}))_{i=1,\cdots,n_I}$ is uniformly continuous. Thus the uniform rate of convergence that holds for $(F_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}))_{p=1,\cdots,n_I}$ remains valid for $(F_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}))_{i=1,\cdots,n_I}$.

Furthermore, from equation (12) and (14), we have on $\mathcal{C}(B)$ where the terms $\widehat{F}_{\mathbf{B_i^*},\mathbf{Z},\mathbf{I}}(b,z,I) - \widehat{F}_{\mathbf{B_j^*},\mathbf{Z},\mathbf{I}}(b,z,I), j \in I\setminus\{i\}$ are bounded away from zero:

$$\begin{aligned} ||\widehat{f}_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}) - f_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})||_0 \leq C_1 \cdot ||\widehat{f}_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}) - f_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})||_0 \\ + C_2 \cdot ||\widehat{F}_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I}) - F_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})||_0 \end{aligned}$$

Thus the uniform convergence rate that holds for $f_{\mathbf{B},\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})$ remains also valid for $f_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}(b,z,\mathrm{I})$.

In any inner compact subset of the support, the pseudo values can be expressed as a continuous function of $f_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}$ and $F_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}$. Furthermore, it is the rate of convergence of $f_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}$ which sets the rate of convergence of $\widehat{X}_{ipl}$ to X_{ipl} in any inner compact subset of the support whereas the estimator for $F_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}$ is converging at a faster rate.

The second issue are the results concerning the uniform rates of convergence of $\widehat{P}_{ipl}$. From equations (22) and (23), the pseudo probabilities can be expressed as a continuous function of $f_{\mathbf{B}^*,\mathbf{Z},\mathrm{I}}$ in any inner compact subset of the support (the denominator stays bounded away from zero). Then uniform the rate of convergence proved by GPV for $\widehat{X}_{ipl}$ also applies for $\widehat{P}_{ipl}$.

Chapitre 6: Bidder Behavior in Multiunit Ascending Auctions: Evidence from cross-border capacity Auctions

Bidder Behavior in Multi-unit Ascending Auctions: Evidence from Cross-border Capacity Auctions

Laurent Lamy*

Abstract

We analyse a unique data set on multi-unit ascending auctions, which contains the whole dynamic of bidders' behavior in the IFA-auctions selling the right to use the electric transmission capacity between France and England. First, we document that daily auctions suffer from a great extent of underpricing and that the winning price and the award concentration are varying a lot across time periods. Second, we fail to explain this evidence by winner's curse-driven arguments. The time periods, which are proxying for small changes in the bidding rules, seem to play a significant role in the extent of underpricing. Our empirical findings are consistent with the view that daily multi-unit ascending auctions among a small number of potential bidders allow a large panel of outcomes, in particular very collusive ones.

Keywords: multi-unit auctions, Ascending auctions, Demand reduction, Transmission, Power Markets
JEL classification: D44, L13, L94, G14

1 Introduction

The use of multi-unit sealed-bid auctions is widespread for a long time. Examples include financial and monetary instruments, as the treasury debt or foreign currency, and wholesale power markets as in the British Pool from 1990 to 2001. The simultaneous ascending auction was first introduced in 1994 for radio spectrum licences in the U.S.. In some European countries as U.K. and Germany, it has been used to auction licences for third generation spectrum (UMTS). Evidence on this format is scarce and mixed: its performance remains largely to be discussed. We analyse the performance of multi-unit auctions for transmission capacity in Power Markets using a unique data set provided by the Commission de Regulation de l'Energie

*Laboratoire d'Economie Industrielle, CREST-INSEE, 28 rue des Saints-Pères 75007 Paris. e-mail: laurent.lamy@ensae.fr

(CRE), the french regulator of electricity and natural gas markets. The data set contains the chronology of the actual bids submitted by each bidder as well as the auction awards to the bidders in over 1500 IFA-auctions that were held during the period 2001-2005 for the France-England Interconnector.[1] Our data set consists of multi-unit ascending auctions for which the complete set of submitted bids (about 40,000) is available.

IFA-auctions sell the rights to use the transmission capacity in one given direction. IFA products are then essentially financial options that are valuable to use if the price differential has the suitable sign. From the perspective of small pure financial traders, it can be viewed as a pure common value auction. In a similar framework, studies of securities markets have found that new issues of debt or equity tend to be underpriced in primary markets relative to its underlying 'true' value [9, 36, 37, 30, 29]. From a theoretical point of view, this gap has been modelled as an equilibrium behavior by Milgrom and Weber [28]. With a large number of bidders, Pesendorfer and Swinkels [33] show however that the price converges in probability to the true value of the object in a uniform sealed bid auction, which is consistent with the empirical evidence that underpricing remains small in security primary markets. On the contrary, the distribution of the number of bidders in IFA auctions is much tighter: the average number of participants is 4 in daily auctions. Bidders are also interacting repeatedly on a daily basis. Furthermore, the ascending multi-unit auction also seems to be more prone to collusion than the uniform sealed bid auction as suggested by the very uncompetitive equilibria derived by Ausubel and Schwartz [5] and Bruso and Lopomo [8]. The GSM spectrum auction analyzed by Grimm et al [18] is a clear cut example of a low price outcome in a multi-unit ascending auction, whereas similar incidents were observed in the FCC spectrum auctions in the US [10]. Finally, prices in power markets are much more volatile which suggests a greater importance of the winner's curse. Altogether, opportunities for tacit collusion are large in IFA-auctions.

There is a huge empirical literature on auction data. Since the pioneering work of Donald and Paarsch [12], a new strand identifies and estimates the Bayes-Nash equilibria derived from the auction theory literature. In most sealed-bid formats, the distribution of bidders' types can be recovered from the auction data under weak assumptions, allowing therefore counterfactuals and thus comparisons between different possible auctions formats.[2] This approach has been very fruitful for single-item auctions. In multi-unit auctions, identification first comes up against the existence and uniqueness of equilibria. Nevertheless, for uniform and discriminatory multi-unit auctions,

[1]The term IFA corresponds to the abbreviation for France-England Interconnector in French.

[2]See the survey of Athey and Haile [3] about structural econometrics of auction data using nonparametric approaches that do not impose any structure on the distribution of types.

some methods have been developed from the share auction model of Wilson [38]: structural elements are estimated from the Euler-Lagrange necessary condition from bidders' optimization problem in Hortaçsu [19] and Février, Préget and Visser [15]. For electricity markets, Wolak [39] develops a similar methodology for the supply function equilibrium concept of Klemperer and Meyer [24]. On the other hand, auction theory does not provide any solvable equilibrium predictions nor first order conditions leading to exploitable moment restrictions for the multi-unit simultaneous ascending auction. The non-stationary nature of this dynamic auction game prevents any structural approach as in Jofre-Bonnet and Pesendorfer [21] where bidders are assumed to adopt markovian strategies. In some application, it could be reasonable to reduce the set of strategies used by the bidders in the multi-unit simultaneous ascending auction, e.g. by restricting attention to proxy-bidding (also called straightforward bidding, see Milgrom [27]) where strategies are isomorphic to a decreasing demand curve as in the uniform price auction. Nevertheless, the quantities demanded by the bidders are not generally a decreasing function of the price in the IFA-auctions excluding thus any structural analysis where the ascending auction is considered as strategically equivalent to the uniform price sealed-bid auction.

For those reasons, we adopt a reduced form approach where our objective is to provide a descriptive analysis of the auction data and to discuss whether the regularities match with the intuitions provided by auction theory. Our aim is to shed some light on the striking features of the data: first, the gap between the final prices for daily and long term contracts, the average prices in the former being about 30% smaller than in the latter ; second, the great variability across periods of time of the average theoretical profit per MW made by the winning arbitrageurs.

The paper is organized as follows. Section 2 describes the IFA products, how they are sold and their position in the french and british power markets. In section 3 we provide a description of the data set and some summary statistics. Section 4 presents some intriguing evidence. Long-term auctions considerably outperform daily auctions in term of revenue. Furthermore, daily auctions present a great inter-temporal instability in term of profitability but also in the way the price differentials explain the winning price. Section 5 reviews various theoretical scenarii that may explain the data. Section 6 provides descriptive statics of the data. We break the time period studied into two categories that proxy for the changes in the bidding rules. We analyse separately six endogenous variables: four concern the outcome of the auction (the weighted average winning price, the winning prices' spread, the number of winners and the HHI index) and two concern the dynamic of the auction (the average number of bids submitted per participant and the average number of bids whose last digit is zero or five). We then run regressions of these endogenous variables on the exogenous variables -consisting of price differentials, a corresponding volatility measure, the quantity auctioned and time dummies- and also on the participation decisions. The total number of participants and more generally each individual participation decisions are arguably endogenous. Lagged values of these regressors are employed as instruments in two-stage least squares estimation. We test whether time period classifications have a significant impact on the endogenous variables after having controlled not only for the exogenous variables (in particular the ex-post theoretical value of the capacity) but also for the heterogenous participation's decisions of the bidders across time periods, our data set allowing us to follow a bidder not only within an auction but also from auction to auction. The results are broadly consistent with switches in the bidders behaviors that could be rationalized by tacit collusion and by small changes of the bidding rules. Section 7 discusses the links with the previous literature on the role of interconnectors in power markets and the empirical and experimental literature on multi-unit ascending auctions. Section 8 concludes.

2 Auctions Rules and Institutional Details

The economic and finance literature is not familiar with the auctions for interconnector capacity. We then present in details the institutional setting.

The IFA is a 2,000 MW high voltage interconnector connecting the French and British transmission systems. It is jointly operated by National Grid and RTE (Réseau de Transport d'Electricité), respectively the English and the French Transmission System Operator. Historically, interconnectors in Europe have been build for reliability reasons: an unforeseen shock in one country could be absorbed by his neighbors. However, the IFA has been the first interconnector that has been used to exploit the price differential between both countries: due to an overcapacity in nuclear energy, the marginal cost in France was traditionally lower than in the rest of Europe, making thus France a natural exporter all over Europe and so in England. Historically, the transmission capacity was attributed through a long term contract that gave EDF exclusive rights over export to the UK. This contract expired in March 2001. RTE and National Grid have opened the interconnector to third parties by auctioning rights to use the interconnector from April 01 of 2001. The capacity is sold simultaneously but separately in the directions 'England to France' and 'France to England' under different contracts covering different durations: tri-annual, annual, seasonal, quarterly, monthly, two days (for the weekend) and daily. The two latter are referred to as short-term auctions, whereas annual, seasonal, quarterly and monthly auctions are referred to as long-term auctions. Table 1 reports the number of auctions held for each duration and both directions. We also report the number of strategies and the number of bids observed for each category. For example, the 73 monthly auctions in our database correspond to the observation of

480 bidding dynamics and 7,419 bids. The amount to be sold, the date of auctions and the planned outages are announced each year. The availability has consistently exceeded 95% per year. Two open and non-discriminatory processes are used to sell capacity right: "tenders" and auctions. "Tenders" are what is referred to as the first price sealed bid auction in the economic literature. The first and unique tri-annual contract is the only contract that has been auctioned in this way. "Auctions" are a multi-unit ascending share auction that is described below in more details. Participation is restricted to eligible users that have signed the IFA User Agreement. Eligibility basically requires that the user has arranged access to the respective transmission system of UK and France. Up to June 2005, 21 different users have participated in IFA auctions in the 1,581 auctions recorded in our database.[3] Participants include firms with generation capacity, distributors and pure traders.

The purchaser of 1MW of transmission capacity for one day and a given direction owns the right to use the capacity in this direction according to the load profile he wants for that day provided that the power transmitted never exceed 1MW. Thus capacity rights are essentially a bundle of options for each time increment of the load profile. Those rights are subject to a "use-it-or-lose-it" rule: by 07:00 a.m. France local time[4] on the day of the daily auction selling the rights for the contract day, owners of units for that contract day (previously sold in the various periodic auctions) announce whether they intend to use the capacity and lose their rights if they indicate that the capacity will be unused. Nevertheless, the IFA access rules do not include any provisions intended to ensure that actual use reflects the notified level of use.[5] Consequently, the actual rules seem to leave some room for capacity preemption as in Joskow and Tirole [22]. Unfortunately, the usage of the purchased capacity is not recorded in the data. Owners of rights for long-term capacity are able to resell some part of it either through a reassignment procedure (a subletting contract that should be notified not later than 05:00 p.m. the day before the daily auction) or a reallocation procedure. In this latter, the owner of a contract that covers the duration contract that will be sold can reallocate some of his units at the auction and receive the prices achieved in the auction.[6] Nevertheless, for the years 2003 and 2004, only a total negligible amount of 500MW has been reauctioned in this way in the direction France to England.

[3]The original users 18 and 19 in the database have been bundled together as a single user identified below as user 18*. The reason is that those users are belonging to a common consortium since april 2000 and that their participation decisions in the IFA-auctions seem to be coordinated: user 18 has participated in the auctions from february 2002 to december 2005, whereas user 19 participates from january 2005. Moreover, the bidding statistics of those users are very similar.

[4]Henceforth, all time reference are expressed in France local time.

[5]see the IFA User Guide, Issue 5, December 2005.

[6]e.g. the owner of an annual contract can reauction his rights in monthly or daily auctions

Table 1: IFA Auctions for the period April 01 of 2001 to July 01 of 2005

	France to England			**England to France**			**Subtotal**		
	Auctions	Participants	Bids	Auctions	Participants	Bids	Auctions	Participants	Bids
Daily	899	3,841	17,138	489	1,513	6,055	1,388	5,354	23,193
Weekend	61	293	2138	1	1	1	62	294	2,139
Monthly	36	247	2,694	37	233	4,725	73	480	7,419
Quarterly	14	105	1,700	14	95	1,287	28	200	2,987
Seasonal	4	27	415	4	23	497	8	50	912
Annual	10	68	1,360	10	62	1,177	20	130	2,537
Tri-annual	1	2	2		0		1	2	2
Subtotal	1,026	4,583	27,478	555	1,927	11,711	1,581	6,510	39,189

Note.-The columns Auctions, Participants and Bids are reporting respectively the total number of Auctions, Participants and Bids in the database.

2.1 Chronology of the IFA-Auctions in the French and the British Power Markets

Each weekday, an auction is held for the capacity available for the following day, simultaneously for each direction. No auction is held on weekends. On Friday, three auctions take place sequentially for capacity available for the following Saturday, Sunday and Monday. By 08:00 a.m. on the day of a daily auction, information on interconnection capacities for the day to be contracted are published. In particular, the capacity sold in previous long term auctions and that is not used is made available for the daily auction process. Then the auction takes place between 08:45 and 09:15 a.m.. The results of the auction are published within 30 minutes of the close of the auction. Contrary to the former English Pool (1990-2001), there is no mandatory market in France and UK nowadays: bilateral contracts can be freely signed. On such a liberalized market, participants can act on a variety of markets. The bulk of the transactions comes from the over-the-counter (OTC) market (about 20,000 MWh per month in the first quarter of 2006 for France). Due to this high volume of transactions, prices data are supposed to be more accurate in those markets and it is those utilized by the CRE from the cotation agency Platts. The alternative is the organized markets or Power Exchanges that have been build in some countries to promote competition. In this perspective, Powernext has been launched in 2001: a day in advance, electricity is traded according to a uniform double-auction. In 2006, Powernext Day-ahead and Futures exceed 50 traders and the traded volumes are increasing steadily (about 2,000 MWh per month in the first quarter of 2006). The fixing of the Day-ahead market is at 11:00 a.m., i.e. after the disclosure of IFA auctions results. In the same way, APX UK is an organized market providing a continuous trading system which closes at 10:30 a.m.. Finally, from 12:00 am to 02:00 p.m., interconnector's users submit and revise their nomination for the IFA capacity.

2.2 The value of the Interconnector's capacity: a call option on price differentials

In this subsection, we present a simple theory of the value of the Interconnector in term of price differentials. The naive approach, that has been used in the first studies of the European Interconnectors as in Boisseleau [6] (pp. 275-286), considers that the value of capacity rights corresponds to a call option on the differential of average daily prices. This approach leaves out the fact that a capacity buyer is free to use it according to his desired intra-day profile and that the price differential fluctuates deeply intra-day. Actually there are large intra-day fluctuations between imports and exports as shown in Figure 1 where a typical business day transfer is depicted. In the same way, it would be absurd to gauge the price of monthly auctions with the monthly average price differentials. Indeed, in the explicit auctions organized by RTE at the other french interconnections, daily capacities are put up for auction per hourly period, whereas Power Exchanges (e.g. Powernext, APX UK) are organized on a 24 or 48 daily slots basis.[7]

Daily capacity rights are essentially a bundle of (European) call options on the hourly price differentials between France and England for a given day and with a null strike price. In the following, we consider that the day is fixed and we drop any reference to it in the notation. Denote by $(p_t^i)_{1\leq t\leq 24}$ the vector of hourly prices per MWh in zone i where i equals to E and F respectively for England and France. Let $d_t^{i\Rightarrow j} = p_t^j - p_t^i$ be the price differential between zone i and zone j. The expected payoff for a daily right on 1MW of transmission capacity from zone i to zone j is:[8]

$$\Delta^{i\Rightarrow j} = \mathbb{E}[\sum_{t=1}^{24}(d_t^{i\Rightarrow j})^+]$$

Without information asymmetry between bidders and if the explicit auctions were competitive, then the winning price, denoted by $P^{i\Rightarrow j}$, should be equal to the theoretical value of the Interconnector $\Delta^{i\Rightarrow j}$.

A monthly capacity right for 1MW is formally equivalent to the bundle of the daily capacity rights for 1MW for each day. Thus, if daily and monthly capacity rights were sold simultaneously, then the arbitrage condition would imply that the price for the monthly auction should be equal to the sum of the prices for the daily auctions. The same applies for the other long term auctions.

2.3 The Auction mechanism

The auction mechanism is an ascending auction through a screen-based system that guarantees the anonymity of the bids submitted. At any time, a bidder is free to submit any price-quantity pair. Those offers are unconditional and irrevocable offers among which the auctioneer selects the best competing ones: the provisional winning allocation is such that capacity is awarded to the bidder offering the highest price, then to the bidder offering the next highest price, and so on until the quantity offered is fully subscribed. Moreover, each successful bidder pays the price he has bid. Note that the seller may select several offers from the same bidder as it happens in about 10% of the daily auctions. This winning bidder has then to pay a different price for each of his selected offers. Thus the provisional allocation rule corresponds to a pay-as-bid pricing rule. Nevertheless, it is a bit misleading since the dynamic structure of the mechanism makes it correspond

[7]See the various capacity access rules for the France-Belgium, France-Spain, France-Italy and French-German interconnections published by RTE (Réseau de Transport d'Electricité).

[8]For a real number x, x^+ designates the maximum between 0 and x.

to a uniform pricing rule. As regularly as possible, the auctioneer publicly reveals the prices and volumes submitted by all parties and the provisional allocation. Then the bidders are permitted to submit new bids at any time provided that the auction is not closed. A bidder is allowed to resubmit a bid not only because one of his previous offers has been outbid but also because he wishes to bid for more units. Thus bidders can submit any demand schedule.

Various rules have been experimented for the closing rules for the long and short-term auctions whose closing rules may have differed in some periods. At the beginning, the end of the auction was fixed in time. Since March 2002, long term auctions last 30 minutes and continue as long as bids are placed every two minutes. For daily auctions, a random closing rule has also been used.[9] The last bid to be accepted may receive only a part of the quantity demanded. In the case of a tie between several offers at the lowest price, then the allocation is made according to a first-come-first-served basis, the time at which each bid is submitted being recorded.[10] Each bid submitted is published anonymously in the electronic system and the new selected allocation is computed. The quantities are expressed in MW with the increment of 1MW. The prices are submitted in Euros per MW for the length of the contract and must be to a maximum of two decimal places. In the daily auctions, a reserve price of 3 Euros per MW applies.

We emphasize that the ascending multi-unit auctions analysed previously in the empirical and experimental literatures are differing from the IFA auctions in an important way. In the latter, bidders are not constrained to submit nondecreasing demand schedule. On the contrary, the experimental literature [2, 13, 23] implements clock auctions for homogenous items where bidders should reduce their demand as the price raises. Similarly, in FCC or UMTS spectrum auctions, some 'activity rules' are prevailing which are roughly imposing that as the price raises the demand of a buyer should fall, as it is also usually imposed in multi-unit sealed-bid uniform auction.

3 Data

3.1 Bid Distribution Data

For this study, the auction data set has been obtained from the CRE under a confidentiality clause. The raw data set contains 39,189 bids that have been submitted for the period April 01 of 2001 to July 01 of 2005. Thus our database contains the whole dynamic of each auction. For example, 2,987

[9] We have tried unsuccessfully to obtain the precise dates where closing rules have changed.

[10] Unfortunatly, our database does not contain the time stamp of the bids. This dimension of the auction dynamic may reveal interesting patterns as in Roth and Ockenfels [35, 31].

bids have been submitted in the 28 quarterly auctions, which also correspond to 200 bidding strategies. The number of participants and the number of bids in the various durations are reported in Table 1 which shows a clear gap between long- and short-term auctions: there are much more participants in long-term auctions and bidders submit more bids. Each auction is characterized by the period of validity of the contract and its direction and also the date on which the auction was issued (this information is provided at least in the case where the same contract was sold sequentially at different dates). Each bid is characterized by the identity of the bidder, the price offered in Euro per MW, the quantity demanded in MW, the quantity obtained and also the rank of the time stamp of the bid. The bidder identifier remains constant not only within the auction but also from auction to auction. The two-digit bidder identifier from 1 to 22, denoted by ID, protects the identity of the bidder.

We then construct variables that characterizes the bidding dynamics and its outcome. For each auction, we construct the variables PRICE and QUANTITY which are respectively the average winning price and the total quantity sold. The average winning price PRICE is the average price of the highest offers of each winning bidder.[11]

Our data set is a record of all bids submitted and not of the IFA auctions themselves. In particular, for some directions and some days, no bids have been submitted and we can not distinguish whether the auction has been cancelled (e.g. due to a planned outage) or the reserve price has been actually binding. Since an outage involves a cancellation of the auction in both directions whereas a binding reserve price in one direction is related to a low price differential in that direction and thus a high differential in the opposite direction since price differentials are inversely correlated, we decide to classify the data in a way such that: the event "no bids in both directions" for one day corresponds to a period of REPAIR. This case corresponds to 222 days for the period from February 02 of 2002 to May 31 of 2006. On the contrary, if the auction in the opposite direction is held then we consider that the event "no bids submitted" corresponds to a null participation.[12]

The number of participants in the auction is measured by PARTICIPANTS which reports the number of different IDs that have submitted bids if the auction has been held.[13] We should be cautious with this measure since a bidder can participate in the auction without submitting any bid

[11]Thus it is not weighted by the quantity obtained by each bidder. Specifications with PRICE being equal to the lowest or the highest winning offers are leading to similar results.

[12]The 222 days with REPAIR equal to 1 may seem to raise a contradiction with the claimed level of availability of the link. Nevertheless, the link comprises of four individual cables of 500MW such that the cancellation of a daily auctions does not mean that the whole link is unavailable. See the appendix C for more details.

[13]If the identity of some bidders is missing, we assume that it corresponds to a unique bidder that has not been previously identified in the auction.

since no activity rules have been set as in the FCC auctions. As an example of an erratic bidding activity, in the annual auction held November 03 of 2004 for the capacity from France to England, bidder 14 has submited the price-quantity offer $(700, 100)$ and then has stopped to bid until 57 bids have been submitted and finally has re-entered the auction as an active bidder between 102,000 and 108,460 where he has submitted 5 bids for the quantity 40 MW. This is a low bidding activity for an annual auction where the average number of bids per participants is above 16.

The aggregate auction purchases in the daily auctions for the direction France to England correspond to 15 Millions of Euros for the period in our data set. As shown in Table 2 at a disaggregate level, the total amount paid and quantity purchased by each bidder in daily auctions are very concentrated: about three quarter of the market both in terms of prices and quantities are in the hand of four bidders 5, 8, 13 and 18. This asymmetry will lead us to test in section 6 whether each of these bidders has an influence on the final outcome.

Table 2 reports additional aggregate information for each bidder. The date of entry and the date of exit -left empty if the bidder has participated to an IFA auction in june 2006- are given. The column 'proportion won conditional on participation' gives the ratio of the auctions where a given bidder has purchased some capacity among the auctions he has submitted an offer. The broad pattern is a ratio in the range 30-50% and such that 'big purchasers' have a higher probability to purchase capacity conditional on participation.

Table 2: IFA Auctions April 2001-May 2005, France to England

ID	Number of Competitive Auctions	Proportion won conditional on participation	Aggregate auction purchases (Euros)	Aggregate quantities (MW)	Weighted average estimated profit	Date of Entry	Date of Exit
18*=18+19	559	0.419	4,588,912	50,347	42.8	02/02/2002	
8	424	0.441	2,782,807	33,212	61.1	02/02/2002	
13	636	0.601	2,386,200	64,398	47.4	01/04/2003	
5	424	0.474	1,917,982	22,192	58.2	01/01/2003	
7	272	0.290	482,746.9	7,739	91.9	05/02/2004	
11	94	0.319	418,140.2	4,492	54.4	02/02/2002	01/01/2004
10	144	0.222	408,226.9	3,564	46.4	24/10/2002	
3	123	0.309	368,833	3,412	43.8	23/04/2004	
2	133	0.376	338,678.4	6,620	58.2	02/02/2002	
12	232	0.220	292,548.3	3,748	80.3	20/04/2002	
9	40	0.375	224,111.9	2,345	74.2	03/02/2002	23/10/2002
16	75	0.44	180,202.3	3,008	65.1	17/12/2004	
1	145	0.221	77,116.36	2,513	41.6	05/03/2004	
4	208	0.139	46,107.26	2,022	61.4	03/02/2002	
17	20	0.75	32,582.22	2,890	25.0	16/02/2002	01/07/2002
21	27	0.556	22,894.61	970	31.2	15/03/2002	01/07/2002
20	16	0.25	20,264.46	248	48.0	16/03/2002	01/04/2005
22	1	1	63	1	771.9	24/03/2005	
6	2	0.5	3.5	1	103.3	26/02/2005	
14	3	0	0	0	.	01/04/2002	
15	0	.	0	0	.	01/06/2004	
missing	263	0.293	336,471.8	4,715	37.7		
TOTAL			14,924,936	218,737			

Note.-Bidders are ranked according to their total purchases in the daily auctions. However that it does not correspond to the rank of their total purchase in all IFA auctions: some of the largest bidders in the long-term auctions, as bidder 2, are "small" in the daily auctions.

3.2 Ex-post Profit Estimation from Market Prices

Since the markets are decentralized, there is no single measure for the prices. In a preliminary step, we have used the hourly prices from Powernext and APX UK corresponding respectively to the 11 a.m. fixing and the closing price at 10 p.m..[14] The ex-post theoretical profits based on those prices were a bad predictor for the auction price. We then decide to use the prices from the cotation agency Platts that are considered as being more accurate and thus used by the CRE. Nevertheless, it means that we give up the use of hourly prices.

Let us introduce a proxy for $\Delta^{i\Rightarrow j}$ that we will use in the regressions. Denote respectively by $\overline{p}^i_{Peak}$ and $\overline{p}^i_{OffPeak}$ the average prices in the Peak and Off-Peak period in zone i where the Peak period designates the period from 08:00 a.m. to 08:00 p.m. and the OffPeak period designates the complementary period. If the sign of the price differential remains constant intra-period then the following equalities would be satisfied:

$$\Delta^{i\Rightarrow j} = 12 \cdot \left((\overline{p}^j_{Peak} - \overline{p}^i_{Peak})^+ + (\overline{p}^j_{OffPeak} - \overline{p}^i_{OffPeak})^+ \right)$$

In general the sign of the price differential may change intra-period. Indeed, the sign of the price differential changes between Peak and OffPeak periods in 30.4% of the days in our data set. From the convexity of the function $x \to x^+$, the following inequality is satisfied very generally:

$$\Delta^{i\Rightarrow j} \geq 12 \cdot \left((\overline{p}^j_{Peak} - \overline{p}^i_{Peak})^+ + (\overline{p}^j_{OffPeak} - \overline{p}^i_{OffPeak})^+ \right)$$

In particular, the difference of the average daily prices is a lower bound for the value of the interconnector. In this study, we report the results with the proxy $12 \cdot \left((\overline{p}^j_{Peak} - \overline{p}^i_{Peak})^+ + (\overline{p}^j_{OffPeak} - \overline{p}^i_{OffPeak})^+ \right)$ which is from now on referred to as VALUE. The daily price differential from zone i to j is defined as $\frac{\left((\overline{p}^j_{Peak} - \overline{p}^i_{Peak})^+ + (\overline{p}^j_{OffPeak} - \overline{p}^i_{OffPeak})^+ \right)}{2}$: it corresponds to the average value in Euros per MWh of transfers from zone i to zone j.

VALUE is estimated from the indicative quotes of OTC transactions reported from Platts. Those quotes are provided to us by the CRE: for each weekday we have the prices in Euros per MWh of a Base contract and a Peak contract for both France and England. For saturday and sunday, only the base contract prices are available: we then set $\overline{p}^i_{Peak} = \overline{p}^i_{OffPeak} = \overline{p}^i_{Base}$ for greater convenience. The base contract (respectively the peak contract) corresponds to a constant profile over the whole day (from 08:00 am to 08:00 pm). In France, the transmission use of system charge is a postage charge for the whole network. Thus the available prices correspond to 1MWh anywhere in the network. On the other hand, the transmission use of system

[14] We would have preferred to use hourly prices fixed just after the closure of the IFA auction for both countries. Such data is not available.

Table 3: Day ahead prices from Platts, price differentials

	2001	2002	2003	2004	2005	2002/ 2005	Std. Dev.	Min.	Max.
Daily Price (MWh):									
England	30.4	25.1	28.7	32.8	44.6	30.8	11.3	9.84	149.1
France	29.1	21.5	29.4	28.3	39.8	28.0	11.8	6.25	150
Price differential (Euros/MWh):									
France to England	4.47	4.64	2.86	5.01	5.77	4.1	5.1	0	40.0
England to France	3.37	1.08	3.54	0.49	0.94	1.4	3.5	0	56.7

charges under the NETA (New Electricity Trading Arangements since 2001) contains a locational varying element supposed to reflect the marginal costs of investments in the transmission system and contributing to 25% of the total use of system charge. This gap between the price cotations and the price at the end of the Interconnector may introduce some bias in some of our regressions if it is not constant over time. We neglect this aspect in our analysis.

Table 3 reports summary statistics of the daily prices and the price differentials between both countries. In the period 2001-2005, the average price in England exceeds the one in France expect in the year 2003 due to an exceptional hot summer in France. The price differential is then greater in the direction from France to England (4.1 Euros per MWh for the period 2002-2005) than from England to France (1.4 Euros per MWh).

We define the variable PROFIT as the ex-post estimated profit due to the acquisition of 1MW of capacity, i.e. $PROFIT = VALUE - PRICE$. The fifth column of table 2 reports for each bidder the average profit he makes per MW purchased in the daily auctions. The huge heterogeneity of this variable among bidders even for bidders of the same size is a striking feature. As an example, in the 272 auctions he participated, bidder 7 bought 7,739 MW, his total theoretical profit is 710,000 Euros which corresponds to an average profit of 92 Euros per MW. On the other hand, bidder 18* participated in 559 auctions and bought 4,273 MW. His theoretical profit is 2,2 Millions of Euros, which corresponds to an average profit of 43 Euros per MW. There is no clear pattern between average profits and the sizes of the bidders. Altogether, the stakes at hand in the daily auctions are small, especially compared to the 90 Millions bid of bidder 18* in the tri-annual auction.

4 Intriguing Evidence about Prices and ex-post Profits

4.1 Failure of the arbitrage condition

Table 4 provides a raw data summary for each duration and both directions. A clear pattern emerges: short-term auctions raise a much lower revenue for the seller or equivalently a much higher profit for the bidders than the long-term ones. Recall that it is exactly the same capacity rights that are sold at long and short term auctions. A mild difference could come from the payment dates. Nevertheless, long term auctions are payable in equal monthly instalments. Such 'yield curve' effect would have then a very mild impact on the outcomes. It means that the arbitrage condition equation fails: it costs less to buy capacity at short-term auction and the Operator seems to have interest to sell only through long-term contracts.

The average price received by the seller for 1MW per day was respectively 172.5, 96.0 and 59.6 Euros in the (single) tri-annual auction, the long-term auctions and daily auctions for the direction "France to England". This represents a difference in term of revenue between long-term and daily auctions of 61%. The corresponding difference equals to 33% for the direction "England to France". In the same way, the number of participants, the number of submitted bids and also the performance measures as the number of winners and the HHI index strongly differ in long-term and daily auctions.[15, 16]

We consider the average profits per day of the winning bidders. Apart from the seasonal auctions, the average profits are inversely related to the

[15]Note that the average of the ex-post value differ between long-term and daily auctions. In a stationary auction timetable, as the one prevailing at the first years on the IFA auctions where there is for example one daily auction and one monthly auction, the average ex-post values should be exactly the same. Nevertheless, the auction timetable has changed from October 2004 where, instead of one monthly auction, two different sequential monthly auctions are run the month before the month to be contracted. The other long-term contracts are been sold with this sequential pattern. Then in the 35 monthly auctions for the direction "France to England" present in our database, months after November 2004 are represented by two points whereas months before November 2004 are represented by a single point. Since the price differential from England to France was higher than usual, then the average ex-post value is higher from monthly auctions and more generally for long-term auctions than the daily auctions that are uniformly captured by a point. Since the price differential from France to England was lower than usual from November 2004, the same argument implies that the average ex-post value is higher in daily-auctions than in long-term auctions.

[16]Note also that the difference between the ex-post value of rows Daily and All Day, REPAIR=0 is significantly positive. A nonparametric Wilcoxon (Mann-Whitney) rank-sum test indicates that the distributions are different at the $p < 0.0001$ level for both directions. As expected, the days where no bids have been submitted and where we presume that an auction has been held according to the classification REPAIR are corresponding to a lower expected value, which gives more support to the classification REPAIR detailed in the appendix C.

duration of the contract auctioned, ranging from 47.9 Euro for a daily auctions[17] to -80.7 Euro for the tri-annual auction. The difference of 128.6 Euro is bigger than the average value of the capacity. On the one hand, we can not reject the null hypothesis that the profits of a winner in the monthly, quarterly, seasonal and annual auctions are drawn from the same population in the direction France to England (a Kruskal-Wallis test yields $p = 0.92$). On the other hand, we can reject the null-hypothesis that the profits from the duration above are drawn from the same distribution as the ones from the daily auctions (a Kruskal-Wallis test yields $p < 0.001$). On the whole, we can not reject that the arbitrage condition equation holds between long-term auctions, but we can reject that it holds between daily and long-term auctions.

[17]The profit of a day without repair and a null participation is assumed to be null.

Table 4: Descriptive Statistics of the IFA Auctions

	Exogenous		Bidding Variables		Performance			
	Ex-post value per day	Quantity sold	# of Participants	# of bids	Price per day	Profit per day	# of winners	HHI
France to England								
Daily (882)	121	244	4.2	19	67.0	53.9	1.6	0.77
All Day, REPAIR=0 (994)	109				59.6	47.9		
Monthly (35)	118	277	8.7	133	89.3	29.3	3.9	0.33
Quarterly (13)	116	192	8.5	121	96.7	24.2	3.4	0.31
Seasonal (4)	135	175	8.8	104	85.3	49.9	3.3	0.36
Annual (9)	117	267	9.3	151	120.4	-9.4	4.4	0.29
Tri-annual (1)	91.8	650	2	2	172.5	-80.7	2	0.86
England to France								
Daily (482)	71.6	482	3.1	12.4	27.5	43.4	2.0	0.68
All Day, REPAIR=0 (978)	37.5				13.5	21.8		
Monthly (36)	33.7	299	7.3	72.6	18.5	15.5	3.9	0.33
Quarterly (13)	32.6	281	7.6	89.1	16.3	16.9	3.4	0.42
Seasonal (4)	12.6	175	7	124	21.1	-8.5	4	0.32
Annual (9)	32.5	310	8.3	131	17.7	14.4	4.4	0.32

4.2 Time Instability of the Margins

A first sign of instability appears roughly in the first sequential auctions that have been held for annual contracts. Those auctions took place November 03 and 30 of 2005 for a contract covering the year 2005. On the one hand, the (average) price for the direction "France to England" was 108,500 Euros/MW (4,500 for the opposite direction) in the first auction. On the other hand, those prices falls respectively to 65,800 and 3,900 Euros in the second auction. We may doubt that the fall of about 2% of the annual contract for the year 2005 in Powernext Futures in November 2005 is the single cause of the fall of 66% in the price of the transmission capacity from France to England especially since the fall concerns also the opposite direction. Furthermore, the fall is also surprising since more bidders have participated in the second auction (14) than in the first (10). The pattern of the price dynamics depicted in Figure 3 suggests that the ascending nature of the auction may play a key role. In both cases, we can observe two phases in the dynamic. In a first phase of about 100 offers, the bidders are using big increments. In particular, there is a jump bid from 57,500 to 75,000 in the first auction whereas, few weeks later, in the second auction for the same good, nearly 300 offers have been needed to raise the price from 60,000 to 66,001 the highest winning offer. It seems that those jump bids are used because it is common knowledge that the value of the capacity is much higher than the current price since we do not observe any exit of participants in this phase. Nevertheless, we can not view this phase as totally uninformative since some bidders reduce the quantities in their offer. The second phase corresponds much more with the button auction formalization of the English auction with small bid increment (see Figure 3).

The main sign of instability that will be carefully analyzed in section 6 concerns the link between the price and the theoretical ex-post value. Let us define the average profit ratio $APV(t,T)$ in period t over $T+1$ auctions as the percentage of the profit captured by the bidders in the period $[t-\frac{T}{2};t+\frac{T}{2}]$:

$$APV(t,T) = 1 - \frac{\sum_{i=t-\frac{T}{2}}^{t+\frac{T}{2}} PRICE_i}{\sum_{i=t-\frac{T}{2}}^{t+\frac{T}{2}} VALUE_i}.$$

In the exploratory phase of the data, we run regressions of the price on our exogenous variables (especially the value) and on the participation and asked whether those relations were robust by running them on different time periods and with different models (OLS, 2SLS, autoregressive time series). What has emerged is a great instability of the estimated coefficients and in particular of the one corresponding to the profit which explains the main part of the variability in our data. As an illustration, consider the simple regression model:

$$PRICE_i = \alpha + \beta \cdot VALUE_i + \epsilon_i \quad (1)$$

The time-series distribution of the variables $APV(t, 90)$ and the $\beta(t)$ and $R^2(t)$ coefficients obtained from a 180-days rolling-window regression of the model above are shown in Figure 2. The difference between those estimations in the summers 2002 and 2003 is spectacular: in September 2002, β is around 0.9 whereas it nearly reaches 0.1 in August 2003 with a standard deviation around 0.04.[18] The coefficient R^2 follows the same pattern as β ranging from 10% to 88% and APV follows an opposite pattern. On the whole, periods where bidders capture a small (respectively big) part of the potential profits, i.e. periods that can be qualified as competitive (collusive), are characterized by highly (less) predictable prices given the ex-post value and the price is (not) very responsive to the expected value. We will see next that this kind of instability goes much beyond the specification (1).

5 Various Theoretical Scenarii of Bidders Behavior

There are several possible explanations why the average profits differ in the different auctions, i.e. the arbitrage condition fails. Risk aversion is one but it would imply that the premium is in favor a long term contracts since they involve more risk due to the huge stakes at work and the incertainty of future prices. Anyway, we doubt that risk aversion plays a role in this market: the aggregate auction purchases of the more active bidders are about 10 millions Euros per year which represents 2% of EDF Trading's annual turn-over (408 Millions in 2004), the department of EDF (Electricité de France) for Energy Trading, the French historic public utility operator which has an annual turn-over of 47 Billions (2004). Five main determinants may predict the pattern of the expected profits: volatility of the underlying asset, asymmetric information, transaction costs, collusion and differences in the auction mechanism. Let us consider the different predictions implied by those theories.

5.1 Volatility of the underlying asset

Capacity rights are a specific financial instrument, a European call option, and the insights of the finance literature thus apply (see Demange and

[18]France experienced an unusual spike in the summer 2003 making electricity flows from England to France much more profitable than usual. Nevertheless, this spike had much less impact on the value of the interconnector from England to France. The time-series distribution of the 180-day moving average of this price differential is shown in Figure 4. Anyway, by adding the square of the profit in the regression, we have controlled that the robustness of the instability of the coefficient β is not linked to the union of nonlinearities in the relation between PRICE and PROFIT and different distribution of PROFIT across time.

Laroque [11] for an introduction). From the Black-Scholes formula, the value of an option is increasing in the volatility of the underlying assets for a given expected value of the underlying assets. At first glance, it does not matter here since we compare directly winning prices with ex post realizations of the profits. Nevertheless, the bias of our estimator of the ex-post value of the option may depend on the intra-day volatility. In more volatile periods, the sign of the price differential is more likely to switch inside the peak or the off-peak periods and our proxy may underestimate the real value of the capacity rights. It could give some rational for the instability of the margins across time periods. However, it is not clear in practice whether the buyers benefits from such switches, the market of base and peak contract being much more liquid than the market for hourly contracts. The goodness of fit coefficient of our unreported regressions with ex post values based on hourly prices was reduced by half compared with our proxy.

5.2 Asymmetric Information

As formalized by the pioneering work of Milgrom and Weber [28], in a common value auction, asymmetric information creates a gap between the winning price and the expected value. The more a bidder cares about his opponents information, the more he shades his bids to avoid the winner's curse, i.e. he bids less than the expected value conditional on his own signal by taking into account the fact that winning the good means that the opponents received low signals. In the literature, e.g. Nyborg et al [29], the volatility is supposed to proxy the precision of bidders' signals. The idea is that a high volatility is indicative of a large winner's curse and should be associated with cautious bidding. The failure of the arbitrage condition could be rationalized by more asymmetric information in daily auctions.

5.3 Transaction Costs

Contrary to asymmetric information which is probably highly volatile as suggested by the high volatility of the electricity prices, transaction costs should rather be constant over time. For a participant in this market, it should be related to the cost of employing a trader that participates at the IFA auctions and at the Power Exchanges in France and UK. Note that the dynamic nature of the ascending auction make it more time consuming (between 15 and 30 minutes per Day for a Daily auction and 1 and 2 hours one a year for an annual auction) and thus involves at least higher transaction costs than standard sealed bids.

As shown in Table 4, the arbitrage between monthly and daily auctions involves a total arbitrage of 25 Euros per MW and per day (18.6 Euros for the direction France to England and 6.3 Euros from England to France). With the average quantity of 150 MW obtained by a winning bidder, it corresponds to a benefit of 3,750 Euros per day.

Anyway, transaction costs can not be the whole story. In that case, it would mean that bidders are making profit when there is a single participant in the auction whereas the auction is competitive and leads to a null profit when there are at least two participants. It stands in contradiction with the evidence that the benefit of a winning bidder is much bigger when there are at least two participants.

5.4 Collusion

A game that is repeated on a daily basis is prone to collusion. Strategies that depend not only on the current game but also on the past outcomes are likely to be played, even if past actions are imperfectly observed as it is the case here due to the anonymous nature of the auction. Folk theorems (see Fudenberg and Tirole [14]) establish that the set of equilibrium outcomes is very rich. In repeated game with imperfect public information, the pioneering work of Porter [34], that has been generalized by Abreu et al [1], has introduced trigger-price strategies for a model where price cuts are secret. It leads to switching regimes models where, at equilibrium, the competition alternates between collusive periods and price wars. It could give some rationale from the instability of the margins across time.

5.5 Small changes of the Bidding Rules

The literature insists on the fact that seemingly innocuous changes of the bidding rules may have a huge impact on the outcome: the different rules for ending an auction may leave more or less scope for some "collusive" equilibria as in the theory of late bidding developed by Ockenfels and Roth [31].[19] They compare the closing rules in the second price Internet auctions on eBay and Amazon. In the former, a fixed end time is used, as in the earliest days of IFA-auctions. In the latter, the auction extends automatically 10 minutes after the last submitted bid whereas a 2-minutes automatic extension rule applies nowadays in the long-term IFA-auctions. They state that under a fixed time closing rule where last-minute bids may be lost, late bidding lowers prices in particular through collusive equilibria that avoid bidding wars. Here, the effect of a fixed time closing rule is different: bidders are submitting bids at the last minute, which makes the ascending auction more similar to the standard sealed-bid discriminatory auction.

Another closing rules has been used in the IFA-auctions: a random closing time, which is now the rule for the daily auctions. We claim that this rule opens the door to seemingly collusive equilibria where participants are

[19]On the contrary, results as the Revenue Equivalence Theorem insists on the fact that seemly big changes of the bidding rules may have not impact on the outcome

bidding very slowly by adding only the price increment to the current winning bid and where they alternately hold the highest bid that could win if the auction closes immediately.[20] Otherwise, whenever some bidders deviate from this equilibrium strategy, bidders are returning to a less profitable equilibrium with high increments. Thus the random and the automatically extended closing rule suggest different predictions in term of the final price. Nevertheless, the multi-unit and repeated nature of the interaction in the IFA auctions leaves scope for collusive equilibria under each closing rules.

In the following, to capture the changes in the closing rules, we divide the data in two periods: the year 2002, where it is supposed that a fixed time closing rule was prevailing and the period from the beginning of the year 2003, where it is supposed that the daily auctions are actually a real dynamic auction.[21]

6 Empirical Findings on Bidder Behavior and Auction Performance

In this section, we report on the broad patterns in bidder behavior and, in particular, how bidders respond to, and how auction performance is related to the value of the interconnector, the volatility of the underlying value, the auction size and the participation decisions. We first present the differences between the year 2002 and the period from the beginning of 2003 with descriptive statistics. A more rigorous analysis of those differences is made in the regression part of this section.

6.1 Descriptive Statistics

Tables 5 and 6 report summary statistics on the bidding variables and the auction performance in the sample as a whole and according to our proxy for the changes in the closing rules. Summary statistics on the value of the interconnector, the uncertainty of this value and the auction size are also reported.

Added to PRICE and PARTICIPANTS, three additional variables are characterizing the strategy profile of the bidders. The aggressiveness of the bidding profile is captured by the variable SPREAD which is equal to the difference between the highest accepted bid and the lowest best offer of a bidder that wins some items. Thus SPREAD is null if there is only one winning bidder. In the sub-sample of auctions with at least two winning bidders, the average SPREAD equals 2.8 Euros from 2003 to 2005 which should be compared to the average winning price 55.9 Euros. On the other hand, this

[20] A formal statement as in [31] is left to the reader.

[21] The choice of the beginning of the year 2003 is consistent with our discussions with IFA-experts and what we observe from the dynamic of the auctions.

average is equal to 26.3 Euros in the year 2002, i.e. about one third of the average winning price. SPREAD is very high in 2002 consistent with the view that the dynamic nature of the auction is canceled by the closing rule. On the contrary, the low values for SPREAD from 2003 are consistent with a dynamic auction. In this last case, it is a measure of the pace of the auction or the aggressiveness of the bidders. The drawback of a alternative measure of aggressiveness that considers the average of the submitted increments is that it would take account of the early bids in the auction which may not be very meaningful. On the contrary, with our measure, a high SPREAD reflects for example jump bids, a sign of competitiveness. On the other hand, collusive bidders may prefer to push the price at a slow pace such that the spread remains low. The average number of bids submitted by each participating bidder is captured by the variable NUMBER OF BIDS PER PART, which has an average of 3.89 and a maximum of 29. The difference is striking between the two periods: in 2002, the average number of submitted bids per bidders is 1.44 in auctions with one participant and 1.66 in auctions with at least two participants. From 2003, those figures are equal respectively to 1.15 and 4.89. A standard two sample mean comparaison test with unequal variance reveals that all those means are statistically different at the $p=0.01$ level. This evidence is again consistent with our proxy for the change in the closing rules.

Under an ascending auction -the average winning price is 3.4 Euros - bidders are submitting roughly only one bid at the reserve price when there is only one participant.[22] However, if bidders are entering the auction at the last moment, they bid a demand curve with several bids and with offers strictly above the reserve price -the average winning price is 22.4 Euros in auctions with only one bidder. When there are several bidders, the difference is striking before and after 2003: the average number of bids per participants is about three times higher in the later period, confirming again its dynamic nature. The extent to which a bidder uses the ascending nature of the auction to communicate is measured by DIGIT, the proportion of bids submitted whose last digit is neither 0 nor 5. Cramton and Schwartz [10] have shown how the simultaneous ascending auction used by the FCC to sell the spectrum has been vulnerable to signaling techniques by the bidders through the trailing digits of their bids. Indeed, more than a communicating device for collusive behavior as in [10], the last digit of a bid may be a proxy for the pace of the auction: digits like 0 and 5 are reflecting a more aggressive strategy profile using possibly jump bids.

The auction performance is first measured by the variable PROFIT

[22] Submitting more than one bid is rational if the first bid covers less than the quantity offered at the auction, which happens in two auctions where the second bid were also at the reserve price. Nevertheless, in six auctions, bidder 18^* outbids itself slightly though there was no other bidder which explains why the winning price is not stuck at the reserve price.

equal to the difference between the VALUE and the PRICE of the capacity. PROFIT is large compared to VALUE: its average is 53.9 Euros which roughly corresponds to the half of the VALUE of the capacity. The cross-section variation is large going from -300 to 807.8 Euros with a standard deviation of 74.5 Euros. The difference of PROFIT of 3.4 Euros between the period after 2003 and the earliest days of the IFA-auctions is not statistically significant (p=0.51 in a t-test with unequal variance). Moreover, it remains true if we condition on the number of participants. It is not clear how to interpret that pattern. First, the average number of participants widely differs between the two periods, rising from 3.2 to 4.56. Thus, since the quantity sold has not changed much, the average profit per participants has clearly dropped. It means that it should not be interpreted as an auction with endogenous entry where bidders are making a constant profit to compensate their participation costs. Second, since the theoretical value of the capacity rights has fallen over time, the share of the VALUE converted into PROFIT falls meaning that the auction price was more competitive under the fixed time closing rule though there were less participants at that time. The variables VALUE and then PRICE are increasing in the number of participants, which confirms the endogenous nature of the participation decisions. On the contrary, PROFIT displays no clear participation pattern, except that the average profit of the winning bidders is much lower in the case where there is a single participant.

WINNING is equal to the number of winning bidders. Its average is equal to 1.68, i.e. 40% of the average number of participants, and increases with the number of participants. HHI is the Herfindahl-Hirschman Index relative to the capacity allocated in the daily auction.

Tables 5 and 6 exhibits three exogenous variables: VALUE, VOLATILITY and QUANTITY. We measure uncertainty as the differential price volatility using an ARCH-in-mean process.[23] The one-day volatility has an average of 14.1, which corresponds to 11.6% of the average value of the interconnector, and a maximum of 330.3, which is thus 23.4 standard deviations larger than the mean. The auction size, QUANTITY, has an average of 245.4 MW and is a little higher than the minimal quantity (200MW) that has to be available in daily auctions.

[23]The details of the time-series modelling are in the Appendix A

Table 5: Descriptive Statistics of the IFA Daily Auctions, France to England

Period	Total Sample				YEAR 2002		YEAR ≥ 2003	
	Mean	Stand. Dev.	Minimum	Maximum	Mean	Stand. Dev.	Mean	Stand. Dev.
Exogenous Variables								
VALUE	121.5	124.8	0	959.8	146.2	140.6	114.2	118.9
VOLATILITY	14.1	25.1	.39	330.3	16.0	31.7	13.5	22.8
QUANTITY	245.4	100.1	5	650	255.2	94.1	242.5	101.7
Participation Decisions								
PARTICIPANTS	4.25	2.17	1	10	3.20	2.0	4.56	2.1
PART5	0.47	0.50	0	1	0	0	0.61	0.49
PART8	0.48	0.50	0	1	0.52	0.50	0.46	0.50
PART13	0.71	0.45	0	1	0	0	0.92	0.27
PART18*	0.55	0.55	0	1	0.69	0.46	0.60	0.49
PART OTHER	2.05	1.65	0	8	1.99	1.66	1.97	1.57
Bidding Variables								
PRICE	67.5	95.8	3	1000	94.9	131	59.4	80.9
SPREAD	2.74	11.64	0	196.7	7.2	22	1.4	5.0
DIGIT	.78	.20	0	1	.75	0.28	.78	0.17
NUMBER OF BIDS PER PART	3.89	3.14	1	29	1.6	1.5	4.6	3.2
Performance								
PROFIT	53.9	74.5	-300	807.8	51.3	60.3	54.7	78.3
WINNING	1.68	.89	1	5	1.4	0.69	1.8	0.92
HHI	.78	.26	.22	1	0.88	0.21	.74	.27
Observations	877				200		677	

Table 6: Descriptive Statistics of the IFA Daily Auctions, France to England

Period	All periods							YEAR = 2002		YEAR ≥ 2003	
Observed Number of Participants	1	2	3	4	5	6	≥7	1	≥2	1	≥2
Exogenous											
VALUE	37.9	76.9	90.5	140.6	143.1	163.1	173.6	49.2	176.9	28.8	122.4
VOLATILITY	10.5	11.0	12.3	15.7	17.7	15.5	14.9	11.4	17.5	9.9	13.9
QUANTITY	187.6	260.6	244.5	257.4	255.1	252.5	250.6	194.9	274.3	181.7	248.3
Bidding Variables											
PRICE	11.9	26.8	38.2	70.8	84.3	103.4	117.5	22.4	117.8	3.4	64.7
SPREAD	0	1.14	2.23	2.76	3.50	5.70	3.44	0	9.5	0	1.5
DIGIT	.88	.85	.81	.75	.74	.73	.71	0.84	.72	.92	.77
NUMBER OF BIDS PER PART	1.28	2.89	3.52	4.21	4.73	4.85	5.09	1.44	1.66	1.15	4.89
Performance											
PROFIT	26.0	50.1	52.0	69.8	58.8	59.7	56.0	26.8	59.1	25.4	57.5
WINNING	1	1.48	1.59	1.65	1.84	2.05	1.98	1	1.49	1	1.84
HHI	1	.79	.79	.78	.74	.69	.69	1	.85	1	.72
Observations	107	119	123	131	130	112	155	48	152	59	618

6.2 Regression Analysis

Our empirical approach is fairly straightforward. For a given outcome variable Y, such as the PRICE or the award concentration measures, we suppose that:

$$Y = f(RULES, EXO, PART, \epsilon)$$

where f is an unknown function, $RULES$ is a discrete variable capturing the auction rules, EXO is a vector of exogenous variables, $PART$ is the vector characterizing the participation decisions of the potential bidders, and ϵ is an unobservable white noise.

In our regression, the main explicative variable is the estimated ex-post value of the transmission capacity VALUE. Following the aforementioned empirical finance literature, we include in our regressors the volatility of the underlying asset VOLATILITY and the quantity sold QUANTITY. If bidders are neither risk-averse nor capacity constrained, then the auction should be invariant to QUANTITY after having controlled for the set of participants. The variable QUANTITY should only affect the participation decisions since higher quantities involve higher potential benefits in play. The remaining endogenous variables are time dummies capturing the seasonal effects: we only control for the day of the week.

The number of participants and more generally the participation decisions of each potential bidder are arguably endogenous. Thus the vector PART should be instrumented. The first lag of these regressors are employed as instrument in a two-stage least squares estimation.[24] The inclusion of the whole participation vector leads to insignificant results. Thus we decide to include only the four biggest bidders - PART5, PART8, PART13, PART18- and to aggregate the participation decision of the remaining bidders (PART OTHER).

Our regressions are reported in Tables 7-10. They have three purposes: first, the estimation of the impact of the exogenous variables ; second, the estimation bidders-specific effects and third the identification of the effect of the auction rules. Under the assumption that the unobserved component ϵ is independent of the time period, this last effect is equal to:

$$\tau = \mathbb{E}_{EXO,PART,\epsilon}[f(1, EXO, PART, \epsilon) - f(0, EXO, PART, \epsilon)].$$

Then the most obvious approach is to consider a linear specification when restricting to auctions with at least two participants:

$$Y = \alpha \cdot RULES + EXO \cdot \beta + PART \cdot \gamma + \epsilon \tag{2}$$

[24]We do not include additional lags as instruments because the time series contains some gaps.

At this point, this approach has some caveats. It does not allow the effect of the auction rules -proxied by the time period- to vary as a function of the profitability of the capacity. To remedy this, we first perform separate regressions for each auction rules that are reported in Table 7. Next, in the nested regression displayed in Table 8, we also report estimates from specifications where we interact RULES with the exogenous variables VALUE, VOLATILITY and QUANTITY and also the participation decisions.

We first comment the effect of the exogenous variables on the price formation. The results are quite different according to the auction rules. In 2002, both VOLATILITY and QUANTITY have not significant effect on the final price. From 2003, those variables have a significant effect at the 1% level and with the predicted sign: an increase in VOLATILITY (resp. QUANTITY) by a one standard deviation results in a typical decrease of 10.0 Euros (6.2 Euros) in the final price, which corresponds to less than 10% of the price's standard deviation. Those results are robust to several specifications: OLS, 2SLS and ARIMA. We reject exogeneity of the variable NUMBER OF PARTICIPANTS with a Davidson-MacKinnon test for misspecification ($p<0.0001$) for both periods. In the following, our analysis is devoted then to the 2SLS regressions.

Table 8 reports the results for the nested regression for both time periods. It allows us to test whether the small changes in the auction rules had a significant impact in the formation of the price. In the first column, we interact RULES with the exogenous variables. The variable RULES has a significant effect on the way the exogenous variables influence the price. We can reject that the slopes for VALUE and VOLATILITY and also the intercept are equal for both periods: the p-values are respectively equal to 0.0007, 0.0222 and 0.0007. From the year 2003, the price appears to be much more reactive to a change in VALUE: a 1 Euro's increase in VALUE translates in a 0.83 Euro's increase in 2002 and only a 0.49 Euro's increase from 2003. It stands in contradiction with the intuition of the linkage principle which suggests a better aggregation of bidders' private information in a dynamic format than in a sealed bid format. Since the auction rules make it closer to an ascending auction from 2003, we should expect that the relation between the final price and VALUE would be flatter in 2002 than from 2003. Nevertheless, this informational linkage is valid for single-unit auction and is not longer true in multi-unit auctions (see Perry and Reny [32]). In particular, the above intuition does not hold for seemingly collusive equilibria.

The significant effect of VOLATILITY from 2003 gives some support to the asymmetric information story with bidders shading their bid to avoid the winner's curse, which is coherent with the low slope (0.487) for VALUE relative to the slope equal to 1 in the theoretical benchmark. Table 9 displays the regression with another time period classification: we consider the years 2003 and 2005 versus the year 2004. The slope on VALUE and on VOLATILITY is significantly different in the 2004 at the 1% level, giving support to the changes of regimes hypothesis. In particular, the estimated slope for VOLATILITY is positive in 2004. In all those regressions, note that we control for the participation decisions, i.e. that bidders' probability of participation are varying differently in the different periods.

One of the main issues raised by the descriptive statistics is the discrepancy of 53.1 Euros per MW between the average price in 2002 and the average price in the period from 2003. In the same spirit, there is a discrepancy of 30.7 Euros per MW between the average price in 2004 and the one for the years 2003 and 2005 tough the exogenous variables were quite similar in these two panels. It is not clear wether the price formation is more competitive in 2002 and 2004 than in 2003 and 2005, whether it is due to differences in the identities of the participants or finally to changes in the exogenous variables. The regression in the first column of Table 8 leads to the following decomposition: 51 Euros are due to bidder heterogeneity, namely more aggressive bidders are bidding more in 2002, 26 Euros are due to the higher value of the interconnector in 2002, the effect of the changes in VOLATILITY and QUANTITY are negligible, the residual is the effect of the period 2002, which comes from the changes in the intercept and in the slopes on the exogenous variables, and is thus negative. Thus we find that, having fixed the number of participants and the set of exogenous variables, then the modification of the closing rules seems to have a positive effect on the price. Nevertheless, if we fix only the set of exogenous variables, then the modification of the closing rules τ has a negative effect on the price. In the second column of Table 8, we also interact the variable RULES with the participation decisions to test whether some bidders are bidding differently according to the time period: we can not reject that the coefficients on PART8*YEAR02, PART18**YEAR02 and PART OTHER*YEAR02 are jointly equal to zero (Wald test, p= 0.7038).

Next, we comment the effect of the participation variables on price formation and on the other endogenous variables from 2003. The results are reported in Table 10. The issue is whether some bidders have a specific impact on the price formation, e.g. by means of specific strategies. We can not reject that the coefficient on the participation decisions are jointly equal for the variables PRICE, SPREAD and NUMBER PER PART. The p-values of the corresponding Wald test are respectively equal to 0.26,0.44 and 0.21. Nevertheless for the variables DIGIT and WINNING, identity-specific effects are highly significant. Bidder 18* seems to play a bit differently in the auction: he raises more bids with less digits which leads to a more concentrated outcome. Nevertheless, his effect on the price is not significant. We run the various regressions on the endogenous variables separately though a game-theoretic model of the ascending auction should involve interactions between those variables. We test whether the residuals of the above regressions are correlated. As it was conjectured, most of the coefficient of correlation between the residuals are significantly different from zero. As an example, the coefficient of correlation between the residuals of the regressions for PRICE and DIGIT is equal to -0.1957. It means that an auction dynamic with more digits is positively correlated with a lower price after having controlled for the exogenous and participation variables.

Table 7: Regression Analysis of the winning price

Period	YEAR 2002				YEAR $\geq$ 2003			
	OLS	**2SLS**		**ARIMA**	**OLS**	**2SLS**		**ARIMA**
		PRICE	NUMBER OF PARTICIPANTS			PRICE	NUMBER OF PARTICIPANTS	
VALUE	0.799**	0.837**	0.003**	0.777**	0.473**	0.478**	0.005**	0.435**
	(0.088)	(0.093)	(0.001)	(0.089)	(0.057)	(0.066)	(0.001)	(0.057)
VOLATILITY	0.149	0.022	0.009†	-0.004	-0.417**	-0.402**	-0.008*	-0.336**
	(0.227)	(0.241)	(0.004)	(0.124)	(0.147)	(0.155)	(0.004)	(0.124)
QUANTITY	0.087	0.088	-0.003†	0.080	-0.058**	-0.062*	0.002**	-0.057**
	(0.077)	(0.092)	(0.002)	(0.092)	(0.022)	(0.024)	(0.001)	(0.022)
NUMBER OF PARTICIPANTS	9.504**	13.735*		8.557**	9.165**	9.881†		8.423**
	(3.037)	(6.101)		(3.155)	(1.465)	(3.318)		(1.311)
FIRST LAG OF THE NUMBER OF PARTICIPANTS			0.426**				0.416**	
			(0.073)				(0.034)	
N	152	127	127	152	621	587	587	621
R^2	0.824	0.836	0.405		0.643	0.644	0.35	

Significance levels : † : 10% * : 5% ** : 1%. Robust standard errors are in parentheses. The regressions are run for observations with at least two participants. The coefficients for time dummies and the intercept are not reported.

Table 8: Regression Analysis of the price: YEAR 2002 versus YEAR $\geq$ 2003

	PRICE	PRICE
PART8	28.478**	28.468*
	(10.887)	(12.247)
PART8*YEAR02		-24.427
		(36.030)
PART13	-61.914†	-60.625†
	(34.419)	(35.424)
PART5	21.940	23.634†
	(13.904)	(14.177)
PART18*	6.634	13.215
	(15.823)	(16.525)
PART18**YEAR02		-88.210
		(78.473)
PART OTHER	9.306**	9.040*
	(3.430)	(4.275)
PART OTHER*YEAR02		4.189
		(8.141)
YEAR02	-122.179**	-83.539
	(36.022)	(54.659)
VALUE*YEAR02	0.827**	0.876**
	(0.092)	(0.105)
VALUE*(1-YEAR02)	0.487**	0.478**
	(0.060)	(0.064)
VOLATILITY*YEAR02	0.069	0.051
	(0.212)	(0.216)
VOLATILITY*(1-YEAR02)	-0.504**	-0.452**
	(0.148)	(0.157)
QUANTITY* YEAR02	0.070	0.171
	(0.084)	(0.124)
QUANTITY* (1- YEAR02)	-0.044	-0.047
	(0.035)	(0.035)
N	738	738
R^2	0.711	0.700

Significance levels : † : 10% $*$: 5% $**$: 1%. Robust standard errors are in parentheses.
The regressions are run for observations with at least two participants.
The coefficients for time dummies and the intercept are not reported.

Table 9: Regression Analysis of the price: YEAR 2004 versus YEAR 2003 and 2005

	PRICE
PART8	13.680
	(12.562)
PART13	-36.207
	(31.907)
PART5	10.798
	(17.552)
PART18*	-4.323
	(16.978)
PART OTHER	5.021
	(4.070)
YEAR04	-37.476
	(24.460)
VALUE*YEAR04	0.275**
	(0.080)
VALUE	0.388**
	(0.073)
VOLATILITY*YEAR04	0.607**
	(0.217)
VOLATILITY	-0.497**
	(0.142)
QUANTITY* YEAR04	0.047
	(0.075)
QUANTITY	-0.042
	(0.031)
N	604
R^2	0.67

Significance levels : † : 10% * : 5% ** : 1%. Robust standard errors are in parentheses. The regressions are run for observations with at least two participants. The coefficients for time dummies and the intercept are not reported.

Table 10: Regression Analysis of the endogenous variables, YEAR $\geq$ 2003

	PRICE	SPREAD	DIGIT	NUMBER PER PART	WINNING	HHI
PART8	27.596*	1.555†	-0.059*	0.189	-0.097	0.045
	(12.135)	(0.920)	(0.029)	(0.645)	(0.202)	(0.058)
PART13	-59.638†	-3.670	-0.036	2.589	0.455	-0.143
	(35.336)	(3.012)	(0.086)	(1.760)	(0.585)	(0.180)
PART5	23.197	0.820	0.059	-0.707	-0.168	0.050
	(14.165)	(1.260)	(0.041)	(0.916)	(0.232)	(0.072)
PART18*	14.135	-0.590	-0.151**	1.892*	-0.637*	0.146†
	(16.380)	(1.723)	(0.043)	(0.955)	(0.270)	(0.081)
PART OTHER	8.830*	0.405	-0.042**	0.321	0.118†	-0.025
	(4.326)	(0.323)	(0.010)	(0.225)	(0.066)	(0.019)
VALUE	0.482**	0.008†	0.000†	0.000	0.000	0.000
	(0.067)	(0.005)	(0.000)	(0.002)	(0.001)	(0.000)
VOLATILITY	-0.492**	-0.016	0.000	-0.007	-0.001	0.001
	(0.152)	(0.012)	(0.000)	(0.006)	(0.002)	(0.000)
QUANTITY	-0.051	0.002	0.000	-0.004*	0.003**	-0.001**
	(0.034)	(0.003)	(0.000)	(0.002)	(0.001)	(0.000)
R^2	0.61	0.05	0.22	0.04	0.081	0.026

Significance levels : † : 10% $*$: 5% $**$: 1%. Robust standard errors are in parentheses. The regressions are run for observations with at least two participants. The coefficients for time dummies and the intercept are not reported.

6.3 Check of robustness

VALUE is the main variable that drives about 90% of the explained variance for PRICE in our regression. Since there is no particular reason for the relationship between the endogenous variables and VALUE to be linear, we have included the square of VALUE in the regressions as a check of robustness and we have estimated the model without the 10 and 40 observations with the highest VALUE. The results are similar though the squared terms differs significantly from zero in both periods ($p<0.0001$). The coefficients on VALUE are modified significantly relative to the original regression. However, the impact on the other covariates are similar and the same comments apply.

All our regressions can be criticized as been driven by the fact that VALUE may be an imperfect proxy of the ex-post value of the Interconnector. In particular, the relationship between those two values may depend on the day of the week (e.g. the profiles of the export and import flows depend on the weekday) or may experience large fluctuations across years. We have run some complementary regressions to check the robustness of our results relative to this issue. We run 2SLS regressions with a specific slope for VALUE for each day of the week. For the full sample, the estimated slopes are lying between 0.35 and 0.72 and we can not reject that the slopes are jointly equal (Wald-test, $p = 0.1829$). Nevertheless, in some specific periods, it is no longer the case and it seems that the measurement error problem matters though it does not induce strong bias in the broad pattern of estimated coefficients. E.g., for the years 2003 and 2005, we can reject that the slopes are equal (Wald-test, $p < 0.0001$). Moreover, for friday, the slope is not statistically different to zero at the 10% level.

7 Discussion

7.1 Theoretical literature on the allocation of transmission capacity

One of the main assumptions of the literature on the allocation of transmission capacity (Joskow and Tirole [22], Gilbert et al. [16]) is that efficient arbitrage forces the price paid for transmission contracts to equal the spot price difference between the nodes. On the contrary, our empirical analysis shows that the price paid for transmission is very significantly below spot price differences. We also show that the auction mechanism itself seems to play a role: the closing rule (fixed time versus random) or the frequency of the auction (daily versus long term).

A second important aspect of this literature are the different incentives of the actors in this market relative to their respective market power in the importing and the exporting markets. Furthermore, this literature does not consider the additional motive for market manipulation resulting from the coincidence that half of the auction revenue goes to RTE, whereas EDF, her parent company, participates in the auction. Our analysis finds that bidders may have identity-specific effects on the concentration outcome and on the auction dynamic. Nevertheless, we do not find evidence that specific bidders influence significantly the final price. Thus our analysis gives no support to the view that bidders may have heterogenous motives for capacity rights.

7.2 The Empirical and Experimental Evidence on multi-unit Ascending Auctions

The empirical analyses of simultaneous ascending auctions are scarce. The European UMTS auctions happen only once preventing any statistical inference. Jehiel and Moldovanu [20] discuss the auction results across different countries in the light of their theories of auctions with externalities. Considering the whole dynamic of the auction in the U.K., Börgers and Dustmann [7] are trying to elucidate why several bidding behaviors deviate strongly from straightforward bidding with private values. Cramton and Schwartz [10] analyse some revenue-reducing strategies that are specific to simultaneous ascending auctions by means of code bids or retaliating bids. They find evidence that those signalling techniques were effective.

The experimental literature on auctions has asked whether the ascending multi-unit auction outperforms its sealed-bid counterparts. More precisely, in [2, 13, 23], the ascending multi-unit auction studied is a clock auction which implements dynamically the traditional uniform-price auction. As the price is increasing, a bidder may drop out on some units he has bid for. Furthermore, dropping out is irrevocable: there is no room for re-entry. In a private-value framework with two units demand, all those experiments show that demand reduction is stronger in the ascending format. As a consequence, both efficiency and revenue are substantially lower than in other auctions. What kind of conjectures give those experiments for the IFA auctions? There are numerous differences between those experiments and the environment in the IFA-auctions. First, transmission capacity entails an important common-value component, which is supposed to benefit to ascending formats. With flat multi-unit demand and with informational externalities, Ausubel [4] has proved that the Ausubel auction outperforms the Vickrey auction in term of revenue. Nevertheless, the hypothetic theoretical gains from the linkage principle may overestimate the practical benefits of open formats as shown in Kagel, Levin and Richard [26] for a single object, where the English auction outperforms the first price auction only for super experienced bidders and on a much smaller scope. Second the lack of any activity rules leaves more scope for collusion: it costs nothing to a bidder to reduce temporally his demand and see how his opponents react because he can increase his demand again if they do not play the "demand reduction equilibrium". The early phases of the auction can then be used as a room for cheap talk. In the same spirit, Goswami et al [17] have shown how preplay communication facilitates the adoption of low price equilibria in uniform-price share auctions. Thus, apart from the possibility of intertemporal strategies, the daily repetition gives time to learn how to play low price equilibria. Finally instead of two bidders and two units auctions as in [13, 23], IFA-auctions are share auctions with more participants. On the one hand, the extreme divisibility of IFA-auctions suggests strong underpricing may happen.[25] On the other hand, the number of participants is greater but not that much in daily auctions. The number of units is thus much larger than the number of bidders.

On the whole, the previous points suggest that the gap between the ascending and the sealed bid uniform-price auctions should be worse in the IFA-auctions than in the multi-unit ascending uniform-price auction experimented in the lab.

8 Conclusion

The simultaneous ascending auction has been proposed to auction items that are very sensitive to the winner's curse for long-term licences. Anyway, the main drawback of this format is that it is more prone to seemingly collusive low price equilibria especially if it is played on a daily basis as in the IFA-auctions. Moreover, the specific details of the ascending auction in IFA daily auctions - the absence of any activity rules and the closing rules - are making this ascending format less prone to competitive equilibria.

Our results are not clear-cut. On the one hand, the winner's curse may play some role since the volatility of the underlying market - our proxy for the precision of bidders' signals- has a significant negative effect on the price from 2003. Nevertheless, its influence is positive and not significant at the 10% level in 2002. Moreover, the effects on the other endogenous variables are not significant. On the contrary, studies as Nyborg et al [29] have found a statistically negative effect of the volatility on the award concentration. On the other hand, the IFA daily auctions seem to experience different regimes with a significant effect on the endogenous variables as the price or the award concentration. The effect of the period is significant after having controlled for the identity of the participants in the auction.

We have tried to analyse the factors and the path that drive the bidders to a low price equilibrium. In particular, we consider several endogenous variables relative to the dynamic of the auction as the last digits of the bids. It appears that some of the big bidders have a significant influence on those variables. Nevertheless, the heterogeneity in the bidders' strategies and their

[25] See Kremer and Nyborg [25] for an analysis of the impact of price and quantity tick size.

Table 11: Volatility Coefficients

	Main Equation			Volatility Equation			
	β_0	β_1	σ^2	γ_1	γ_2	γ_3	γ_7
	France to England						
Mean	.112	.710	-.050	.375	.157	.040	.046
Standard deviation	.071	.023	.006	.031	.027	.008	.016
	England to France						
Mean	.207	.752	-.046	.132	.111	.244	.259
Standard deviation	.026	.014	.003	.017	.017	.019	0.023

identity-dependent influences on the outcome of the auction deserves more attention, especially in a more controlled framework as a lab experiment.

Appendix

A Volatility Estimation

As in Wolak [39], we first eliminate the multi-seasonal patterns of the price-differential dynamic that are first order in electricity markets through dummies for each days, months and years. Let $\delta_t^{i \Rightarrow j}$ be the deseasonalized price differential between zone i and zone j at time t. From now on, we drop the index i and j to alleviate notation.

We assume that the price differential δ_t follows the autoregressive process such that the conditional variance σ_t^2 influences the conditional mean:

$$\delta_t = \beta_0 + \beta_1 \cdot \delta_{t-1} + \psi \cdot \sigma_t^2 + \epsilon_t$$

where ϵ_t is a white gaussian noise. The volatility σ_t^2 of the error term is:

$$\sigma_t^2 = \exp(\lambda_0 + \sum_{i=1}^{6} \lambda_i \cdot weekday\_i) + \gamma_1 \cdot \epsilon_{t-1}^2 + \gamma_2 \cdot \epsilon_{t-2}^2 + \gamma_3 \cdot \epsilon_{t-3}^2 + \gamma_7 \cdot \epsilon_{t-7}^2 + \nu$$

Multiplicative heteroskedasticity has been introduced relative to the day of the week. The estimated coefficients and the related robust standard errors are reported in Table 11.

Remark A.1 *Various simpler time series models have been tested, i.e. the simple ARCH model used in [29]. The results were quite unsatisfactory since the estimators of the sum of the γ_i were greater than 1 and thus involved a non-stationarity of the volatility. Therefore, we have used a more complex*

ARCH-in-mean model. The estimated coefficients are more satisfactory insofar as the stationarity condition is satisfied and the estimated coefficients are similar for both series. Nevertheless, this model for the volatility remains inconsistant since the Portmanteau test rejects the hypothesis of independence of the residuals at any conventional level.

B Detailed description of the raw data set

The raw data contains 39,189 bids that has been submitted for the period April 01 of 2001 to July 01 of 2005. The data set contains all the bids submitted for 1,581 IFA auctions held in this period. Each bid is characterized by five variables and by the contract he stands for.

Each auction is coded by 5 variables: the first, called "Direction", is a binary variable equal to 1 for the direction "France to England"; the second, called "deb", is the first day of the validity period of the capacity; the third, called "fin", is the last day of the validity period of the capacity; the fourth, called "emission", corresponds to the date of the auction[26]; finally, the variable "type" specifies the type of the auction (from daily to tri-annual).

Each bid is characterized by a set of 5 variables: the identity "ID" of the bidder coded by a number between 1 and 23 where 23 corresponds to the event where this value is missing (about 4% of the whole submitted bids), the price "PRICE ASK" in Euros per MW, the number of Units demanded in MW "QUANTITY ASK", the number of units allocated corresponding to this bid "QUANTITY OBT" and finally the variable "NUMBER" which is supposed to code for the chronology of the whole set of recorded bids in the IFA auctions. The highest recorded number is 43,524, which suggests that more than 3,000 bids have been lost or that some jumps occurred.

C Details of the variables construction

REPAIR is a binary variable that have been constructed in order to capture the days where the auction have been cancelled due to an outage. Unfortunately, our database does not report the reason why no bids have been submitted one day: either because potential buyers have decided not to participate or because the auction has been cancelled. Nevertheless, since it is the same link that is used in both direction, an outage should imply a cancellation of the auctions in both directions. On the other hand, it seems very unlikely that the capacity remains unsold in both direction even with the reserve price of 3 Euros. If the reserve price is binding in one direction because the price differential is expected to be low, then bidders should expect a high price differential in the other direction, which is significantly

[26]This value is missing most of the time. Nevertheless, it has been filled when the same validity contract has been auctioned at different dates.

negatively correlated with the previous one,[27] and then should find profitable to bid. Those arguments lead us to set the variable REPAIR to 1 if no bids have been received in both direction and 0 otherwise. REPAIR equal to 1 is interpreted as the cancellation of the auction by the Operator.

We test that the so-defined variable REPAIR is not random suggesting that outages mostly take place when the value of the interconnector is low (which is reasonable for planned outages). The average ex-post value per MW of capacity is 87.7 (respectively 44.0) Euros for the direction "France to England" (resp. "England to France") if REPAIR equals to 1. Those values significantly differ from the averages with REPAIR equal to 0: the value of the direction "France to England" (resp. "England to France") is greater (resp. lower) in the periods with REPAIR equal to one. On the whole, the total average value of the interconnector is lower in period of repair (131.7 Euros) than in period where REPAIR equals to 0 (144.7 Euros). This nearly 10% difference is statistically significant at the $p < 0.05$ level using a one-tailed Wilcoxon nonparametric test.

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Figure 1: Source: National Grid Transco

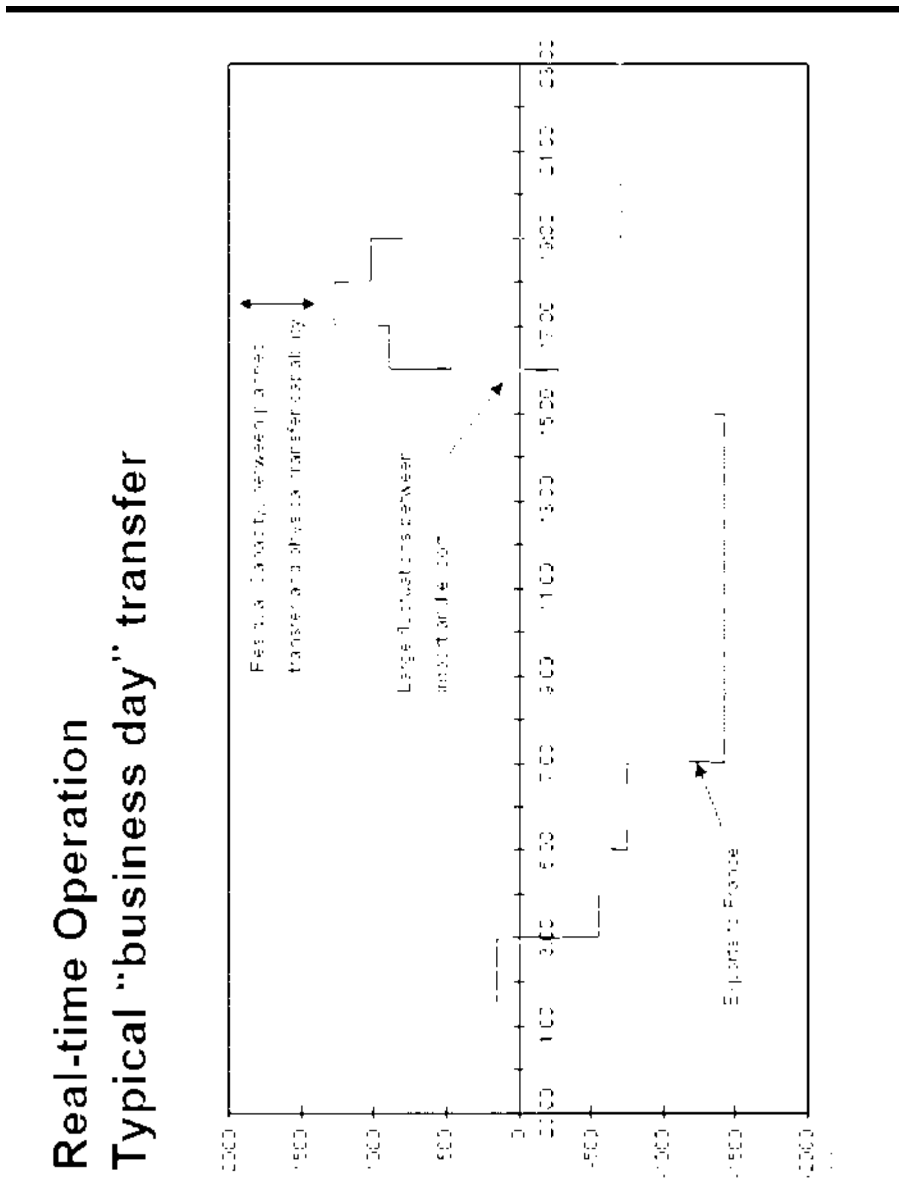

Figure 2: Instability of the margins

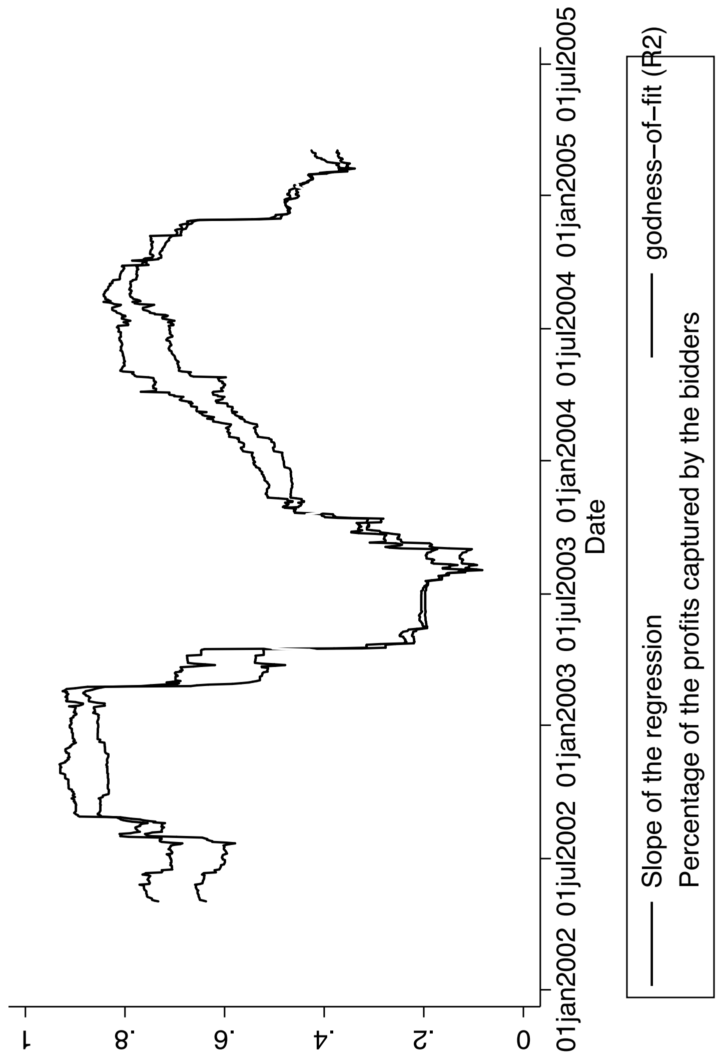

Figure 3: Dynamic of the sequential auctions for the annual contract for the year 2005 in the direction "France to England"

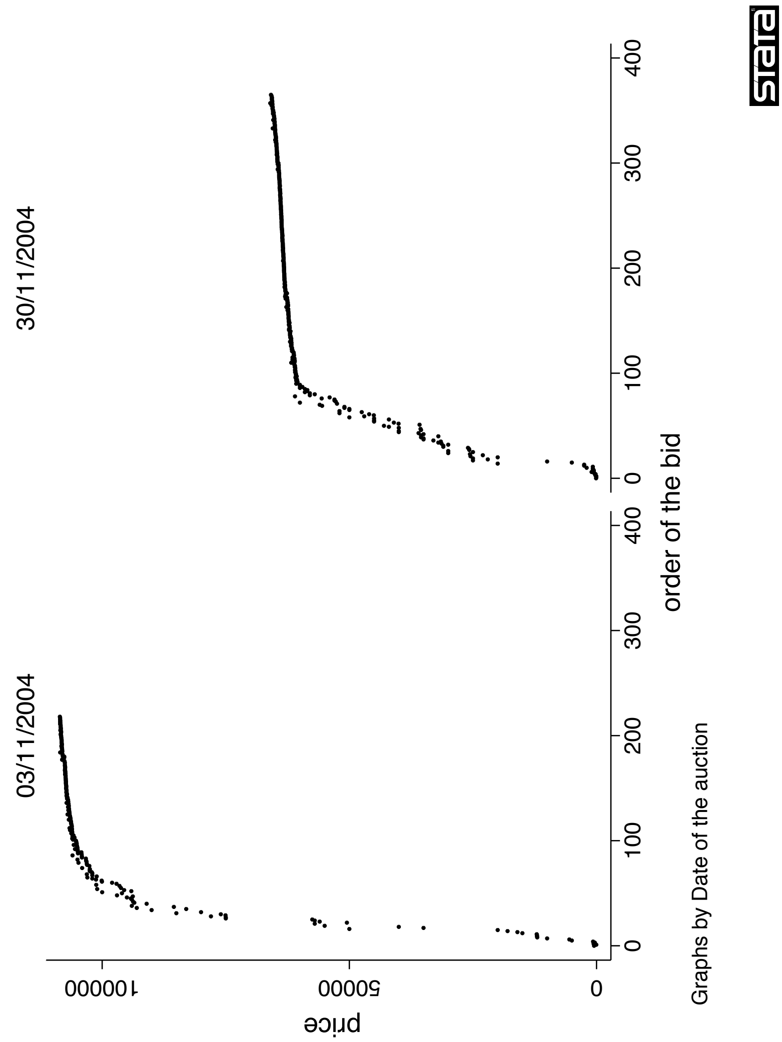

Figure 4: Price differential from France to England: January 2002 - June 2005